Channel Coding at Low Capacity

Mohammad Fereydounian[1]
Mohammad Vahid Jamali[2] Hamed Hassani[1] Hessam Mahdavifar[2]
[1]University of Pennsylvania [2]University of Michigan

Motivation

- Narrowband Communication: NB-IoT, eMCT

IoT devices $\Longrightarrow$ 1.5 billion by 2021
$\Longrightarrow$ 170 dB coupling loss
$\Longrightarrow$ -13 dB effective SNR
$\Longrightarrow$ Capacity $\simeq$ 0.03

- Wideband Communication: mm-Wave

$$C = B\log(1 + \frac{P}{N_0 B})$$

$$B \to \infty \Longrightarrow SNR \to 0 \Longrightarrow C \to 0$$

Low-capacity Regime

- Formally: $C < n^{s-1}, s \in [0,1)$
- Informally: $\kappa := nC = o(n)$

Challenges

- In the moderate-capacity regime [1]:

$$\log_2 M^*(n, p_e) = nC - \sqrt{nV}Q^{-1}(p_e) + \mathcal{O}(\log_2 n)$$

- There are numerically off estimates and neglected significant terms in the low-capacity regime.

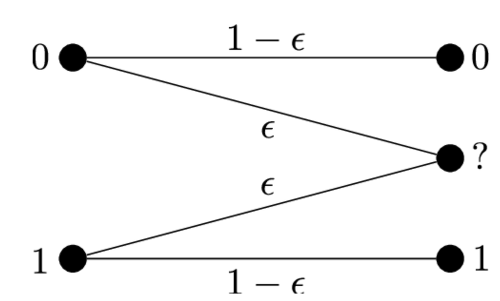
Our Contribution

- Non-asymptotic laws for the low-capacity BEC, BSC
- Practical code design for the low-capacity regime

Binary Erasure Channel (BEC)

- In mod-cap regime: moderate ϵ e.g., $\epsilon = 0.3, 0.6$
  $R = n(1-\epsilon) + \mathcal{O}\left(\sqrt{n}\right)$
  Gaussian convergence laws

- In low-cap regime: high ϵ e.g., $\epsilon = 0.95, 0.99$



...1???0???????1???...

$$P\{R < r\} = \mathcal{P}_{n(1-\epsilon)}(r)$$

Rare information $\Longrightarrow$ Poisson convergence laws

Theorem. $M_1 \le M^*(n, p_e) \le M_2$

$$\mathfrak{P}_1(M_1) + \alpha\sqrt{\mathfrak{P}_1(M_1)} - p_e = 0,$$

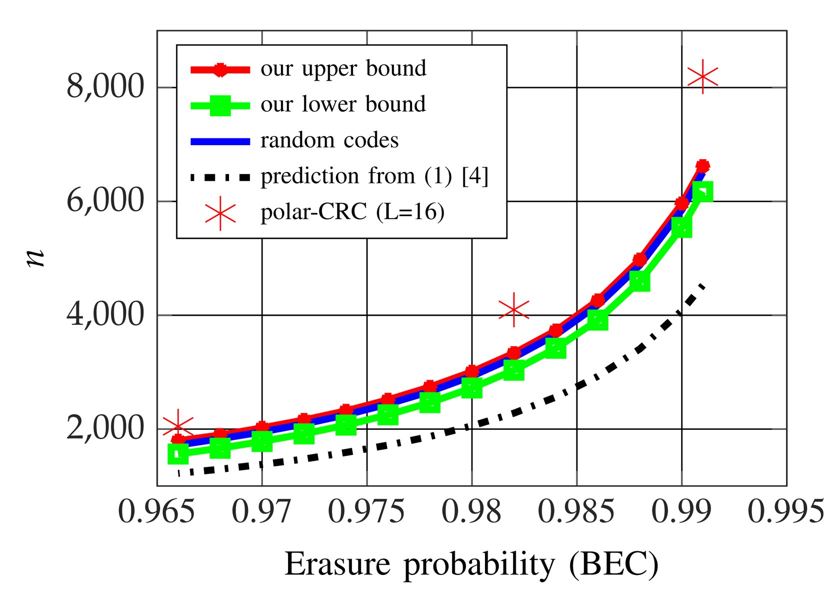
$$\mathfrak{P}_2(M_2) - \alpha\sqrt{\mathfrak{P}_2(M_2)} - \alpha\sqrt{\mathcal{P}_{,\kappa}\left(\log_2 M_2\right)} - p_e = 0,$$

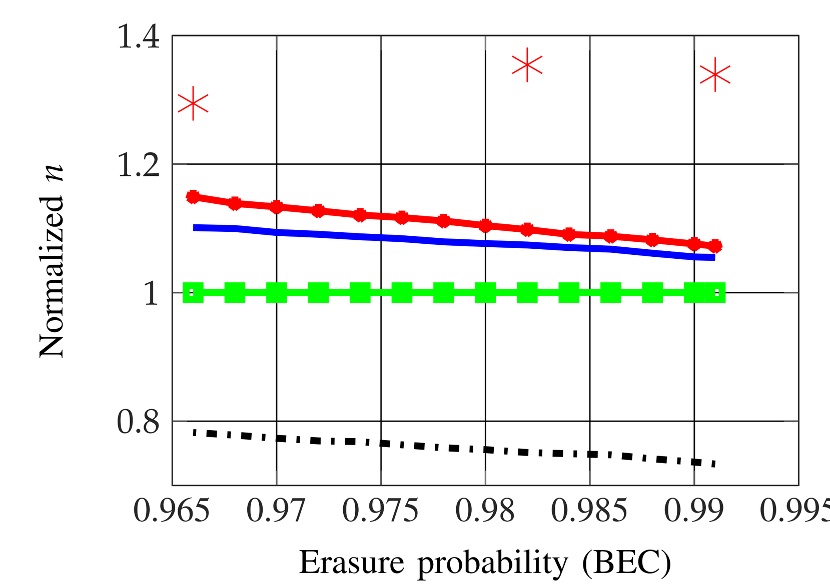
$$\mathfrak{P}_1(M_1) = \mathcal{P}_\kappa(\log_2 M_1) + M_1 e^{-\kappa/2}\left(1 - \mathcal{P}_{\kappa/2}(\log_2 M_1)\right),$$

$$\mathfrak{P}_2(M_2) = \mathcal{P}_\kappa(\log_2 M_2) - \frac{e^\kappa}{M_2}\mathcal{P}_{2\kappa}\left(\log_2 M_2\right),$$

$$\alpha = \frac{\sqrt{2}}{\epsilon^{3/2}}\left(1 + 2\sqrt{\frac{3}{\epsilon\kappa}}\right)\left(\sqrt{e} - 1\right)(1-\epsilon).$$

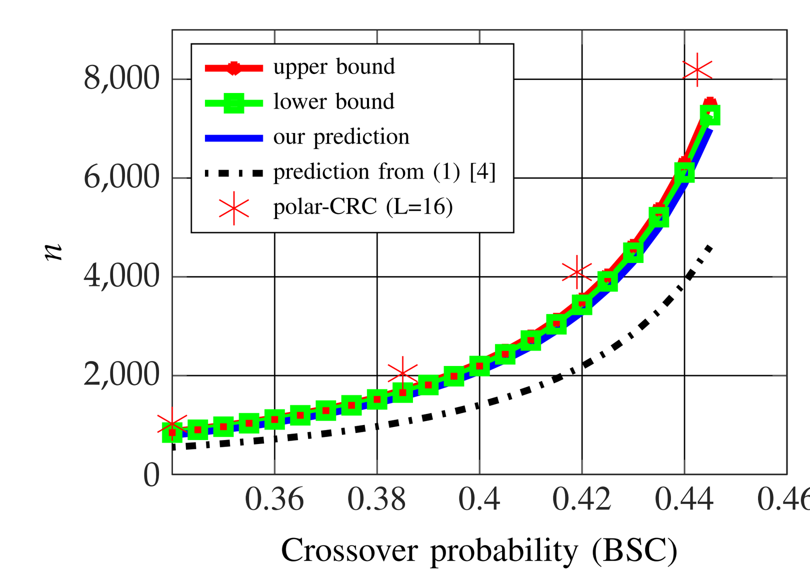
- We use RCU bound [1] to derive M_1 and raw BEC converse bound [1] to derive M_2 .

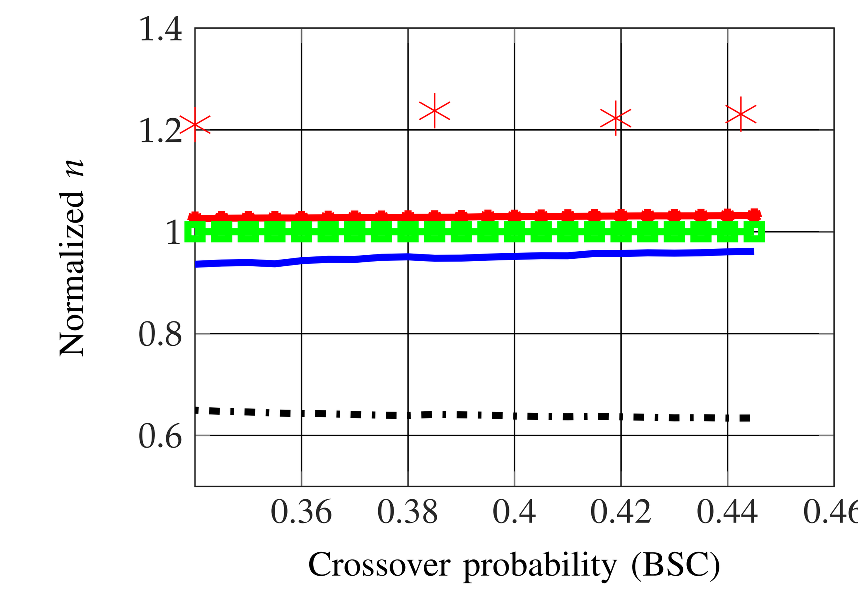




Binary Symmetric Channel (BSC)

Theorem. $\log_2 M^*(n, p_e) = \kappa - 2\sqrt{\frac{2\delta(1-\delta)}{\ln 2}} \cdot \sqrt{\kappa}\, Q^{-1}(p_e) + \frac{1}{2}\log_2 \kappa - \log_2 p_e + \mathcal{O}(\log\log\kappa)$.



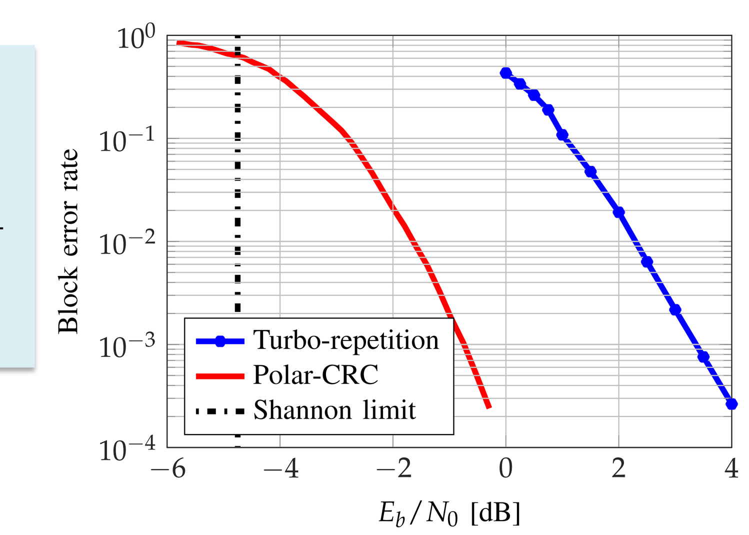


Practical Code Design

- Current standards use repetition + some off-the-shelf moderate-rate codes like Turbo codes/ LDPC.
- Using iterative codes + repetition results in mediocre performances.
- Polar codes implicitly apply the optimal repetition length. We obtain this optimal length as below.

Theorem.

$$\frac{n(1-\epsilon)\ell}{2\left(1-\frac{\beta}{\ell}\right)} \cdot \left(\frac{\beta}{\ell}\right)^2 \le \frac{n}{r_\beta} \le \frac{n(1-\epsilon)\ell}{2\left(1-\frac{\beta}{\ell}\right)}$$



References

[1] Polyanskiy, H. V. Poor, and S. Verdú, "Channel coding rate in the finite blocklength regime," IEEE Trans. Inf. Theory, vol. 56, no. 5, 2010.

[2] M. Fereydounian, M. V. Jamali, H. Hassani, and H. Mahdavifar, "Channel coding at low capacity," arXiv preprint arXiv:1811.04322, 2018.