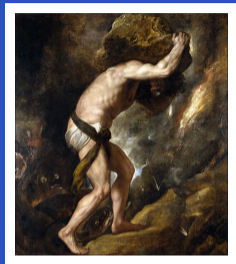


# Sisyphus: Mostly Automated Proof Repair for Verified Libraries



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National University of Singapore

*Pictured: OCaml programmer fixing a broken Coq proof*

Let's write a program!

Let's write a *verified* program!

Let's write a *verified* program!

**Q:** Convert a sequence to an array.

# OCaml's 'a Seq.t datatype

```
type 'a t    = unit -> 'a node
and 'a node = Nil
            | Cons of 'a * (unit -> 'a node)
```

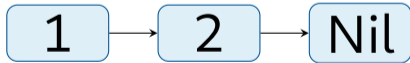
# OCaml's 'a Seq.t datatype

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type 'a t    = unit -> 'a node
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```

*A thunked tail*

# OCaml's 'a Seq.t datatype

```
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# OCaml's 'a Seq.t datatype

```
type 'a t    = unit -> 'a node
and 'a node = Nil
            | Cons of 'a * (unit -> 'a node)
```



```
fun () -> Cons (1, fun () -> Cons (2, fun () -> Nil))
```



Let's write a verified program!

**Q:** Convert a sequence to an array.

Let's write a verified program!

**Q:** Convert a sequence to an array.

*Let's write  
some code!*

```
let to_array s =
```

```
let to_array s =  
  match s () with
```

```
let to_array s =  
  match s () with  
  | Nil ->
```

```
let to_array s =  
  match s () with  
  | Nil -> [| |]
```

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->
```

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->  
    let sz = length s in
```



```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->  
    let sz = length s in  
    let a = make sz h in
```

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->  
    let sz = length s in  
    let a = make sz h in  
    iteri (fun i vl ->  
  
          a.(i) <- vl  
  
    ) s;
```

```
let to_array s =  
  match s () with  
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  | Cons (h, _) ->  
    let sz = length s in  
    let a = make sz h in  
    iteri (fun i vl ->  
  
        a.(i) <- vl  
  
    ) s;  
  a
```

Let's write a verified program!

**Q:** Convert a sequence to an array.

Let's write a **verified** program!

**Q:** Convert a sequence to an array.

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**Q:** Convert a sequence to an array.

$\forall s \ell, \{s \mapsto \text{Seq } \ell\}$

$(\text{to\_array } s)$

$\exists a, \{a \mapsto \text{Array } \ell\}$

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**Q:** Convert a sequence to an array.

$\forall s \ell, \{s \mapsto \text{Seq } \ell\}$

`(to_array s)`

$\exists a, \{a \mapsto \text{Array } \ell\}$

**"a" points-to** an array

**Q:** Convert a sequence to an array.

$\forall s \ell, \{s \mapsto \text{Seq } \ell\}$

`(to_array s)`

$\exists a, \{a \mapsto \text{Array } \ell\}$

Let's write  
some proofs!

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
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    iteri (fun i vl ->  
  
        a.(i) <- vl  
  
    ) s;  
  a
```

```

let to_array s =
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    iteri (fun i vl ->
      a.(i) <- vl
    ) s;
  a

```



Using CFML2  
verification  
library

Charguéraud (2023)

```

let to_array s =
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```

```

xcf.
xapp; case  $\ell$  as [| h tl]
  - xvaemptyarr.
  -
  xapp.
  xalloc.
  xapp (iteri_spec ( $\lambda t \rightarrow$ 
     $a \mapsto$  Array (
       $t \text{ ++ drop (length } t)$ 
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  ).
xval.

```

```
let to_array s =  
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```

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  -
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       $t ++ \text{drop} (\text{length } t)$ 
       $(\text{make} (\text{length } \ell) h)$ 
    )
  ).
xval.

```

```

let to_array s =
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    let a = make sz h in
    iteri (fun i vl ->
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    ) s;
  a

```

```

xcf.
xapp; case ℓ as [| h tl]
  - xvaemptyarr.
  -
  xapp.
  xalloc.
  xapp (iteri_spec (λt →
    a ↦ Array (
      t ++ drop (length t)
      (make (length ℓ) h))
    )).
xval.

```

*prefix*



```

let to_array s =
  match s () with
  | Nil -> [| |]
  | Cons (h, _) ->
    let sz = length s in
    let a = make sz h in
    iteri (fun i vl ->
      a.(i) <- vl
    ) s;
  a

```

```

xcf.
xapp; case  $\ell$  as [| h tl]
  - xvaemptyarr.
  -
  xapp.
  xalloc.
  xapp (iteri_spec ( $\lambda t \rightarrow$ 
     $a \mapsto \text{Array} ($ 
       $t ++ \text{drop} (\text{length } t)$ 
       $(\text{make} (\text{length } \ell) h))$ 
    )).
  xval.

```

<pre> let to_array s =   let rec loop i with       Cons (h, _) -&gt;       let sz = length s in       let a = make sz h in       iteri (fun i vl -&gt;         a.(i) &lt;- vl       ) s;       a </pre>	<pre> xcf. xapp_spec (fun [l h t]   -   xapp.   xalloc.   xapp (iteri_spec (λt →     a ↦ Array (       t ++ drop (length t)       (make (length ℓ) h))     ).   xval. </pre>
---	--

$a \mapsto \text{Array}(t \text{ ++ drop } (\text{length } t) (\text{make } (\text{length } \ell) h))$

```

let to_array s =
  | Cons (h, _) ->
    let sz = length s in
    let a = make sz h in
    iteri (fun i vl ->
      a.(i) <- vl
    ) s;
    a
  | Nil ->
    xcf.
    xapp.
    xalloc.
    xapp (iteri_spec (λt →
      a ↦ Array (
        t ++ drop (length t)
          (make (length ℓ) h))
      )
    ).
    xval.

```

*a* →



```

let to_array s =
  | Cons (h, _) ->
    let sz = length s in
    let a = make sz h in
    iteri (fun i vl ->
      a
      a. (
        ) s;
        a
  ) s;
  a

```

$a \mapsto \text{Array}(t \ ++ \ \text{drop} \ (\text{length} \ t) \ (\text{make} \ (\text{length} \ \ell) \ h))$

```

    xcf.
    xapp_spec (as [[ h t]]
    xapp.
    xalloc.
    xapp (iteri_spec (\t ->
      a \mapsto \text{Array} (
        t \ ++ \ \text{drop} (\text{length} t)
        (\text{make} (\text{length} \ell) h))
    ).
    xval.

```

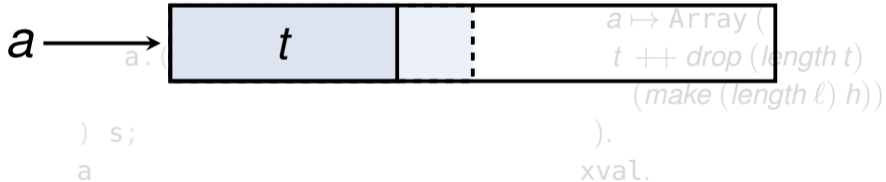


```

let to_array s =
  | Cons (h, _) ->
    let sz = length s in
    let a = make sz h in
    iteri (fun i vl ->

```

$a \mapsto \text{Array}(t \text{ ++ drop } (\text{length } t) \text{ (make } (\text{length } \ell) \text{ h)})$



```

let to_array s =
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  | Cons (h, _) ->
    let sz = length s in
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    iteri (fun i vl ->
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  a

```

```

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  xapp (iteri_spec ( $\lambda t \rightarrow$ 
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      ( $make \ (length \ \ell) \ h$ ))
    )).
xval.

```

```

let to_array s =
  match s () with
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    iteri (fun i vl ->
      a.(i) <- vl
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  a

```

Qed.

```

xcf.
xapp; case ℓ as [| h tl]
  - xvaemptyarr.
  -
  xapp.
  xalloc.
  xapp (iteri_spec (λt →
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      t ++ drop (length t)
      (make (length ℓ) h))
    )).
xval.

```

Let's write a **verified** program!

Let's write a **verified** program!

**Conclusion:**

Writing verified code is hard

Writing verified code is hard

Writing verified code is hard...

A problem arises...



# A problem arises...

 [c-cube / ocaml-containers](#)

 440 stars

# A problem arises...

## Make Seq.to\_array behave better with stateful sequences #390

 Merged c-cube merged 4 commits into `c-cube:master` from `shonfeder:state-friendly-seq-to-array`  on Dec 12, 2021

 Conversation **8**  Commits **4**  Checks **0**  Files changed **2**



**shonfeder** commented on Dec 12, 2021 • edited ▾

Contributor  ...

2 participants

This PR suggests a change to `Seq.to_array`.



The change is motivated by the surprising behavior of `Seq`.

# Old

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->  
    let sz = length s in  
    let a = make sz h in  
    iteri (fun i vl ->  
      a.(i) <- vl  
    ) s;  
  a
```

# New

```
let to_array s =  
  let sz, ls = fold_left  
    (fun (i, acc) x ->  
      (i+1, x::acc))  
    (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init :: rest ->  
    let a = make sz init in  
    let idx = len - 2 in  
    List.fold_left  
      (fun i vl ->  
        a.(i) <- vl; i-1)  
      idx rest;  
  a
```

## Old

```
let to_array s =  
  match s () with  
  | Nil -> [| |]  
  | Cons (h, _) ->  
    let sz = length s in  
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    iteri (fun i vl ->  
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```

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let to_array s =  
  let sz, ls = fold_left  
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    let a = make sz init in  
    let idx = len - 2 in  
    List.fold_left  
      (fun i vl ->  
        a.(i) <- vl; i-1)  
      idx rest;  
  a
```

# Old

```
let to_array s =  
  match s () with  
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```

# New

```
let to_array s =  
  let sz, ls = fold_left  
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        a.(i) <- vl; i-1)  
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  a
```

# Old

```
let to_array s =  
  match s () with  
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```

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```



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      a.(i) <- vl  
    ) s;  
  a
```

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let to_array s =  
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*Completely different implementation...*

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*Completely different implementation...*

*...proof must be redone.*

Writing verified code is hard...

Maintaining

~~Writing~~ verified code is hard...

Maintaining

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**Proof Repair**

Maintaining

~~Writing~~ verified code is hard...

## Proof Repair

Ringer (2021)



Maintaining

~~Writing~~ verified code is hard...

*...can we make it easier?*

Maintaining

~~Writing~~ verified code is hard...

*...can we make it easier?*

**Yes!**

# This Work



*Mostly automated proof-repair* for OCaml programs

# Outline

- 1 Motivation
- 2 Key Challenges & Solutions
- 3 Evaluation

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# New Program:

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## New Program:

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let to_array s =  
  
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## How to generate a proof script?

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    (fun () ());  
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## New Program:

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## How to generate a proof script?

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  (0, []) <+>  
  match ls with  
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```

## Observation: proofs are syntax-directed

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List.fold_left  
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case l as [| init rest].
  - xmatch 0. xvalemptyarr.

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xcf.
xapp (...).
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```

## How to fill in holes?

# Key challenges for proof repair

- ① *Generating* candidate invariants
- ② *Choosing* valid invariants



# Key challenges for proof repair

① *Generating* candidate invariants

② *Choosing* valid invariants

How to generate invariants?

How to generate invariants?

Use the old program and proofs!

# Generating candidate invariants

```
let to_array l =  
  match l () with  
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    let len = length' l in  
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# Generating candidate invariants

*Programs are different—*

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*—but have similarities.*

# Generating candidate invariants

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# Generating candidate invariants

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# Generating candidate invariants

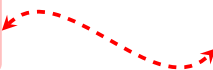
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  | Nil -> [| |]  
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# Generating candidate invariants

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    let a = make len x
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```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

Similar behaviour...

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->
```

```
          (i + 1, x :: acc))
```

```
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
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```
  | init::rest ->
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    let a = make sz init in
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    let idx = sz - 2 in
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```
      List.fold_left
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# Generating candidate invariants

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```
    a
```

..similar invariants?

# Generating candidate invariants

**Q:** How to discover these similarities automatically?

```
let to_array l =
```

```
  match l () with
```

```
    Nil      -> []
```

```
  | Cons (x, _) ->
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    let len = length' l in
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    let a = make len x
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```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

```
let to_array l =
```

```
  let sz, ls =
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```
    fold (fun (i, acc) x ->
```

```
          (i + 1, x :: acc))
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```
        (0, []) l in
```

```
  match ls with
```

```
    [] -> []
```

```
  | init::rest ->
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```
    let a = make sz init in
```

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    let idx = sz - 2 in
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```
      List.fold_left
```

```
        (fun i x -> a.(i) <- x; i - 1)
```

```
        idx rest in
```

```
    a
```

Similar behaviour...

..similar invariants?

# Generating candidate invariants

**Q:** How to discover these similarities automatically?

```
let to_array l =
```

```
  match l () with
```

```
    Nil -> []
```

**A:** Instrumentation based dynamic analysis

```
    let len = length' l in
```

```
    let a = make len x
```

```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

```
let to_array l =
```

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  let sz, ls =
```

```
    fold (fun (i, acc) x ->
```

```
          (i + 1, x :: acc))
```

```
        (0, []) l in
```

```
    [] -> []
```

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      let a = make sz init in
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      let idx = sz - 2 in
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```
        List.fold_left
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          (fun i x -> a.(i) <- x; i - 1)
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```
          idx rest in
```

```
      a
```

Similar behaviour...

..similar invariants?

# Generating candidate invariants

**Q:** How to discover these similarities automatically?

```
let to_array l =
```

```
  match l () with
```

```
    Nil -> []
```

**A:** Instrumentation based dynamic analysis

```
    let len = length l in
```

```
    let a = make len x
```

```
    iteri
```

```
      (fun i x a l => x) a l
```

```
    a
```

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->
```

```
      (i + 1, x :: acc))
```

```
      (0, []) l in
```

```
    [] -> []
```

```
  'init::rest ->
```

```
    let a = make sz init in
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    let _ =
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      fold (fun (i, acc) x ->
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        (i + 1, x :: acc))
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```
        (0, []) a in
```

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    idx rest in
```

```
    a
```

Similar behaviour...

Identify "similar" program points through traces.

..similar invariants?

```

let to_array l =
  match l () with
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```



## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
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    let len = length' l in  
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Seq.of\_list [1;2;3]

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let to\_array l =

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List.fold\_left

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## Seq.of\_list [1;2;3]

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let to_array l =
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    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    3 let len = length' l in  
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    iteri  
      (fun i x -> a.(i) <- x)  
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    a
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      (fun i x -> a.(i) <- x)  
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```

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        idx rest in  
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let to_array l =  
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      (fun i x -> a.(i) <- x)  
      l;  
    a
```

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let to_array l =  
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        idx rest in  
    a
```

## Seq.of\_list [1;2;3]

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  | Nil      -> [| |]  
  | Cons (x, _) ->  
    3  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

[|1;1;1|]

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let to_array l =  
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    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil          -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
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        (fun i x -> a.(i) <- x; i - 1)  
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    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =
```

```
  match l () with
```

```
  | Nil          -> [| |]
```

```
  | Cons (x, _) ->
```

```
    let len = length' l in
```

```
    let a = make len x in
```

```
    iteri  
      (fun i x -> a.(i) <- x)  
      l;
```

```
    a
```

3

[[1;1;1]]

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
```

```
  | init::rest ->
```

```
    let a = make sz init in
```

```
    let idx = sz - 2 in
```

```
    let _ =
```

```
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

```
    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil          -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;
```

3

[|1;1;1|]

[|1;2;3|]

a

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

a

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil          -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
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  match ls with  
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    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```



## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
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    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil          -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

a

## Seq.of\_list [1;2;3]

```
let to_array l =
```

```
  match l () with
```

```
  | Nil          -> [| |]
```

```
  | Cons (x, _) ->
```

```
    let len = length' l in
```

```
    let a = make len x in
```

```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
```

```
  | init::rest ->
```

```
    let a = make sz init in
```

```
    let idx = sz - 2 in
```

```
    let _ =
```

```
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

```
    a
```

3

## Seq.of\_list [1;2;3]

```
let to_array l =
```

```
  match l () with
```

```
  | Nil          -> [| |]
```

```
  | Cons (x, _) ->
```

```
    let len = length' l in
```

```
    let a = make len x in
```

```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
```

```
  | init::rest ->
```

```
    let a = make sz init in
```

```
    let idx = sz - 2 in
```

```
    let _ =
```

```
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

```
    a
```

3

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

[|3;3;3|]

## Seq.of\_list [1; 2; 3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1; 1; 1|]

[|1; 2; 3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

[|3; 3; 3|]

## Seq.of\_list [1; 2; 3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1; 1; 1|]

[|1; 2; 3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
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  match ls with  
  | [] -> [| |]  
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    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

3

[|3; 3; 3|]

a



# Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
  | [] -> [| |]  
  | init::rest ->  
    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

3

[|3;3;3|]

[|1;2;3|]

a

## Seq.of\_list [1; 2; 3]

```
let to_array l =
```

```
  match l () with
```

```
  | Nil      -> [| |]
```

```
  | Cons (x, _) ->
```

```
    let len = length' l in
```

```
    let a = make len x in
```

```
    iteri
```

```
      (fun i x -> a.(i) <- x)
```

```
      l;
```

```
    a
```

3

[|1; 1; 1|]

[|1; 2; 3|]

```
let to_array l =
```

```
  let sz, ls =
```

```
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
```

```
  | init::rest ->
```

```
    let a = make sz init in
```

```
    let idx = sz - 2 in
```

```
    let _ =
```

```
      List.fold_left
```

```
        (fun i x -> a.(i) <- x; i - 1)
```

```
        idx rest in
```

```
    a
```

3

[|3; 3; 3|]

[|1; 2; 3|]

## Seq.of\_list [1; 2; 3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1; 1; 1|]

[|1; 2; 3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
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    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

[|3; 3; 3|]

[|1; 2; 3|]

a

## Seq.of\_list [1; 2; 3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1; 1; 1|]

[|1; 2; 3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
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    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

[|3; 3; 3|]

[|1; 2; 3|]

a

## Seq.of\_list [1;2;3]

```
let to_array l =  
  match l () with  
  | Nil      -> [| |]  
  | Cons (x, _) ->  
    let len = length' l in  
    let a = make len x in  
    iteri  
      (fun i x -> a.(i) <- x)  
      l;  
    a
```

3

[|1;1;1|]

[|1;2;3|]

```
let to_array l =  
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with  
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    let a = make sz init in  
    let idx = sz - 2 in  
    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in  
    a
```

3

[|3;3;3|]

[|1;2;3|]

a

Seq.of\_list [1; 2; 3]

**Q:** How to discover these similarities automatically?

```
let to_array l =
```

```
  match l () with  
  | Nil          -> [| |]  
  | Cons (h, t) -> [| h; to_array t |]
```

```
  let len = length' l in
```

```
  let a = make len x in
```

```
  let rec loop i x =  
    a.(i) <- x;  
    if i < len - 1 then loop (i + 1) x  
  in loop 0 x
```

a

```
let to_array l =
```

```
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in  
  match ls with
```

```
  | [] -> [| |]
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```
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    let idx = sz - 2 in
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    let _ =  
      List.fold_left  
        (fun i x -> a.(i) <- x; i - 1)  
        idx rest in
```

a

**A:** Instrumentation based dynamic analysis

Identify "similar" program points through traces.

[[1; 1; 1]]

[[1; 2; 3]]

[[1; 2; 3]]

[[1; 2; 3]]

Seq.of\_list [1; 2; 3]

**Q:** How to discover these similarities automatically?

```
let to_array l =
```

```
  match l () with  
  | Nil          -> [| |]  
  | Cons _::_l  -> [| |]
```

**A:** Instrumentation based dynamic analysis

```
  let len = length' l in
```

```
  let a = make len x in
```

Identify "similar" program points through traces.

```
  l;
```

```
  a
```

```
let to_array l =
```

```
  let sz, ls =  
    fold (fun (i, acc) x ->  
          (i + 1, x :: acc))  
        (0, []) l in
```

```
  match ls with
```

```
  | [] -> [| |]
```

```
  | _::rest ->
```

```
    let idx = sz - 2 in
```

```
    let _ =  
      List.fold_left
```

```
        (fun i x -> a i) x i  
      idx rest in
```

```
    a
```

[[1; 1; 1]]

[[1; 2; 3]]

3

3

[[1; 2; 3]]

[[1; 2; 3]]

Use invariants from old proof to synthesise invariants for new one.

Old Invariant:

$a \mapsto \text{Array}(t \text{ ++ } \text{drop}(\text{length } t) (\text{make}(\text{length } \ell) h))$



Old Invariant:

$$a \mapsto \text{Array}(t \text{ ++ } \text{drop}(\text{length } t) (\text{make}(\text{length } \ell) h))$$

New Invariant:

Old Invariant:

$$a \mapsto \text{Array}(t ++ \text{drop}(\text{length } t) (\text{make}(\text{length } \ell) h))$$

New Invariant:

```
(fun acc t => a \mapsto  
  Array (repeat (acc + 1) init ++  
         drop (acc + 1) \ell))
```

(1)

Old Invariant:

$$a \mapsto \text{Array}(t ++ \text{drop}(\text{length } t) (\text{make}(\text{length } \ell) h))$$

New Invariant:

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++      . . .  
         drop (acc + 1)  $\ell$ ))
```

(1)

## Old Invariant:

$$a \mapsto \text{Array}(t \text{ ++ drop } (\text{length } t) \text{ (make } (\text{length } \ell) \text{ h)})$$

## New Invariant:

```
(fun acc t => a ↦  
  Array (repeat (acc + 1) init ++  
         drop (acc + 1) ℓ))
```

(1)

...

```
(fun acc t => a ↦  
  Array ([ ] ++ drop 0  
         (repeat (length t) init)))
```

(n)

# Key challenges for proof repair

- 1 *Generating* candidate invariants
- 2 *Choosing* valid invariants

# Key challenges for proof repair

① *Generating* candidate invariants

② *Choosing* valid invariants

# Choosing an Invariant

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```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
      (fun acc t =>  
        (??)  )).  
...
```



# Choosing an Invariant

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...
xlet idx.
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(fun acc t => a  $\mapsto$ 
  Array (repeat (acc + 1) init ++
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  Array ([ ] ++ drop 0
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  Array ([ ] ++ drop 0  
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(fun acc t => a  $\mapsto$   
 Array (repeat (acc + 1) init ++  
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 Array ([] ++ drop 0  
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# Choosing an Invariant

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xlet idx.  
xapp (fold_left_spec idx rest  
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 Array (repeat (acc + 1) init ++  
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 Array ([] ++ drop 0  
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# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??)  )).  
...
```

Which one?

(fun acc t => a  $\mapsto$   
 Array (repeat (acc + 1) init ++  
 drop (acc + 1)  $\ell$ ))

(fun acc t => a  $\mapsto$   
 Array ([ ] ++ drop 0  
 (repeat (length t) init)))

# Choosing an Invariant

```
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xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
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```

Validate using SMT?

Which one?

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++  
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```
(fun acc t => a  $\mapsto$   
  Array ([ ] ++ drop 0  
    (repeat (length t) init)))
```

# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??) )).  
...
```

Validate using SMT?

Which one?

*Too slow!*

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++  
    drop (acc + 1)  $\ell$ ))
```

```
(fun acc t => a  $\mapsto$   
  Array ([ ] ++ drop 0  
    (repeat (length t) init)))
```

# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??) )).  
...
```

If only we could test...

Which one?

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++  
    drop (acc + 1)  $\ell$ ))
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```
(fun acc t => a  $\mapsto$   
  Array ([ ] ++ drop 0  
    (repeat (length t) init)))
```



# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??) ))).  
...
```

Depends on  
logical parameters!

If only we could test...

Which one?

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++  
    drop (acc + 1)  $\ell$ ))
```

```
(fun acc t => a  $\mapsto$   
  Array ([ ] ++ drop 0  
    (repeat (length t) init)))
```

# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??) )).  
...
```

Depends on  
logical parameters!



Which one?

```
(fun acc t => a ⇨  
  Array (repeat (acc + 1) init ++  
    drop (acc + 1) ℓ))
```

```
(fun acc t => a ⇨  
  Array ([] ++ drop 0  
    (repeat (length t) init)))
```

# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
      (fun acc t =>  
        (??) )).  
...
```

# Choosing an Invariant

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
      (fun acc t =>  
        (??) )).  
...
```

# Verified Fold Left

```
let fold_left f acc ls =  
  
  match ls with  
  | [] ->  
  
    acc  
  | h :: tl ->  
  
    let acc' = f acc h in  
  
    fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t =  
  
  match ls with  
  | [] ->  
  
    acc  
  | h :: tl ->  
  
    let acc' = f acc h in  
  
    fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t / =  
  
  match ls with  
  | [] ->  
  
    acc  
  | h :: tl ->  
  
    let acc' = f acc h in  
  
    fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
  match ls with  
  | [] ->  
  
    acc  
  | h :: tl ->  
  
    let acc' = f acc h in  
  
    fold_left f acc' tl
```



# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
match ls with  
| [] ->  
  {l acc t}  
  acc  
| h :: tl ->  
  
  let acc' = f acc h in  
  
  fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
  match ls with  
  | [] ->  
    {l acc t}  
    acc  
  | h :: tl ->  
    {l acc t}  
    let acc' = f acc h in  
  
    fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
match ls with  
| [] ->  
  {l acc t}  
  acc  
| h :: tl ->  
  {l acc t}  
  let acc' = f acc h in  
  {l acc' (t ++ [h])}  
  fold_left f acc' tl
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
match ls with  
| [] ->  
  {l acc t}  
  acc  
| h :: tl ->  
  {l acc t}  
  let acc' = f acc h in  
  {l acc' (t ++ [h])}  
  fold_left f acc' tl  
  {l acc'' (t ++ [h] ++ tl)}
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
  match ls with  
  | [] ->  
    {l acc t}  
    acc  
  | h :: tl ->  
    {l acc t}  
    let acc' = f acc h in  
    {l acc' (t ++ [h])}  
    fold_left f acc' tl  
    {l acc'' (t ++ [h] ++ tl)}
```

Describes **exactly**  
how *l* is maintained

# Verified Fold Left

```
let fold_left f acc ls t l =  
  {l acc t}  
  match ls with  
  | [] ->  
    {l acc t}  
    acc  
  | h :: tl ->  
    {l acc t}  
    let acc' = f acc h in  
    {l acc' (t ++ [h])}  
    fold_left f acc' tl  
    {l acc'' (t ++ [h] ++ tl)}
```

# Verified Fold Left

```
let fold_left f acc ls t l =  
  assert {l acc t}  
  match ls with  
  | [] ->  
    assert {l acc t}  
    acc  
  | h :: tl ->  
    assert {l acc t}  
    let acc' = f acc h in  
    assert {l acc' (t ++ [h])}  
    fold_left f acc' tl  
    assert {l acc'' (t ++ [h] ++ tl)}
```

# Verified Fold Left

```
let fold_left f acc ls i i =  
  assert {I acc t}  
  match ls with  
  | [] ->  
    assert {I acc t}  
  | _::_ ->  
    assert {I acc t}  
    let acc' = f acc h in  
    assert {I acc' (t++[h])}  
    fold_left f acc' tl  
    assert {I acc'' (t++[h]++tl)}
```

## Proof-Driven Testing



# Proof-Driven Testing

`fold_left_spec`

# Proof-Driven Testing

```
fold_left_spec ?/ f 2 [2; 1]
```

# Proof-Driven Testing

*Instantiate with concrete arguments..*

```
fold_left_spec ?/ f 2 [2; 1]
```

# Proof-Driven Testing

*Instantiate with concrete arguments..*

```
fold_left_spec ?/ f 2 [2; 1]
```

# Proof-Driven Testing

*Instantiate with concrete arguments..*

```
fold_left_spec ?/ f 2 [2; 1]
```

*...with existentials for proof arguments*

# Proof-Driven Testing

```
fold_left_spec ?/ f 2 [2; 1]
```

# Proof-Driven Testing

```
fold_left_spec ?/ f 2 [2; 1]
```

# Proof-Driven Testing

fold\_left\_spec ?/ f 2 [2; 1]

|

**reduce proof term**

↓



# Proof-Driven Testing

```
let fold_left f acc ls t =  
  {l acc t}  
  match ls with  
  | [] ->  
    {l acc t}  
    acc  
  | h :: tl ->  
    {l acc t}  
    let acc' = f acc h in  
    {l acc' (t++[h])}  
    fold_left f acc' tl  
    {l acc'' (t++[h]++tl)}
```

# Proof-Driven Testing

```
let fold_left f acc ls t =  
  {/ 2 []}  
  match ls with  
  | [] ->  
    {/ 2 []}  
    acc  
  | 2 :: [1] ->  
    {/ 2 []}  
    let acc' = f 2 2 in  
    {/ acc' [2]}  
    fold_left f acc' tl  
    {/ acc'' [2, 1]}
```

# Proof-Driven Testing

fold\_left\_spec ?/ f 2 [2; 1]

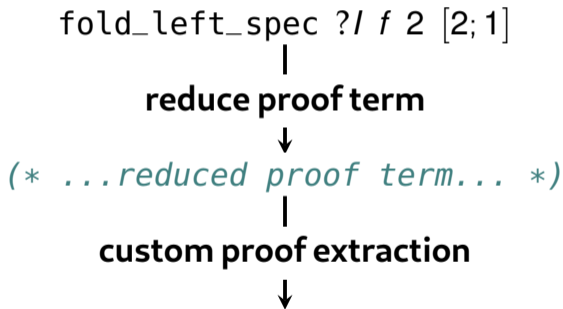
|

**reduce proof term**

↓

(\* ...*reduced proof term*... \*)

# Proof-Driven Testing



# Proof-Driven Testing

```
let fold_left f acc ls =  
  {/ 2 []}  
  match ls with  
  | [] ->  
    {/ 2 []}  
    acc  
  | 2 :: [1] ->  
    {/ 2 []}  
    let acc' = f 2 2 in  
    {/ acc' [2]}  
    fold_left f acc' tl  
    {/ acc'' [2, 1]}
```

# Proof-Driven Testing

```
assert (/ len []);  
let acc = f len 2 in  
assert (/ acc [2]);  
let acc = f acc 1 in  
assert (/ acc [2; 1]);  
( )
```

# Proof-Driven Testing

```
assert (/ len []);  
let acc = f len 2 in  
assert (/ acc [2]);  
let acc = f acc 1 in  
assert (/ acc [2; 1]);  
()
```

Simulates concrete run of  
`List.fold_left`

# Proof-Driven Testing

*Instantiate I with embedding of candidate invariant...*

```
assert (/ len []);  
let acc = f len 2 in  
assert (/ acc [2]);  
let acc = f acc 1 in  
assert (/ acc [2; 1]);  
( )
```



# Proof-Driven Testing

*Instantiate I with embedding of candidate invariant...*

```
assert (/ len []);  
let acc = f len 2 in  
assert (/ acc [2]);  
let acc = f acc 1 in  
assert (/ acc [2; 1]);  
( )
```

*...prune candidate if assertion raised.*

# Testing Candidate Invariants


```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??))).  
...
```

```
(fun acc t => a  $\mapsto$   
  Array (repeat (acc + 1) init ++  
    drop (acc + 1)  $\ell$ ))
```

```
(fun acc t => a  $\mapsto$   
  Array ([] ++ drop 0  
    (repeat (length t) init)))
```

# Testing Candidate Invariants


```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??))).  
...
```


 (fun acc t => a  $\mapsto$   
Array (repeat (acc + 1) init ++  
drop (acc + 1)  $\ell$ ))

(fun acc t => a  $\mapsto$   
Array ([ ] ++ drop 0  
(repeat (length t) init)))

# Testing Candidate Invariants

```
...  
xlet idx.  
xapp (fold_left_spec idx rest  
(fun acc t =>  
  (??))).  
...
```

 (fun acc t => a  $\mapsto$   
 Array (repeat (acc + 1) init ++  
 drop (acc + 1)  $\ell$ ))

 (fun acc t => a  $\mapsto$   
 Array ([] ++ drop 0  
 (repeat (length t) init)))

# Proof-Driven Testing



Proofs are proofs

---



Proofs are programs

---



Proofs are tests

Proof are *programs...*

# Proof-Driven Testing



Proofs are proofs

---



Proofs are programs

---



Proofs are tests

Proof are *programs*...

but, what do they *compute*?

# Proof-Driven Testing



Proofs are proofs

---



CH

Proofs are programs

---



PDT

Proofs are tests

Proof are *programs*...

but, what do they *compute*?

**Curry Howard:** They establish logical facts.

# Proof-Driven Testing



Proofs are proofs

---



Proofs are programs

---



Proofs are tests

Proof are *programs*...

but, what do they *compute*?

**Curry Howard:** They establish logical facts.

**PDT:** HO-proofs describe tests!



# Sisyphus

Old Proof



New Program

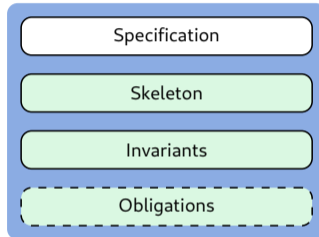
Old Program

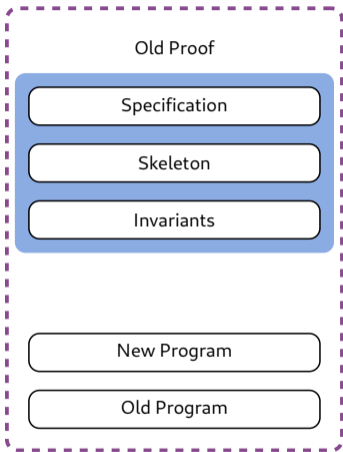
Skeleton Generation

Invariant Synthesis

Invariant Testing

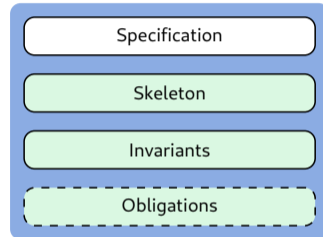
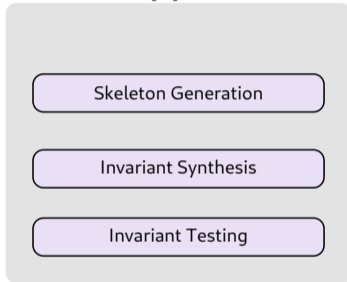
New Proof





Inputs

# Sisyphus



Old Proof

Specification

Skeleton

Invariants

New Program

Old Program

# Sisyphus

Skeleton Generation

Invariant Synthesis

Invariant Testing

New Proof

Specification

Skeleton

Invariants

Obligations

Output

# Sisyphus

Old Proof



New Program

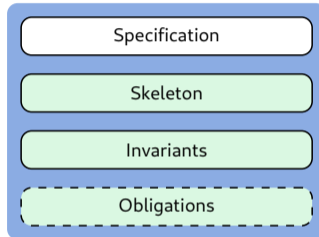
Old Program

Skeleton Generation

Invariant Synthesis

Invariant Testing

New Proof



# Sisyphus

Old Proof



New Program

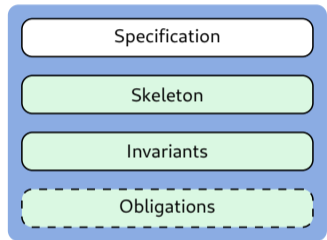
Old Program

Skeleton Generation

Invariant Synthesis

Invariant Testing

New Proof

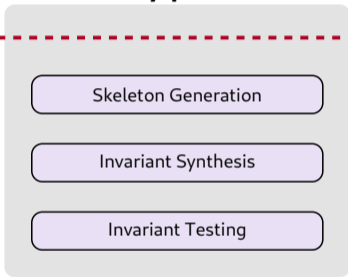
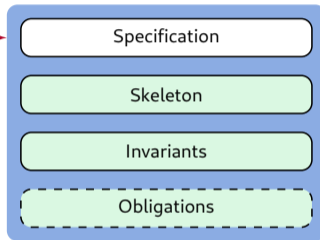


# Sisyphus

Old Proof



New Proof

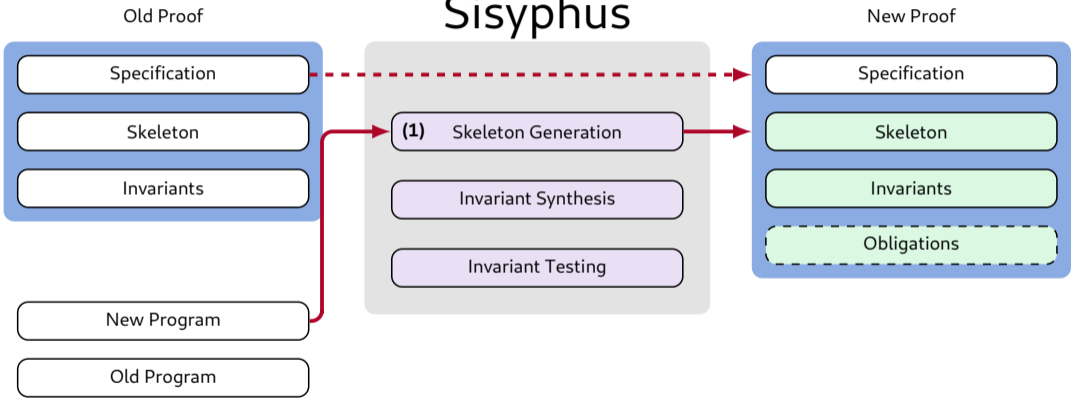


New Program

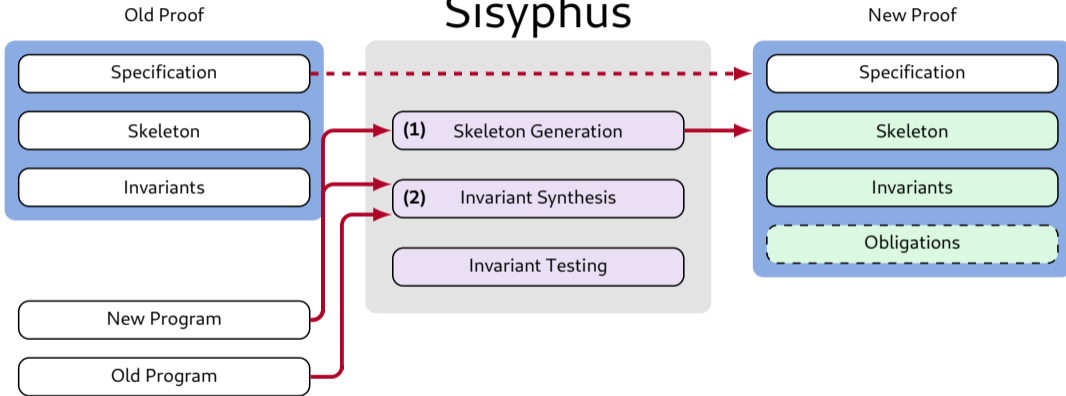
Old Program



# Sisyphus

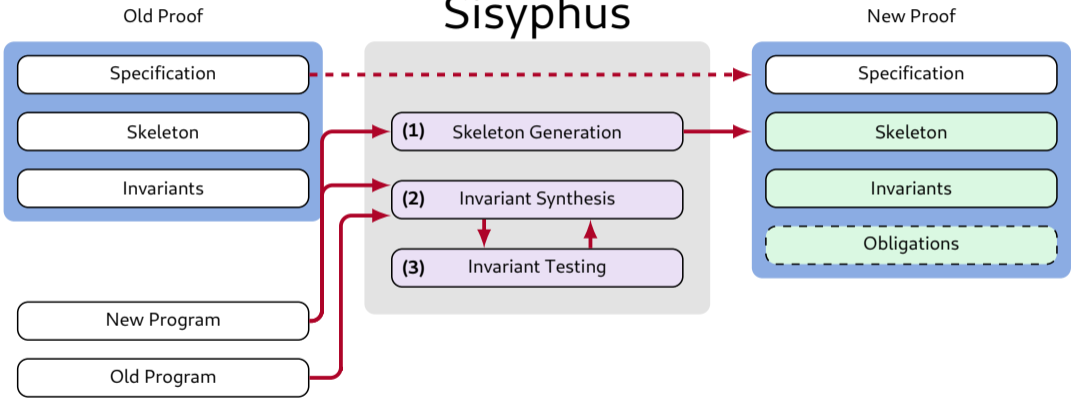


# Sisyphus

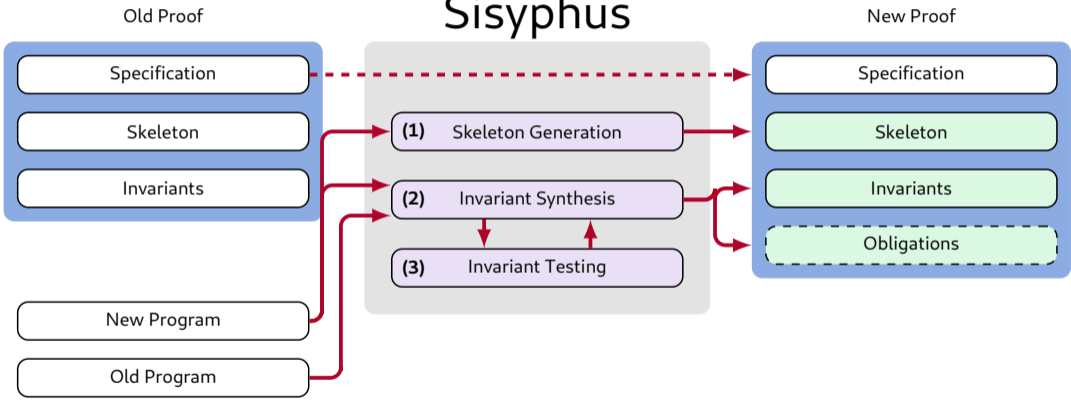




# Sisyphus



# Sisyphus



# Outline

- 1 Motivation
- 2 Key Challenges & Solutions
- 3 Evaluation

# Outline

- 1 Motivation
- 2 Key Challenges & Solutions
- 3 Evaluation**

# Pragmatic Concerns

1 Is Sisyphus effective at repairing proofs?

2 Does Sisyphus repair proofs in reasonable time?




# Benchmark Programs

- 14 OCaml programs and their changes
- 10 from real-world OCaml codebases
- ...such as containers or Jane Street's core

# Benchmark Programs

 [c-cube / ocaml-containers](#)

 440 stars

-  14 OCaml programs and their changes
-  10 from real-world OCaml codebases
-  ...such as containers or Jane Street's core

# Benchmark Programs

 14 OCaml programs and their changes


 10 from real-world OCaml codebases

 ...such as containers or Jane Street's core

 [c-cube / ocaml-containers](#)

 440 stars

 [janestreet / core](#)

 1k stars



# Benchmark Programs

Example	Data Structure	Refactoring
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tree	IterOrd, DataStr
Array exists	Array	Mutable/Pure
Array find mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
Sll partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

# Benchmark Programs

Example	Data Structure	Refactoring
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tree	IterOrd, DataStr
Array exists	Array	Mutable/Pure
Array find mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
Sll partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

# Benchmark Programs

## Array

Example	Structure	Refactoring
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tree	IterOrd, DataStr
Array exists	Array	Mutable/Pure
Array find mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
Sll partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

# Benchmark Programs

## Array

Example	Structure	Refactoring
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tree	IterOrd, DataStr
Seq lists	Array	Mutable/Pure
Seq and mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
Sll partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

## Seq

# Benchmark Programs

Example	Array	Structure	Refactoring	Stack
Seq to array		Array, Seq	IterOrd, DataStr	
Make rev list <sup>†</sup>		Ref	Mutable/Pure	
Tree to array <sup>†</sup>		Array, Tree	IterOrd, DataStr	
Seq lists	Seq	Array	Mutable/Pure	
Seq and mapi		Array, Ref	Pure/Mutable	
Array is sorted		Array	Pure/Mutable	
Array findi		Array	Pure/Mutable	
Array of rev list		Array	DataStr	
Array foldi		Array	Pure/Mutable	
Array partition		Array	DataStr	
Stack filter <sup>†</sup>		Stack	DataStr	
Stack reverse <sup>†</sup>		Stack	DataStr	
Sll partition <sup>†</sup>		SLL	Mutable/Pure, IterOrd	
Sll of array <sup>†</sup>		Array, SLL	IterOrd	

# Benchmark Programs

Example	Structure	Refactoring
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tree	IterOrd, DataStr
Seq lists	Array	Mutable/Pure
Seq and mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
SLL partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

Array

Stack

Seq

SLL

# Benchmark Programs

Example	Array	Structure	Refactoring	Stack	SLL
Seq to array		Array, Seq	IterOrd, DataStr		
Make rev list <sup>†</sup>		Ref	Mutable/Pure		
Tree to array <sup>†</sup>		Array, Tree	IterOrd, DataStr		
Seq lists	Seq	Array	Mutable/Pure		
Seq and mapi		Array, Ref	Pure/Mutable		
Array is sorted		Array	Pure/Mutable		
Array findi		Array	Pure/Mutable		
Array of rev list		Array	DataStr		
Array foldi		Array	Pure/Mutable		
Array partition		Array	DataStr		
Stack filter <sup>†</sup>		Stack	DataStr		
Stack reverse <sup>†</sup>		Stack	DataStr		
Sll partition <sup>†</sup>		SLL	Mutable/Pure, IterOrd		
Sll of array <sup>†</sup>		Array, SLL	IterOrd		

# Benchmark Programs

Example	Structure	Refactoring
Tree	Array	Stack
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tr	DataStr
Seq lists	Array	Mutable/Pure
Seq and mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
SLL partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd



# Benchmark Programs

Example	Structure	Refactoring
Tree	Array	Stack
Seq to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tr	DataStr
Seq lists	Array	Mutable/Pure
Seq and mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array foldi	Array	Pure/Mutable
Array partition	Array	DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
SLL partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Sll of array <sup>†</sup>	Array, SLL	IterOrd

Queue

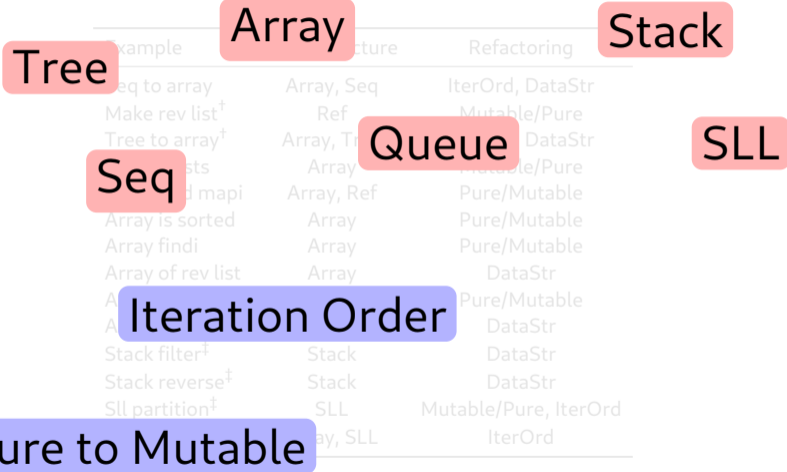
SLL

# Benchmark Programs

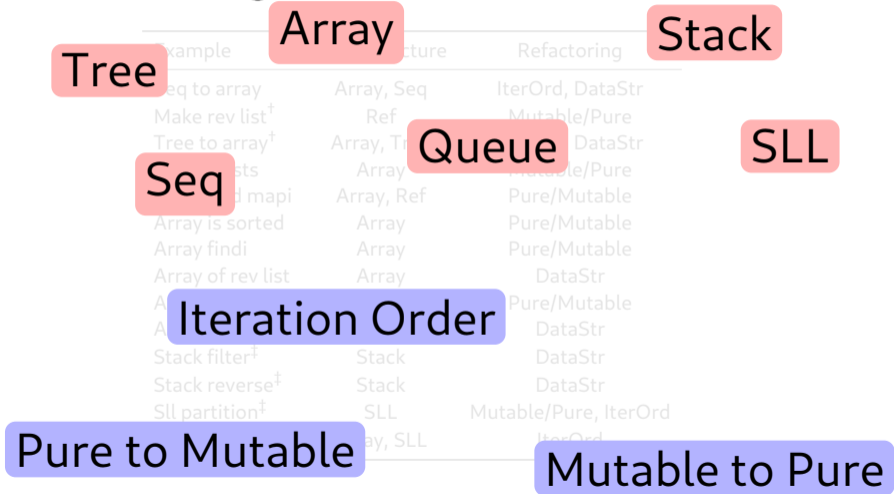
Example	Array	Structure	Refactoring	Stack	SLL
Seq to array		Array, Seq	IterOrd, DataStr		
Make rev list <sup>†</sup>		Ref	Mutable/Pure		
Tree to array <sup>†</sup>		Array, Tr	DataStr		
Seq lists	Seq	Array	Mutable/Pure		
Seq and mapi		Array, Ref	Pure/Mutable		
Array is sorted		Array	Pure/Mutable		
Array findi		Array	Pure/Mutable		
Array of rev list		Array	DataStr		
A			Pure/Mutable		
A			DataStr		
Iteration Order					
Stack filter <sup>†</sup>		Stack	DataStr		
Stack reverse <sup>†</sup>		Stack	DataStr		
Sll partition <sup>†</sup>		SLL	Mutable/Pure, IterOrd		
Sll of array <sup>†</sup>		Array, SLL	IterOrd		

# Benchmark Programs

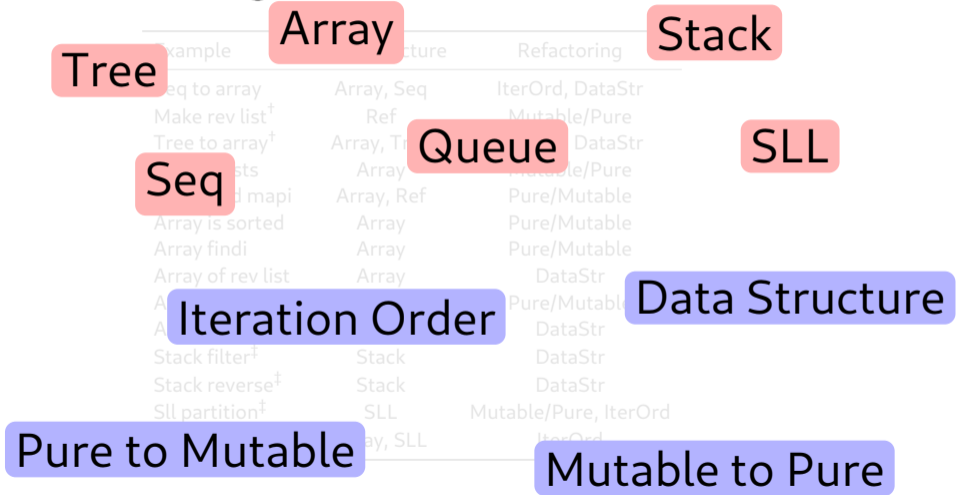
Example	Structure	Refactoring
Tree to array	Array, Seq	IterOrd, DataStr
Make rev list <sup>†</sup>	Ref	Mutable/Pure
Tree to array <sup>†</sup>	Array, Tr	DataStr
Seq lists	Array	Mutable/Pure
Seq and mapi	Array, Ref	Pure/Mutable
Array is sorted	Array	Pure/Mutable
Array findi	Array	Pure/Mutable
Array of rev list	Array	DataStr
Array		Pure/Mutable
Array		DataStr
Stack filter <sup>†</sup>	Stack	DataStr
Stack reverse <sup>†</sup>	Stack	DataStr
SLL partition <sup>†</sup>	SLL	Mutable/Pure, IterOrd
Pure to Mutable	Array, SLL	IterOrd



# Benchmark Programs



# Benchmark Programs



# RQ1: Effectiveness of proof repair

Name	# Admits / # Obligations	Time (old)	Time (new)
Seq to array	3 / 5	2hrs	17m
Make rev list	0 / 2	10m	-
Tree to array	2 / 4	5hrs	18m
Array exists	2 / 4	30m	12m
Array find mapi	2 / 5	1.5hrs	12m
Array is sorted	2 / 5	4hrs	2m
Array findi	3 / 7	1.5hrs	9m
Array of rev list	2 / 3	1hr	3m
Array foldi	0 / 1	15m	-
Array partition	3 / 3	2.5hrs	5m
Stack filter	3 / 3	1.5hrs	11m
Stack reverse	1 / 1	2hrs	30s
SLL partition	0 / 2	2hrs	-
SLL of array	0 / 1	2hrs	-

# RQ1: Effectiveness of proof repair

Name	# Admits / # Obligations	Time (old)	Time (new)
Seq to array	3 / 5	2hrs	17m
Make rev list	0 / 2	10m	-
Tree to array	2 / 4	5hrs	18m
Array exists	2 / 4	30m	12m
Array find mapi	2 / 5	1.5hrs	12m
Array is sorted	2 / 5	4hrs	2m
Array findi	3 / 7	1.5hrs	9m
Array of rev list	2 / 3	1hr	3m
Array foldi	0 / 1	15m	-
Array partition	3 / 3	2.5hrs	5m
Stack filter	3 / 3	1.5hrs	11m
Stack reverse	1 / 1	2hrs	30s
SLL partition	0 / 2	2hrs	-
SLL of array	0 / 1	2hrs	-

# RQ1: Effectiveness of proof repair

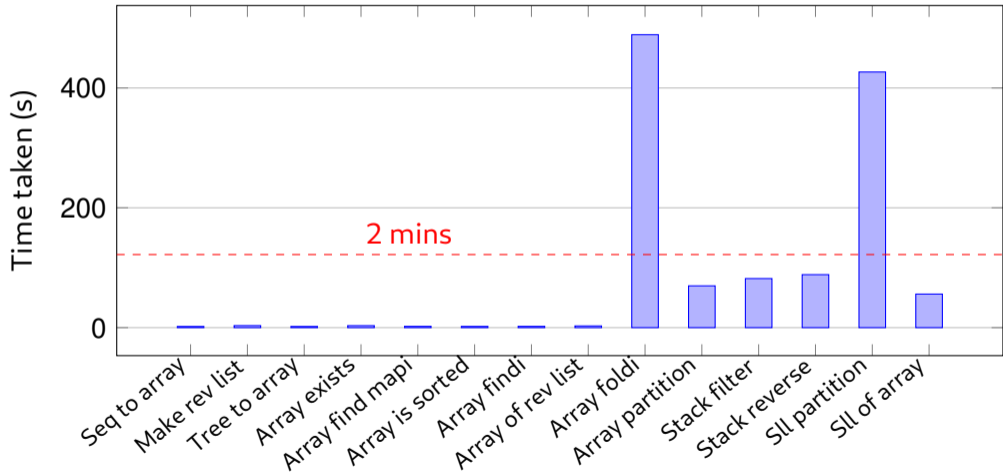
Name	# Admits / # Obligations	Time (old)	Time (new)
Seq to array	3 / 5	2hrs	17m
Make rev list	0 / 2	10m	-
Tree to array	2 / 4	5hrs	18m
Array exists	2 / 4	30m	12m
Array find mapi	2 / 5	1.5hrs	12m
Array is sorted	2 / 5	4hrs	2m
Array findi	3 / 7	1.5hrs	9m
Array of rev list	2 / 3	1hr	3m
Array foldi	0 / 1	15m	-
Array partition	3 / 3	2.5hrs	5m
Stack filter	3 / 3	1.5hrs	11m
Stack reverse	1 / 1	2hrs	30s
SLL partition	0 / 2	2hrs	-
SLL of array	0 / 1	2hrs	-



# RQ1: Effectiveness of proof repair

Name	# Admits / # Obligations	Time (old)	Time (new)
Seq to array	3 / 5	2hrs	17m
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Stack reverse	1 / 1	2hrs	30s
SLL partition	0 / 2	2hrs	-
SLL of array	0 / 1	2hrs	-

# RQ2: Efficiency of proof repair



## RQ2: Efficiency of proof repair

Example	Time (s)				Total (s)
	Generation	Extraction	Testing	Remaining	
seq_to_array	28.57	1.95	20.36	5.28	58
make_rev_list	$\leq 10ms$	3.36	$\leq 10ms$	11.95	15
tree_to_array	6.75	1.95	2.98	13.32	25
array_exists	$\leq 10ms$	3.30	$\leq 10ms$	13.23	17
array_find_map	$\leq 10ms$	2.13	$\leq 10ms$	13.95	17
array_is_sorted	$\leq 10ms$	2.04	$\leq 10ms$	15.38	18
array_findi	$\leq 10ms$	2.13	$\leq 10ms$	19.07	22
array_of_rev_list	1.72	2.82	0.96	15.62	21
array_fold	$\leq 10ms$	488.89	$\leq 10ms$	15.00	504
array_partition	3.51	69.73	2.62	17.53	95
stack_filter	$\leq 10ms$	81.88	$\leq 10ms$	21.53	104
stack_reverse	$\leq 10ms$	88.42	$\leq 10ms$	16.94	105
sll_partition	$\leq 10ms$	426.62	$\leq 10ms$	16.43	443
sll_of_array	0.02	55.98	0.01	13.33	69

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- 1 Building blocks for *new proof* found in *old proof*
- 2 Picking a correct invariant is hard!
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# Backup Slides

## RQ3: Failure Modes

- Repair assumes components from old proof are sufficient for new one.
- Quality of repair degrades when this fails to hold.

e.g. `array_partition`'s pure obligations required fact

$$\text{filter } p (\text{filter } p \ell) = \text{filter } p \ell$$

not present in original proof.

## RQ3: Failure Modes

```
let to_array s =  
  let batches = (* .. *) in  
  let res =  
    Array.make (* .. *) in  
  List.iter (fun batch ->  
    let dst = (* .. *) in  
    Array.copy batch res dst)  
  batches;  
res
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# RQ3: Failure Modes

Invariant requires *flattening* operation...

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