Equivalent-circuit model of the squeezed gas film in a silicon accelerometer

Timo Veijola a,*, Heikki Kuisma b, Juha Lahdenperä b, Tapani Ryhänen b

a Helsinki University of Technology, Faculty of Electrical Engineering, Circuit Theory Laboratory, Otakaari 5A, FIN-02150 Espoo, Finland
b Vaisala Technologies Inc., PO Box 9, FIN-00421 Helsinki, Finland

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Abstract

We present a new electric equivalent circuit for the forces created by a squeezed gas film between vertically moving planar surfaces. The model is realized with frequency-dependent resistors and inductors. Circuit analysis tools are applied to calculate the response of a micromechanical silicon capacitive accelerometer in both the frequency and the time domains. The simulations are shown to match the measured frequency responses in an excellent way. We utilize the circuit model to calculate the effective viscosity in a narrow gap between the moving surfaces. The results are compared with different slip-flow equations discussed in the literature. We present a simple approximate equation of the pressure-dependent viscosity that is valid for both viscous and molecular damping regions, and has 5% accuracy.

Keywords: Accelerometers; Equivalent-circuit models; Squeezed gas films; Slip flow; Effective viscosity

1. Introduction

Silicon microstructures, especially sensors and actuators that make use of capacitive measurement principles or electrostatic driving forces, are characterized by very small gaps between the moving elements and the fixed electrodes. Typically, the gap width is only a few micrometres for bulk micromechanical devices and below one micrometre for surface-micromachined devices. In micromechanical accelerometers, the gas in the gap is often used as the damping medium. The desired frequency response of the micromechanical device can be achieved by carefully controlling the gas pressure. However, damping leads to a complicated frequency response because of the compressibility of the gas.

In the presence of a compressible gas film, the moving surfaces are forced to squeeze the gas from between them to be able to move. The phenomenon has an interesting frequency response: below the cut-off frequency, the gas has enough time to flow away from the gap, thus causing dissipation, whereas above the cut-off frequency, the gas film is trapped and squeezed between the moving plates and behaves like a spring element with low dissipation. At the cut-off frequency the viscous and spring-like forces acting on the surfaces are equal. When designing micromechanical devices, the gas-film behaviour is of great importance.

The isothermal squeezed-film effect has been modelled by Griffin et al. [1] and Blech [2]. In their analysis, the compressible gas-film Reynolds equation [3], governing the gas pressure between the surfaces, has been linearized assuming that the motion of the plates and the pressure variation from ambient are small. As a result, the damping and spring forces of the gas film for rectangular geometry have been solved as a function of frequency, and the cut-off frequency of the gas-film damping has been found. This model has been applied by Andrews et al. [4,5] to predict the frequency response of micromechanical pressure sensors with very good results.

If the gas pressure is low, the molecular mean free path is not negligible compared with the gap width, and the gas cannot be treated as a continuous medium. Several models for the flow of a rarefied gas in a narrow gap have been presented in the literature since 1906 [6–11]. These approaches differ considerably when the ratio of the mean free path to the gap width is large. In gas lubrication theory, the phenomenon is often called slip flow.
In this article, we present a new model of a micromechanical accelerometer that includes the damping and spring forces created by the squeezed gas film. The gas-film properties are described by a series expansion of frequency-independent elements. The squeezed-film model, together with the equation of motion of the seismic mass, is realized using an electric equivalent circuit consisting of linear and nonlinear frequency-independent components. The model is verified by comparing the small-signal frequency response with measured data. The transient behaviour of the micromechanical accelerometer is demonstrated by simulations. Furthermore, we present a comparison of different theories of the gas flow in a narrow gap with our measured data.

2. Structure and model of a micromechanical accelerometer

A micromechanical accelerometer typically consists of a mass suspended with a spring on the sensor body, i.e., a harmonic oscillator. Below its natural resonance frequency, the spring-mass system behaves as an accelerometer so that the displacement of the mass with respect to the sensor body is directly proportional to acceleration; above the resonance frequency the device behaves as an amplitude detector. The damping of the system is determined by the intrinsic dissipations in the mechanical spring and by the dissipations due to the interaction with the surrounding gas or liquid. The nature of the damping depends on the gas pressure: at low pressures the damping is of a molecular nature, whereas at high gas pressures, it is viscous.

In Fig. 1(a) we present the structure of a silicon micromechanical accelerometer that consists of three silicon wafers [12]. The mass and two cantilever beams are anisotropically etched into a 0.38 mm thick silicon wafer. This thin wafer is anodically bonded between two 1.3 mm thick silicon wafers covered by a 150 μm glass layer. The metal electrodes are deposited on top of the insulating glass layer. The bonded wafers are diced, and the contact pads are deposited at one site of the accelerometer chip as shown in Fig. 1(b).

The cross section of the mass-spring system is shown in Fig. 2. The cantilever beams form the spring element. The capacitance is measured between the moving seismic mass and the thin-film electrodes on top of the insulating glass layer. The contact is made through the glass layer to the thick silicon wafer as shown in Fig. 2.

The mass-spring system of the accelerometer in Figs. 1 and 2 has several mechanical resonance modes. The first mode is the normal bending mode with resonance frequency determined by \((\kappa M)^{1/2}/(2\pi)\), where \(\kappa\) is the spring constant and \(M\) is the mass. The second torsional mode appears at a frequency roughly 10 times higher. Thus the dynamics of an accelerometer resemble those of a simple harmonic oscillator characterized by

\[
M \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \kappa x = F_{\text{ext}}
\]

where \(t\) is time, \(x\) is the displacement of the mass and \(\gamma\) is the damping coefficient. The driving force \(F_{\text{ext}}\) is a sum of forces created by the applied acceleration \(Ma\), possible electrostatic forces \(F_{\text{el}}\) and the stochastic force due to the damping gas.

The electrical field resulting from the potential difference \(U\) between the moving mass and the fixed electrode, having area \(A\), causes an electrical attractive force on the mass. Assuming perpendicular motion to the surfaces, the electrostatic force is

\[
F_{\text{el}} = \frac{\varepsilon AU^2}{2d^2}
\]

where \(\varepsilon\) is the dielectric constant of the gas and \(d\) is the gap width.

An electrical equivalent circuit for the mechanical phenomenon is used here. Thus circuit simulation tools can be applied. The parallel resonator circuit shown in Fig. 3 leads to a minimum number of circuit nodes.

Fig. 1. The structure of the micromechanical accelerometer produced by Vaisala Technologies Inc. A thin silicon wafer containing the mechanical structure is anodically bonded between two glass-covered thick silicon wafers.

Fig. 2. The cross section of the micromechanical accelerometer. The cantilever beams support the moving mass, which forms two varying capacitances with the thin-film electrodes. The cross section is not drawn to scale.
At small pressures, when the molecular mean free path $\lambda$ is not negligible compared with the gap width, the gas flow can be modelled by a modified Reynolds equation [7–10]. In the case of the linearized Eq. (4), the modification can be expressed using the effective viscosity

$$\eta_{\text{eff}} = \frac{\eta}{1 + f(K_n)}$$

where $K_n = \lambda/d$ is the Knudsen number and $\eta$ is the viscosity coefficient. The effective viscosity $\eta_{\text{eff}}$ depends on static pressure $P_a$ because the mean free path $\lambda$ is inversely proportional to pressure [6]:

$$\lambda = \frac{P_0}{P_a} \lambda_0$$

where $\lambda_0$ is the mean free path at pressure $P_0$.

Table 2 summarizes different functions $f(K_n)$ used in the literature to model the gas flow in a narrow gap. They differ considerably, especially when the Knudsen number is high. The empirical equation of Knudsen, quoted in Ref. [6], is based on the gas flow in capillary tubes, and it is used in the pressure sensor development by Andrews et al. [4,5]. Seidel et al. [11] have modelled the molecular and viscous damping in an accelerometer structure in terms of the gas mean free path modified by the average mean free path between two parallel plates, i.e., 0.7 times the average gap width. The equation of Burgdorfer [7] is a traditional approach based on the modified Reynolds equation. It is used in the analysis of gas-lubricated bearings, but it is limited to cases where $K_n < 1$. Hsia and Domoto [9] and Mitsuya [10] have extended the feasibility of Burgdorfer’s equation for larger Knudsen numbers by adding a higher-order correction.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Author & Year & Function $f(K_n)$ \\
\hline
Knudsen [6] & 1906 & $\frac{ZK_n}{0.1474}$ \quad $Z = \frac{K_n + 2.507}{K_n + 3.095}$ \\
Burgdorfer [7] & 1959 & $6K_n$ \\
Hsia and Domoto [9] & 1983 & $6K_n + 6K_n^2$ \\
Fukui and Kaneko [8] & 1988 & $\frac{6Q_e(D)}{D} - 1$, \quad $D = \frac{\sqrt{\pi}}{2K_n}$ \\
Mitsuya [10] & 1993 & $6 \left( \frac{2 - \alpha}{\alpha} \right) K_n + \frac{8}{3} K_n^2$, \quad $0 < \alpha < 1$ \\
\hline
\end{tabular}
\caption{Effective viscosity equations given by several authors. These functions are used in Eq. (5) to define the pressure dependency of effective viscosity of the gas in a narrow gap.}
\end{table}
term. Mitsuya uses a surface-correcting coefficient 
\((2 - \alpha)\alpha\) involving the accommodation coefficient \(\alpha\).
The coefficient is defined by the interaction between
the surfaces and the gas. For most engineering surfaces,
the coefficient can be assumed to be nearly unity.

Fukui and Kaneko [8] have used Boltzmann's transport
equation to derive a modified Reynolds equation
for arbitrary Knudsen numbers. When deriving this
equation, diffuse reflection from the surfaces is assumed
(accommodation coefficient \(\alpha = 1\)). The effective
viscosity can be expressed in terms of the Poiseuille flow
rate \(Q_p(\sqrt{\nu/2K_s})\), which is presented in Appendix 2 of
Ref. [8]. The Poiseuille flow rate cannot be presented in
a closed form, and thus we have derived a simple
approximate equation that differs by less than 5% from
the original equation (0 \(\leq K_s \leq 880\)). Our approximation
for the effective viscosity is

\[
\eta_{\text{eff}} = \frac{\eta}{1 + 9.638K_s^{1.159}} \tag{7}
\]

The comparison between experimental and calculated
results, given in Ref. [10], shows that the equation of
Fukui and Kaneko [8] most closely agrees with the
measurement results.

The solution of the linearized Reynolds Eq. (4) has,
in principle, two components: one in phase with the
plate movement and another out of phase, in other
words, the spring term and the damping term,
respectively. The force components can be calculated
simply by integrating over the area of the plates. Blech
[2] has solved the damping and the spring forces for
rectangular plates as an infinite series expansion:

\[
\frac{F_{\text{d}}}{x} = \frac{64\sigma P_\pi A}{\pi^2 d} \sum_{m,n \text{ odd}} \frac{m^2 + c^2 n^2}{(mn)^2(m^2 + c^2 n^2)^2 + \sigma^2/\pi^4} \tag{8}
\]

\[
\frac{F_{\text{t}}}{x} = \frac{64\sigma P_\pi A}{\pi^3 d} \sum_{m,n \text{ odd}} \frac{(mn)^2(m^2 + c^2 n^2)^2 + \sigma^2/\pi^4} \tag{9}
\]

where \(m\) and \(n\) are odd integers 1, 3, 5, 7, ... \(A = w l\)
is the plate area, \(c = \sqrt{\nu/\omega}\) and \(w\) and \(l\) are the width
and the length of the mass planar surfaces, respectively.
The squeeze number \(\sigma\) is given by [1]

\[
\sigma = \frac{12\eta_{\text{eff}} w^2}{P_\pi d^2} \omega \tag{10}
\]

where \(\omega\) is the angular frequency.

4. Electrical equivalent circuit

In order to derive an electrical equivalent circuit for
the forces of Eqs. (8) and (9), we use the combination
of elementary circuit sections shown in Fig. 4. These
sections represent the terms of the series expansion
and thus the same odd indices \((m\) and \(n\)) as in Eqs.
(8) and (9) are used. The sections of the expansion
are connected in parallel with the resonator circuit
shown in Fig. 3. After replacing the voltage \(u\) with
the flux term \(d\psi/dt\), the differential equation

\[
L_{m,n} \frac{di_{m,n}}{dt} + R_{m,n} i_{m,n} = \frac{d\psi}{dt} \tag{11}
\]

defines the relation between the section current \(i_{m,n}\)
and the flux \(\psi\) of the resonator inductance \(L\). In the
steady state, when all signals are sinusoidal having a
single angular frequency \(\omega\), the current and the flux
can be expressed in the form

\[
i_{m,n} = i_{m,n} \exp(\omega t)
\]

\[
\psi = \psi \exp(\omega t)
\]

where \(i_{m,n}\) and \(\psi\) are complex coefficients. In the steady
state, Eq. (11) becomes

\[
\omega L_{m,n} i_{m,n} + R_{m,n} i_{m,n} = j\omega \psi
\]

The total current \(i\) in the squeezed-film equivalent
circuit, corresponding to the total force, is the sum of
all currents of the parallel sections:

\[
i = \sum_{m,n \text{ odd}} i_{m,n} = \psi \sum_{m,n \text{ odd}} \frac{i_{m,n}}{R_{m,n} + j\omega L_{m,n}}
\]

The imaginary and real parts of the ratio \(i/\psi\) are

\[
\text{Im}\left(\frac{i}{\psi}\right) = \sum_{m,n \text{ odd}} \frac{R_{m,n} \omega}{R_{m,n}^2 + \omega^2 L_{m,n}^2}
\]

\[
\text{Re}\left(\frac{i}{\psi}\right) = \sum_{m,n \text{ odd}} \frac{L_{m,n} \omega^2}{R_{m,n}^2 + \omega^2 L_{m,n}^2}
\]

The derived Eqs. (16) and (17) satisfy the frequency
dependency specified in Eqs. (8) and (9), respectively.
Requiring that \(\text{Im}\left(\frac{i}{\psi}\right) = F_{\text{d}}/c\) and \(\text{Re}\left(\frac{i}{\psi}\right) = F_{\text{t}}/\omega\),
the component values for \(L_{m,n}\) and \(R_{m,n}\) in the series
expansion are

\[
L_{m,n} = (mn)^2 \frac{\pi^4 d}{64AP_\pi}
\]

\[
R_{m,n} = (mn)^2(m^2 + c^2 n^2)^2 \frac{\pi^6 d^3}{768 \pi^2 \eta_{\text{eff}}}
\]

The frequency dependence of the real and imaginary
parts in Eqs. (16) and (17) is interesting. The real part,
Eq. (17), corresponding to the effective spring constant, grows rapidly above the cut-off frequency, whereas the imaginary part, Eq. (16), corresponding to the dissipation, rapidly decreases above the cut-off frequency. Griffin et al. [1] and Blech [2] have defined the cut-off frequency as a frequency where the real and imaginary parts are equal.

The complete squeezed-film equivalent circuit together with the resonator circuit of Fig. 3 is shown in Fig. 5. The accelerometer has two air gaps, and this leads to two circuits connected in parallel, denoted here by superscripts A and B. The circuit elements corresponding to the different gaps cannot be combined because the distances $d_A$ and $d_B$ are not supposed to be equal. The coefficients that depend on the indices $m$ and $n$ in Eqs. (18) and (19) grow fast, and, in practice, only a few RL sections are needed to achieve sufficient accuracy.

According to Eqs. (18) and (19), the component values $L_{m,n}$ and $R_{m,n}$ depend on the distance $d$. If the displacement is large, the component values of the squeezed-film circuit must also vary with the displacement $x = L d$. The components are non-linear and can be written as

$$L_{m,n}^A = (mn)^2 \frac{\pi^4 (d_A - x)^2}{64 A P_a} \quad (20)$$

$$R_{m,n}^A = (mn)^2 (m^2 + c^2 n^2) \frac{\pi^6 (d_A - x)^3}{768 A w^2 \eta_{eff}} \quad (21)$$

$$L_{m,n}^B = (mn)^2 \frac{\pi^4 (d_B + x)^2}{64 A P_a} \quad (22)$$

$$R_{m,n}^B = (mn)^2 (m^2 + c^2 n^2) \frac{\pi^6 (d_B + x)^3}{768 A w^2 \eta_{eff}} \quad (23)$$

When an initial static force, which can be either electrical or due to acceleration, is applied to the sensor, the mass will be initially displaced. In the electrical circuit analysis, the initial conditions are solved using operating point analysis (d.c. analysis). With the aid of Eqs. (20)–(23), the operating point is automatically calculated correctly by the simulator.

The non-linear behaviour of the equivalent circuit can be used in analysing the distortion in the sensor response. However, the original assumption of Eqs. (8) and (9) stated that the dynamic pressure change caused by the squeezed-film effect must be negligible when it is compared with static intrinsic gas pressure. The ratios between the squeezed-film forces and the force caused by the static pressure are

$$\alpha_A = \frac{i_{A}}{P_a A} \quad (24)$$

$$\alpha_B = \frac{i_{B}}{P_a A} \quad (25)$$

Here $i_{A}$ and $i_{B}$ are the currents of sections A and B in Fig. 5, respectively. The model is accurate if both $\alpha_A \ll 1$ and $\alpha_B \ll 1$.

5. Frequency-domain measures

To compare measured and simulated frequency responses, we have used several almost identical accelerometers with different intrinsic gas pressures. These accelerometers are early prototypes of the commercially available ±3g devices produced by Vaisala Technologies Inc. The accelerometer dimensions and other known model parameters are shown in Table 3. Viscosity and mean free path are interpolated to room temperature using data from Ref. [13]. The uncertainty of the temperature is taken into account in the viscosity and mean free path.

The motion of the mass in these accelerometers is damped using a well-controlled argon gas pressure inside the closed sensor cavity. The exact value of the pressure is unknown because of the small outgassing and gettering during the fabrication process of the sensors. The intrinsic gas pressure and effective viscosity are thus

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed parameter</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Gap widths $d_A$ and $d_B$</td>
</tr>
<tr>
<td>Length of the cantilever beam</td>
</tr>
<tr>
<td>Width of the moving mass $w$</td>
</tr>
<tr>
<td>Length of the moving mass $l$</td>
</tr>
<tr>
<td>Mass $M$</td>
</tr>
<tr>
<td>Damping coefficient $\gamma$</td>
</tr>
<tr>
<td>Mean free path $l$ at 1 atm</td>
</tr>
<tr>
<td>Viscosity coefficient $\eta$</td>
</tr>
<tr>
<td>Temperature $T$</td>
</tr>
<tr>
<td>Bias voltage</td>
</tr>
</tbody>
</table>
determined by fitting the squeezed-film model to the measured data. The spring constants for all sensors are also found by fitting.

In our simulations six RL sections are used to model both air gaps \((m, n) = (1, 1), (1, 3), (3, 1), (3, 3), (1, 5), (5, 1)\). The component values of the remaining sections are insignificant and can therefore be ignored. The \(Q\)-value of the silicon structure is known to be orders of magnitude higher in vacuum than in the presence of a damping gas. Thus the intrinsic damping coefficient \(\gamma\) is assumed to be negligible.

Fig. 6 presents the set-up used for measuring the frequency responses of the accelerometers. The mass is biased to some fixed d.c. voltage \(U_{\text{bias}}\). One of the electrodes is connected to ground potential via a resistor, and the other electrode is connected to an electrical signal source \(U_{\text{signal}}\) which causes an actuating force to the system. Under this biasing scheme, according to Eq. (2), the electrostatic force affecting the mass is given by

\[
F_{\text{el}} = \frac{\varepsilon AU_{\text{bias}}^2}{2d^2} - \frac{\varepsilon A(U_{\text{bias}} - U_{\text{signal}})^2}{2d^2} \approx \frac{\varepsilon AU_{\text{bias}}U_{\text{signal}}}{d^2}
\]  

if the amplitude of the signal voltage (50 mV) is small compared to the bias voltage (>2 V). The movement of the mass is measured by coupling a 1 MHz carrier signal to the moving electrode. Thus the high-frequency signal is amplitude modulated by the capacitance change, which is detected with a diode detector. The non-idealities of the detector and the filter are measured and corrected on the data before comparing them with the simulations.

The curve fitting is performed using the gradient optimization method implemented in the circuit design program APLAC [14,15]. The optimization variables are the spring constants, the gas pressures and the effective viscosities. The extracted parameters for seven different accelerometers are shown in Table 4. The spread of the values for each sensor is based on several measurements using different bias voltages. The measured and simulated frequency responses of two selected accelerometers A and D are shown in Figs. 7 and 8.

Fig. 7 presents the frequency response of sensor A. Because of the low gas pressure, the \(Q\)-value of the sensor is relatively high, and a clear resonance peak is found at 1 kHz. The resonance occurs at a lower frequency than that predicted on the basis of the mass and the spring constant. This is due to the electrical potential which affects the effective spring constant. Fig. 8 shows the measured frequency response of sensor D, where the gas pressure is considerably higher. The squeezed gas film clearly changes both the amplitude and the phase of the sensor response. The characteristic resonance peak created by the squeezed gas film is found at roughly 2 kHz.

Using the extracted values of Table 4, the ratio \(\eta_{\text{eff}}/\eta\) as a function of the Knudsen number \(K_{\text{n}}\) is presented in Fig. 9. The effective viscosity functions of Table 2 are also shown in Fig. 9. For large \(K_{\text{n}}\), the first-order equations \(f(K_{\text{n}})\) give too high an effective viscosity, whereas the second-order equations lead to values that

<table>
<thead>
<tr>
<th>Sensor</th>
<th>(\kappa) (N m^{-1})</th>
<th>(P_s) (Pa)</th>
<th>(\eta_{\text{eff}}\ 10^{-9}) (N s m^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>179.2–212.1</td>
<td>10.0–11.0</td>
<td>8.28–10.2</td>
</tr>
<tr>
<td>B</td>
<td>174.6–202.0</td>
<td>75.4–77.5</td>
<td>45.3–50.5</td>
</tr>
<tr>
<td>C</td>
<td>149.7–164.9</td>
<td>97.0–101.0</td>
<td>57.1–64.7</td>
</tr>
<tr>
<td>D</td>
<td>159.2–179.4</td>
<td>405.4–456.9</td>
<td>336.2–374.3</td>
</tr>
<tr>
<td>E</td>
<td>153.2–177.2</td>
<td>479.7–499.2</td>
<td>392.5–438.2</td>
</tr>
<tr>
<td>F</td>
<td>172.7–195.7</td>
<td>1716–1763</td>
<td>1362–1471</td>
</tr>
<tr>
<td>G</td>
<td>174.7–194.6</td>
<td>3110–3202</td>
<td>2379–2567</td>
</tr>
</tbody>
</table>
An additional phenomenon that can affect the measured results is the microscopic properties of the surface. If we assume that the single-crystal silicon surface reflects the gas molecules in a way similar to a glass surface, we could use the coefficient that is experimentally confirmed between glass and air, $\alpha = 0.89$ [8]. When the Knudsen number in Eq. (7) is multiplied by the surface correcting coefficient $(2 - \alpha)/\alpha$, the respective curve in Fig. 9 will shift about 25% to the left, towards the measurement results.

6. Simulated transient behaviour

The squeezed-film effect model in Fig. 5 was derived using the frequency-domain steady-state response. Because of the component-level realization, the model is also valid for analysing transient phenomena. In our simulations of frequency- and time-domain responses, we have used the parameters shown in Table 3. The approximate Eq. (7) is used to calculate the effective viscosity coefficient as a function of pressure.

The frequency responses analysed at pressures of 30, 300 and 3000 Pa are shown in Fig. 10. Figure 11 shows the corresponding time-domain responses calculated using transient analysis. A step acceleration of 0.5g is applied to the sensor mass at 0.5 ms. Fig. 12 shows the ratio $\alpha$, defined by Eq. (24), and the time-domain response at 300 Pa.

With 30 Pa pressure, the system remains oscillating at the resonance frequency with an amplitude of almost 200 nm. The higher damping gas pressure of 300 Pa effectively eliminates the high-amplitude oscillations, and the mass receives its final position within 3 ms.
7. Discussion

Fig. 10 indicates two important features of the accelerometer dynamics created by the gas film: compared to the ideal harmonic oscillator, the phase shift at lower frequencies is significantly larger, and the gas film generates a new resonance at a higher frequency, thus causing a spurious response outside the required frequency range. The parameter extraction in Fig. 7 indicated that even at a low gas pressure of about 10 Pa, only the use of the gas-film model can explain the measured response. The frequency response in Fig. 10 showed that even at the relatively modest damping pressure of 300 Pa, the dynamics of the accelerometer are strongly affected by the squeezed gas film. At 3000 Pa, the behaviour is almost completely dominated by the compressible gas. Thus the use of the squeezed-film model is important in designing micromechanical sensors.

The model presented here is, however, valid only for the perpendicular motion of two parallel rectangular plates. In our accelerometers the mass is supported only from one side by cantilever beams. The non-perpendicular motion of the mass, caused by these relatively short cantilever beams, creates an asymmetric pressure distribution on the plates. We assume that the effect of this non-ideality on the calculated frequency response is rather small.

The restriction in the model of Griffin et al. [1] and Blech [2] is that the pressure change between the moving plates must be small. The criterion used in our simulations is that the maximum pressure change in the gap volume must be much smaller than the average absolute pressure. In the simulated responses the pressure change for a 0.5g acceleration step is below 2% of the static pressure. In the case of frequency-response measurements and simulations, the movement of the mass is much smaller. For larger transient signals the model is restricted to frequencies well below the cut-off frequency of the gas film. At higher frequencies and large amplitudes, the model can only be used for qualitative analysis of the dynamics.

The spring terms of the model in Fig. 5 depend on the gas pressure according to Eq. (18), whereas the damping terms are proportional to the effective gas viscosity according to Eq. (19). Thus our model makes it possible to study the relation between the effective gas viscosity and the absolute pressure of the gas in the narrow gap between two rectangular plates.

The results of the gas-lubrication theory [7–10] are not widely utilized in designing micromechanical sensors [4,5,11]. Our comparison between different slip-flow equations shows that the equation of Fukui and Kaneko [8] agrees most closely with the measured data. The extracted effective viscosity is within 30% of our approximate Eq. (7), except for a single measurement,
whereas all other approaches differ considerably from the measured data, especially when the Knudsen number is high. The approximation (7) of the slip-flow equation of Fukui and Kaneko is applied here to the linearized Reynolds equation, but it can also be utilized to derive a new non-linear modified Reynolds equation. We have shown that Eq. (7) is useful in accelerometer design, and we believe that it can be applied in any kind of gas lubrication and squeezed gas film design.

Based on our equivalent circuit, a complete accelerometer component model has been developed and implemented in the circuit simulator tool APLAC [14,15]. The model includes the squeezed-film model presented here together with the non-linear capacitances and electrostatic attractive force sources. The model is documented in detail elsewhere [16].

8. Conclusions

We conclude that the linearized solution of the Reynolds equation for rectangular plates can be extended in series form so that the terms of the expansion are frequency-independent spring and damper elements that can be described by inductors and resistors in an electrical equivalent circuit. In this way, dynamic micromechanical structures can be modelled in electrical-circuit simulators such as SPICE or APLAC. Thus simulation of the transient behaviour of micromechanical structures together with measurement electronics is possible. Using the model presented, we have evaluated several slip-flow equations and present an approximate function that is applicable in the field of gas lubrication and micromechanical sensor design. The model provides a valuable tool for analysing and designing micromechanical sensors and actuators. The electrical equivalent circuit approach to squeezed gas-film analysis can be extended to other structures that are characterized by small air gaps between moving elements.

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References


Biographies

Timo V. Veijola was born in Helsinki on February 4, 1954. He received the Diploma Engineer (M.Sc.) and the Licentiate in Technology degree in electrical engineering from the Helsinki University of Technology, Finland in 1980 and 1986, respectively.

Since 1981 he has been working as Laboratory Manager in the Circuit Theory Laboratory at the Helsinki University of Technology. His current research interests are computer-aided circuit simulation and modelling.

Heikki Kuusma was born in Kalvola on August 3, 1953. He received the Diploma Engineer degree (M.Sc.) in electrical engineering from the Helsinki University of Technology, Finland in 1978. Since 1980 he has been working at Vaisala Oy and Vaisala Technologies Inc. doing research and development on silicon microme-
chanics and microsensors. He holds several patents on silicon microsensors. Currently he is leading the sensor research group at Vaisala Technologies Inc.

Juha Lahdenperä was born in Liperi on March 31, 1959. He received the Diploma Engineer degree (M.Sc.) in applied physics from the Helsinki University of Technology in 1986. Since 1986 he has been working as a research and development engineer at Vaisala Oy working in micromechanical pressure and acceleration sensor development and from 1991 on at Vaisala Technologies Inc. His current activities are the designing of micromechanical acceleration sensors and micromechanical process development.

Tapani Ryhänen was born in Helsinki on July 7, 1959. He received the Diploma Engineer degree (M.Sc.) in technical physics and the Doctor of Technology degree in applied electronics from the Helsinki University of Technology, Finland in 1986 and in 1992, respectively. He has written several publications on the theory, design and characterization of ultra-low-noise superconducting thin-film magnetometers. Since 1992 he has been working as a research and development engineer at Vaisala Technologies Inc. His current research activities are the designing of micromechanical pressure and rotation rate sensors, development of new measurement systems for micromechanical sensors and the modelling of micromechanical structures.