Heat Transfer Enhancement by Fins in the Microscale Regime

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The current literature contains many studies of microchannel and micro-pin-fin heat exchangers, but none of them consider the size effect on the thermal conductivity of channel and fin walls. The present study analyzes the effect of size (i.e., the microscale effect) on the microfin performance, particularly in the cryogenic regime where the microscale effect is often appreciable. The size effect reduces the thermal conductivity of microchannel and microfin walls and thus reduces the heat transfer rate. For this reason, heat transfer enhancement by microfins becomes even more important than for macroscale fins. The need for better understanding of heat transfer enhancement by microfins motivates the current study, which resolves three basic issues. First, it is found that the heat flow choking can occur even in the case of simple plate fins or pin fins in the microscale regime, although choking is usually caused by the accommodation of a cluster of fins at the fin tip. Second, this paper shows that the use of micro-plate-fin arrays yields a higher heat transfer enhancement ratio than the use of the micro-pin-fin arrays due to the stronger reduction of thermal conductivity in micro-pin-fins. The third issue is how the size effect influences the fin thickness optimization. For convenience in design applications, an equation for the optimum fin thickness is established which generalizes the case without the size effect as first reported by Tuckerman and Pease.

Introduction

Fins and microchannels are widely used to enhance heat transfer in heat exchangers. The advent of high-density electronic components has resulted in high energy dissipation and requires more effective heat transfer enhancement techniques. The pioneering work of Tuckerman and Pease (1981) provided a method for cooling a chip by forcing coolant through closed microchannels etched onto the backside of a silicon wafer. Subsequently, many theoretical and experimental investigations of microchannel and micro-pin-fin heat exchangers have been conducted. Some works focused on how to optimize arrays of fins (Harpole and Eninger, 1991; Knight et al., 1992). Silicon and CVD diamond were employed or proposed due to the high thermal conductivity to make microchannel heat exchangers, such as for laser diode array cooling (Mundinger et al., 1988; Missaggia et al., 1989; Goodson et al., 1997). To meet the requirement of high cooling rate, water (Tuckerman and Pease, 1981) and liquid nitrogen (Choi et al., 1992; Cha et al., 1993; Riddle and Bernhardt, 1992) were used as the working fluids. Numerical analyses of the conjugate heat transfer in the silicon micro-pin-fin arrays and microchannels operating with liquid nitrogen were reported by Yin and Bau (1997a, b).

It should be noted that none of the above works consider the size effect (Flik and Tien, 1990) on the thermal conductivity of channel and fin walls. All of the analyses use the conventional macroscale approach in which only the thermal conductivity for bulk materials is used. The size effect is a phenomenon in which the thermal conductivity of a material is less than the bulk value due to the scattering of the primary carriers of energy by its boundaries. This effect is important for systems that are very small or are at low temperatures. The microscale approach may be needed to study the performance of microchannels and microfins because widths as narrow as 10 μm (Harpole and Eninger, 1991; Cha et al., 1993; Joo et al., 1995) are practical today and cooling at cryogenic temperatures is required for the operations of complementary metal-oxide-semiconductor (CMOS) devices (Yin and Bau, 1997a, b) and superconducting magnets (Cha et al., 1993).

This paper studies how to effectively enhance the heat transfer by fins in the microscale regime, despite the size effect-induced reduction of the thermal conductivity of microchannel and fin walls. This study resolves three basic issues: (1) heat flow choking in microfins, (2) the effect of size on the heat transfer enhancement ratio, and (3) the effect of size on the fin thickness (or channel wall thickness in microchannel heat exchangers) optimization.

Heat Flow Choking in Microfins

This issue comes from the question: "Is there an upper limit for the heat transfer enhancement?" A quantitative review of some typical existing works will be helpful to answer the question. Tuckerman and Pease (1981) were able to support a chip heat flux up to 790 W/cm^2, which corresponds to a convective heat transfer coefficient of about $h = 4 \times 10^4$ W/(m^2-K), by using the microchannel shown in Fig. 1(*a*) and single phase water cooling. The channel wall width, t_w, ranges from 44 to 57 μm. Copeland (1996) achieved $h = 2.4 - 49.3$ kW/(m^2-K) for single-phase FC-72 jet impingement cooling of pin fin arrays, and the critical heat flux was about 45–395 kW/(m^2-K) for boiling cooling. The size of the smallest copper fin was 0.1 mm. Riddle and Bernhardt (1992) used liquid nitrogen as the working fluid in a heat sink consisting of 50 μm wide and 800 μm deep channels. A thermal resistance as low as 4.6×10^{-6} K/(W/m^2) was achieved.

Considering a plate fin as shown in Fig. 1(*b*), an estimation of related parameters from existing experimental works is as follows:

$$h = 10^2 - 10^5 \text{ W/(m}^2\text{-K)}$$

$$H = 10^{-4} - 10^{-3} \text{ m}$$

$$k_w = 10^2 - 10^4 \text{ W/(m-K)}$$

where h is the convective heat transfer coefficient, H is the fin height, and k_w is the thermal conductivity of the fin. The heat transfer rate by conduction through the fin base is estimated to be $O[k_w t_w L(T_0 - T_L)/H)$ where T_0 and T_L are the temperatures of fin base and fin tip, respectively. The convection heat transfer rate along the fin surface is $O\{2hHL[(T_0 + T_L)/2 - T_f]\}$. Approx-

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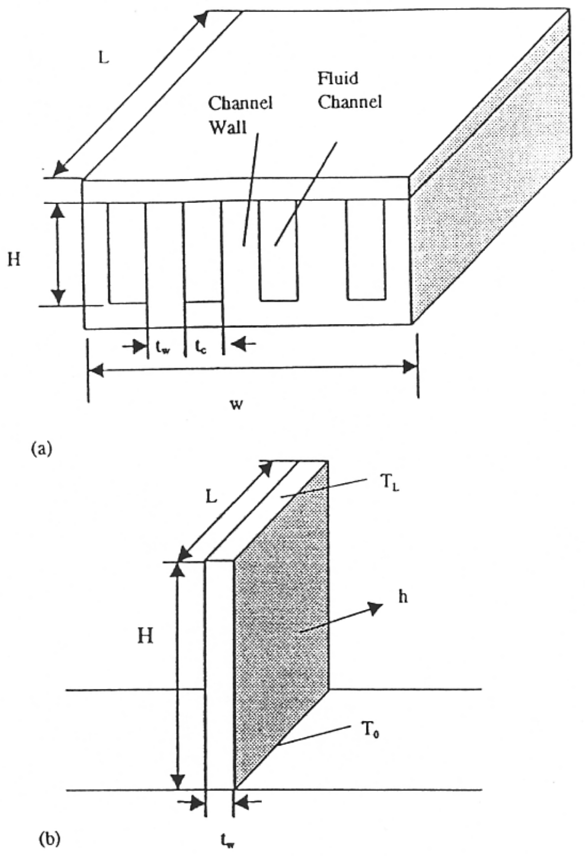

Fig. 1 Schematic of (*a*) a microchannel heat sink and (*b*) a plate fin

imations adopted in this order of magnitude estimation are: the temperature distribution from fin base to fin tip is linear, the temperature of fin tip T_L approaches the temperature of fluid T_f, and h is constant along the fin surfaces. To prevent heat flow choking (Kraus and Bar-Cohen, 1985) of the fin, the amount of heat that can be conducted through the fin base, must exceed the amount of convection along the fin surfaces. Otherwise, the performance of the fin may become limited by the heat flow choking. Therefore, the following equation must be satisfied:

$$O[k_w t_w L(T_0 - T_f)/H] > O[hHL(T_0 - T_f)]. \tag{1}$$

The fin thickness to prevent choking is thus

$$t_{wc} > O(hH^2/k_w) = O[(hH/k_w)H] = O(\mathrm{Bi}\ H), \tag{2}$$

where Bi $(= hH/k_w)$ is the Biot number. From the foregoing estimations, Bi is found to range from 1 to 10^{-6}.

The cases of Bi = 0.14, 0.1, and 0.05 are taken as examples, which may correspond to water cooling ($h = 5.36 \times 10^4$, 3.83×10^4 and 1.91×10^4 W/(m^2-K)) of a silicon fin ($k_w = 1.34 \times 10^2$ W/(m-K) at 325 K) with fin height $H = 350\ \mu$m. The aforementioned cases are those that might occur in Tuckerman and Pease (1981). Then, Eq. (2) becomes

$$t_{wc} > O(49\ \mu\mathrm{m}) \quad \text{for} \quad \mathrm{Bi} = 0.14 \quad \text{and} \quad H = 350\ \mu\mathrm{m}. \tag{3}$$

A regime map for the cases is shown in Fig. 2. Above the horizontal lines at 49, 35, and 17.5 μm, for the cases of Bi = 0.14, 0.1, and 0.05 and $H = 350\ \mu$m, choking will not occur in plate fins. The curved lines, which delineate the regimes where the size effect on thermal conduction in diamond, silicon, and copper should be considered, are from the results of Flik et al. (1992). It should be noted that the lines are not obtained from the bulk mean free path, λ_b, but from seven times the bulk mean free path, $7\lambda_b$. At temperatures and fin thicknesses below these lines, thermal conductivity will be reduced due to the size effect. This reduction of thermal conductivity will make Bi even higher than that with same h and H but without the size effect. The ranges of fin thickness and working temperature in the work of Tuckerman and Pease (1981) and many other similar works are shown in Fig. 2. Yin and Bau (1997a, b) and Krane et al. (1990) reported that the operation of CMOS devices at cryogenic temperatures, such as those obtained by using neon (24.5–44.5 K) and nitrogen (63–77 K), provides a number of significant benefits such as lower electrical resistance of the conductors, lower leakage currents between conductors, and lower component degradation. The case of working temperature 63–77 K and fin thickness 50 μm was studied in the work of Yin and Bau (1997b). A microchannel heat sink with channel wall thickness 10 μm was proposed by Cha et al. (1993) for liquid nitrogen cooling of superconducting magnets. Figure 2

Nomenclature

A = fin base area, m^2
A_c = cross-sectional area of rod, m^2
Bi = Biot number, hH/k_w
c_p = specific heat at constant pressure, J/(kg-K)
d = diameter of pin fin, m
D_h = hydraulic diameter of fluid channel, m
f = flow rate, m^3/s
H = height of fin or microchannel wall, m
h = convective heat transfer coefficient, W/(m^2-K)
k = thermal conductivity, W/(m-K)
L = longitudinal length of fin or microchannel, m
mL = dimensionless parameter for fin, $(hPH^2/k_w A)^{0.5}$
N = dimensionless parameter related to fin efficiency, $(2h/k_w t_w)^{0.5}H$
n = exponent of thermal conductivity reduction in Eq. (18)
Nu = Nusselt number, hD_h/k_f
P = perimeter of fin, m
p = probability of diffuse phonon scattering at boundary
Q = heat flow, W
R = thermal resistance, K/W or K/(W/m^2)
T = temperature, K
t = thickness of fin or microchannel wall, m
w = width of microchannel substrate, m
α = surface multiplication factor, $2H/(t_c + t_w)$
δ = dimensionless plate fin thickness, t_w/λ_b
δ_d = dimensionless pin fin diameter, d/λ_b
η = fin efficiency
λ = mean free path, m
λ_c = mean free path for pure diffuse reflection, m
ρ = fluid density, kg/m^3

Subscripts

0 = fin base
b = quantity for bulk materials
c = convection by the fluid or fluid channel
d = pin fin with diameter d
f = fluid
h = heating of fluid
L = fin tip
r = reference state
t = plate fin with thickness t_w
w = wall of fin or microchannel
wc = wall thickness of fin to prevent choking
wo = optimum wall thickness for microchannel
z = longitudinal direction

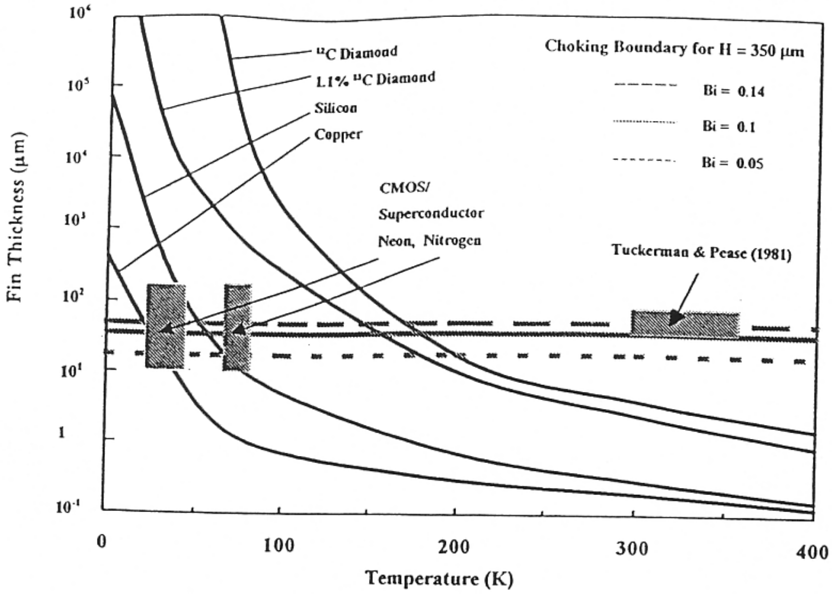

Fig. 2 Regime map for fin heat flow choking and thermal conductivity size effect for diamond, silicon, and copper films (Flik and Tien, 1990)

shows that there is no need to consider the size effect for fins at room temperature. For the nitrogen and neon cooling of CMOS devices and superconducting magnets, however, the consideration of the size effect is absolutely needed.

Figure 2 also shows that the ranges of fin thickness and working temperature in the works of Tuckerman and Pease (1981) and for CMOS/superconducting magnet microchannel cooling are below the choking line for Bi = 0.14 and $H = 350$ µm. Of course, the result of Eq. (3) is only an order of magnitude estimation. The utilization of fin thicknesses of $t_w < 49$ µm may not necessarily induce heat flow choking under the condition of Bi = 0.14 and $H = 350$ µm, but a more careful analysis is required to make a definitive determination. It is worth noting that although choking has usually been found to be caused by the accommodation of a cluster of fins at the fin tip (Kraus and Bar-Cohen, 1995), in the microscale regime, choking may occur even in the cases of simple plate and pin fins (without clusters).

For the case of small Biot number, Bi $= 10^{-5}$, which corresponds to air cooling ($h = 10^2$ W/(m^2-K)) of a metal fin ($k_w = 10^3$ W/(m-K) at room temperature) of height $H = 100$ µm, Eq. (2) becomes

$$t_{wc} > O(2 \times 10^{-9}\ \text{m}) = O(2\ \text{nm}). \qquad (4)$$

This shows that there will be no choking under the condition of small Biot number, regardless of fin thickness.

The Effect of Size on the Heat Transfer Enhancement Ratio

This second section studies how small size influences the effectiveness of heat transfer enhancement by fins. The heat transfer rate Q through the fin base can be found in standard heat transfer texts,

$$Q = k_w A(T_0 - T_f)(hPH^2/k_w A)^{0.5} \tanh mL$$
$$= (T_0 - T_f)(hPH^2 k_w A)^{0.5} \tanh mL, \qquad (5)$$

where T_0 and T_f are the temperatures of fin base and fluid, respectively, $mL = (hPH^2/k_w A)^{0.5}$ is a dimensionless parameter for the fin, P is the perimeter of the fin, and A is area of the fin base. The value of mL is not too much greater than unity in a well-designed fin (Lienhard, 1981). To make a reasonable comparison, the fin number is increased but the total fin base area and fin height are kept fixed. For pin fins, this means that if the fin number increases four times, the diameter of each fin decreases to half of the original one. The convective heat transfer coefficient h is assumed to be unchanged. Then, the heat transfer enhancement ratio can be obtained for pin and plate fins.

Pin Fins. For pin fins, the heat transfer enhancement ratio can be derived as

$$Q_d/Q_r = (k_d/k_r)^{0.5}(d_r/d)^{0.5}$$
$$\times \tanh[(mL_r)(d/d_r)(k_d/k_r)^{0.5}]/\tanh mL_r, \quad (6)$$

where the subscript "d" denotes the quantities or properties of pin fin with diameter d, and the subscript "r" denotes a reference state which may or may not be in the size effect regime. Concerning the phonon transport along an infinite rod, Ziman (1960) showed that the phonon mean free path, λ, limited by diffuse surface scattering is

$$\lambda = [(2 - p)/p]\lambda_c, \qquad (7)$$

where p is the probability of diffuse phonon scattering at boundaries, and λ_c is the mean free path for pure diffuse reflection given as $\lambda_c = 1.12(A_c)^{0.5}$, where A_c is the cross-sectional area of the rod. The value of p depends on the ratio of the phonon wavelength to the characteristic roughness of the surface, but is close to unity for most engineering surfaces at moderate temperatures. Assuming that p is also unity for microfin operation at moderate temperatures, the size effect on thermal conductivity for pin fins can be expressed as $k_d/k_b = \delta_d = d/\lambda_b$, where k_d and k_b are the thermal conductivities for the pin fin with diameter d and for the bulk material. This linear reduction of thermal conductivity with size, however, is only for the case $\delta_d \ll 1$ (Tien et al., 1969). A simple approximate relation: $(k_d/k_b) = \delta_d/(1 + \delta_d)$, which holds for $p = 1$ and all values of δ_d, has been suggested by Nordheim (1934). It shows that k_d/k_b approaches unity for $\delta_d \gg 1$ and becomes a linear reduction with d for $\delta_d \ll 1$. The variation of heat transfer enhancement ratio Q_d/Q_r with diameter ratio d/d_r is shown in Fig. 3 for $mL_r = 1$ and 2 for the cases with and without size effect. The set of curves for $k_d/k_r = 1$ is for the cases without size effect. The reference state for the consideration of size effect shown in Fig. 3 is $d_r = \lambda_b$. For this reference state, $k_r/k_b = 0.5$ and the thermal conductivity ratio k_d/k_r for the cases with size effect is

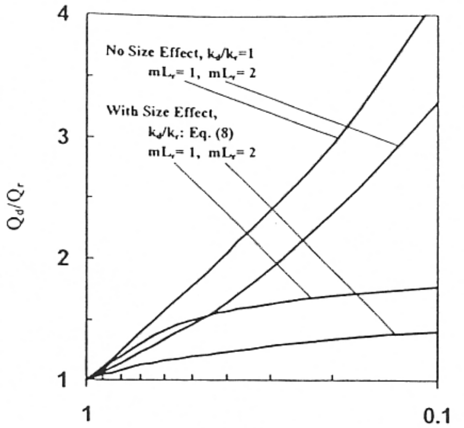

Fig. 3 Pin fin heat transfer enhancement ratio versus diameter ratio

$$(k_d/k_r) = 2(d/d_r)/[1 + (d/d_r)]. \quad (8)$$

A typical reference case for the pin fin is liquid cooling (h = 2.5×10^4 W/(m^2-K)) of a CVD diamond fin ($k_w = 10^3$ W/(m-K) around 70 K) of height $H = 2 \times 10^{-3}$ m and diameter d_r = 400 μm. For this case, $mL_r = 1$. The results for $mL_r = 1$ and 2 with $k_d/k_r = 1$ in Fig. 3 shows that heat transfer will be significantly enhanced by increasing the fin number but keeping the fin base area fixed if the size effect is not accounted for. The results for $mL_r = 1$ and 2 with k_d/k_r according to Eq. (8) are depicted in Fig. 3 and show that the heat transfer enhancement ratio will be significantly reduced and will rapidly reach a saturated value due to the size effect. From the foregoing comparison, one finds that if fins are not in the size effect regime, there is significant heat transfer enhancement by increasing the number of fins while keeping total fin base area fixed. If fins are in the size effect regime, there is only a small enhancement.

Plate Fins. For plate fins, the heat transfer enhancement ratio can be derived as

$$Q_t/Q_r = (k_t/k_r)^{0.5}(t_r/t)^{0.5}$$
$$\times \tanh[(mL_r)(t/t_r)^{0.5}(k_t/k_r)^{0.5}]/\tanh mL_r \quad (9)$$

where the subscript "t" denotes the quantities and properties of the plate fin with fin thickness t_w. Again, the subscript "r" denotes a reference state, which may or may not be in the size effect regime. The match solution for the phonon mean free path along the film (Flik and Tien, 1990) is

$$\lambda_z/\lambda_b = 0.5 - (\cos^{-1}\delta)/\pi - (1 - S^3)/(3\delta\pi)$$
$$+ \delta[1 + \exp(-6\delta)]\ln[(1 + \delta + S)/(1 + \delta - S)]/\pi, \quad (10)$$

where λ_z is the mean free path in the longitudinal direction, $\delta = t_w/\lambda_b$ is the dimensionless plate fin thickness, and $S = (1 - \delta^2)^{0.5}$. The solution derived from uniform origination (Flik and Tien, 1990) is

$$\lambda_z/\lambda_b = 0.5 - (\cos^{-1}\delta)/\pi - (1 - S^3)/(3\delta\pi)$$
$$+ \delta\ln[(1 + \delta + S)/(1 + \delta - S)]/\pi, \quad (11)$$

and the reduction of the thermal conductivity (Flik and Tien, 1990) is $k_t/k_b = \lambda_z/(\lambda_b/2)$. The variation of heat transfer enhancement ratio Q_t/Q_r with plate fin thickness ratio t_w/t_r is shown in Fig. 4 for $mL_r = 1$ and 2 for the cases with and without size effect. The set of curves for $k_t/k_r = 1$ is for the cases without size effect. The reference state for the consideration of size effect shown in Fig. 4 is $t_r = \lambda_b$. For this reference state $k_r/k_b = 0.7878$, and the thermal conductivity ratio is

$$k_t/k_r = 2.539\lambda_z/\lambda_b. \quad (12)$$

Figure 4 shows that the reduction of the heat transfer enhancement ratio due to the size effect is much smaller for plate fins than for pin fins. This is caused by the fact that the reduction of thermal conductivity for plate fins described by Eqs. (10)–(12) is smaller than that for pin fins described by Eq. (8). This result shows that, in the microscale regime, the shape effect should be considered in the design of micro heat exchangers as well as the size effect. This result also suggests that plate fins are more desirable than pin fins for heat transfer enhancement.

The Effect of Size on the Optimization of Plate Fin Thickness

Several works on microchannel thickness optimization have been reported by Tuckerman and Pease (1981) and many other investigators. Most results from later investigators compare favorably with the original results of Tuckerman and Pease (1981), although a few studies that do not closely agree. None of these studies, however, consider the size effect. The present analysis of fin thickness optimization basically follows that of Tuckerman and Pease (1981), but includes the size effect. Therefore, it will be described only briefly. Considering the microchannel heat sink shown in Fig. 1(a), the thermal resistance, $R = \Delta T/Q$ where ΔT is the temperature difference, can be divided into the resistances due to convection from the heat sink to the coolant fluid, R_c, and due to heating of the fluid as it absorbs energy passing through the heat exchanger, R_h:

$$R_c = 1/(\alpha h L w) = D_h/(\alpha \text{ Nu } k_f L w), \quad \text{and} \quad (13)$$

$$R_h = 1/(\rho c_p f) = [24\mu L/(\rho c_p w \Delta P)](\alpha^{-1} t_c^{-3}). \quad (14)$$

Here $\alpha = 2H/(t_c + t_w)$ is the surface multiplication factor, L is the channel length in the longitudinal direction, w is the width of the microchannel substrate, Nu $= hD_h/k_f$ is the Nusselt number, D_h is the hydraulic diameter of the flow channel, k_f is the thermal conductivity of the fluid, c_p is the heat capacity of the fluid, f is the flow rate, and ΔP is the pressure drop. To account for the fin efficiency η, Eq. (13) becomes

$$R_c = D_h/(\alpha \text{ Nu } k_f L w \eta) = 2t_c/(\alpha \text{ Nu } k_f L w \eta). \quad (15)$$

Approximating D_h as $2t_c$ for a high-aspect ratio channel, the fin efficiency can be obtained by assuming that the heat flow in the channel wall is one-dimensional,

$$\eta = \tanh N/N, \quad (16)$$

where

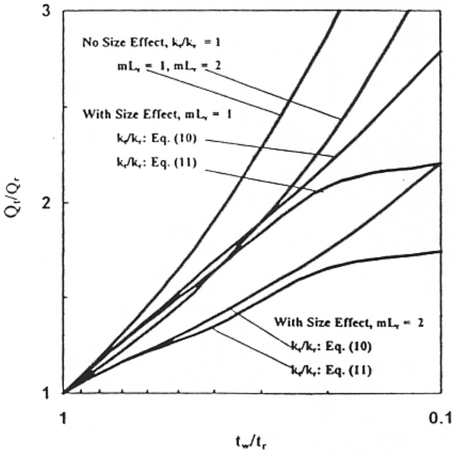

Fig. 4 Plate fin heat transfer enhancement ratio versus fin thickness ratio

$$N = (2h/k_w t_w)^{0.5} H = (\mathrm{Nu}\, k_f/k_w t_c t_w)^{0.5} \alpha(t_c + t_w)/2. \quad (17)$$

From Eqs. (14) and (15), one can see that for any t_c and α, the thermal resistance can be minimized by maximizing η. Since η is a monotonically decreasing function of N, maximizing η can be obtained by minimizing N.

The size effect on the thermal conductivity of plate fin walls is approximated by a power law of fin thickness ratio,

$$k_w = k_b (t_w/\lambda_b)^n. \quad (18)$$

Minimizing N can be obtained from

$$dN/dt_w = d[k_b(t_w/\lambda_b)^n]^{0.5}(t_c t_w)^{0.5}/[\alpha(t_c + t_w)]/dt_w = 0. \quad (19)$$

Then the optimum fin thickness can be obtained as

$$t_{wo} = (0.5 + 0.5n)t_c/(0.5 - 0.5n). \quad (20)$$

The optimum fin thickness can also be obtained by holding H instead of α constant. For this case, it is important to note that an identical result to that of Eq. (20) will be obtained. The result $t_{wo} = t_c$, which was found by Tuckerman and Pease (1981), corresponds to the result without size effect: $n = 0$ in the present result. If k_w linearly decreases with t_w, such as the behavior of pin fins for $\delta_d \ll 1$, $n = 1$ is another special case: a limiting case. Equation (20) shows that there is no realistic optimum value for t_{wo} under the condition of $n \geq 1$. This is illustrated in Fig. 6. For $n = 0.5$, Eq. (20) indicates that $t_{wo} = 3t_c$.

Two methods can be used to calculate the value of n. The first method is to fit the curve of the following effective thermal conductivity variation (Flik and Tien, 1990),

$\underline{\delta > 1 (t_w > \lambda_b)}$

$$k_w/k_b = 1 - 2/(3\delta\pi) \quad (21)$$

$\underline{\delta \leq 1 (t_w \leq \lambda_b)}$

$$k_w/k_b = 1 - 2(\cos^{-1}\delta)/\pi - 2(1 - S^3)/(3\delta\pi)$$
$$+ 2\delta[1 + \exp(-6\delta)] \ln[(1 + \delta + S)/(1 + \delta - S)]/\pi. \quad (22)$$

The results are shown in Fig. 5. One can see that the three segments $n = 0.0352$ for $4 \leq \delta(= t_w/\lambda_b) \leq 10$, $n = 0.133$ for $1 \leq \delta \leq 4$, and $n = 0.35$ for $0.1 \leq \delta \leq 1$ can fit the curve. This result might be easier to use than the second method but is less accurate.

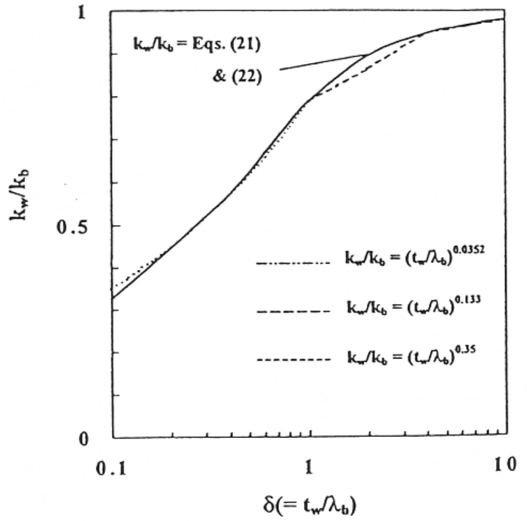

Fig. 5 Curve fitting of the thermal conductivity ratio by the power law of the dimensionless plate fin thickness δ

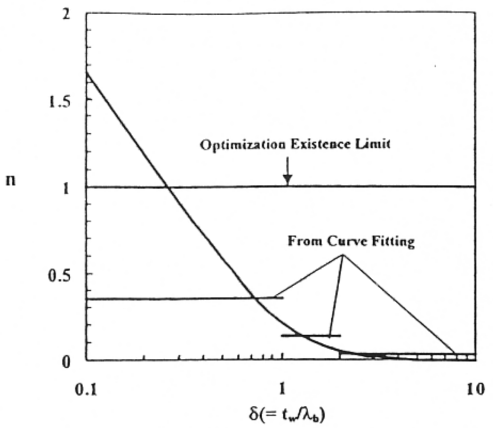

Fig. 6 Variation of the power-law exponent n with the dimensionless plate fin thickness δ

In the second method, by differentiating Eqs. (21) and (22) with respect to δ, one can obtain the n variation as follows:

$\underline{\delta > 1 (t_w > \lambda_b)}$

$$n = 2/(3\delta^2\pi). \quad (23)$$

$\underline{\delta \leq 1 (t_w \leq \lambda_b)}$

$$n = 2/(\pi S) + 2(1 - S^3)/(3\delta^2\pi) - 2S/\pi$$
$$+ 2[1 + \exp(-6\delta)] \ln[(1 + \delta + S)/(1 + \delta S)]/\pi$$
$$- 12\delta \exp(6\delta) \ln[(1 + \delta + S)/(1 + \delta - S)]/\pi$$
$$+ 2\delta[1 + \exp(-6\delta)]\{[1 - (\delta/S)]/(1 + \delta + S)$$
$$- [1 + (\delta/S)]/(1 + \delta - S)\}/\pi. \quad (24)$$

The following results can be compared with those of the first method: $n = 0.0021$ for $\delta = 10$, $n = 0.012$ for $\delta = 4$, $n = 0.212$ for $\delta = 1$, $n = 0.465$ for $\delta = 0.5$, and $n = 1.656$ for $\delta = 0.1$. The last result, $n = 1.656$ for $\delta = 0.1$, means that for plate fins, there is a regime for δ in which there is no optimum fin thickness. The variation of n with δ for the aforementioned two methods is shown in Fig. 6. The results of the second method are much more accurate than the first method, especially in the regime of low δ.

It is worth noting that an iterative process is required to determine the optimum fin thickness t_{wo} in the size effect regime. Now, some discussions and quantitative examples on t_w and t_c are needed. As discussed in the Introduction, liquid nitrogen was proposed or used as a working fluid by Choi et al. (1992), Cha et al. (1993), and Riddle and Bernhardt (1992). Going back to Fig. 2, one can see that at cryogenic temperatures (below 90 K), there is no optimum fin thickness if ^{12}C diamond is used to fabricate the microchannels. This occurs because the frequently used microfin thicknesses 50–500 μm are far smaller than the bulk mean free path indicated by the curved line shown in Fig. 2, making δ very small and thus yielding $n > 1$ in Fig. 6. The fin thickness optimization may be found in certain ranges for silicon and 1.1 percent ^{13}C diamond. The selection of t_c is mainly based on the allowable pressure drop. Since the viscosity of nitrogen is about one-fifth of water, $t_c = 50$ μm as used by Tuckerman and Pease (1981) is a reasonable choice. The 1.1 percent ^{13}C diamond is chosen as the material for thickness optimization because it has higher bulk thermal conductivity than silicon at 70 K. From Fig. 2, one can find that λ_b is about 100 μm for 1.1 percent ^{13}C diamond at 70 K. The following procedures for iteration are followed.

1 Begin the iteration by assuming $t_w = t_c$, then $\delta = 0.5$ and $n = 0.464$ from Eq. (24).
2 From Eq. (20), $t_{wo} = 2.73t_c$, then $\delta = 1.36$ and n is about 0.16 from Eq. (23).

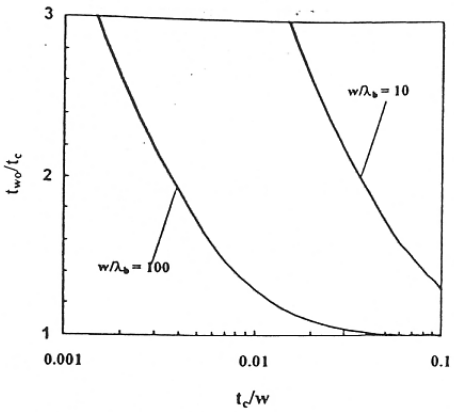

Fig. 7 Optimum wall thickness versus dimensionless fluid channel width for two values of dimensionless substrate width

3 From Eq. (20), $t_{wo} = 1.38t_c$, then $\delta = 0.69$ and n is about 0.3 from Eq. (24).

After many more iterations, the result is found to be $t_{wo} = 1.724t_c = 86.2\ \mu\text{m}$ for $t_c = 50\ \mu\text{m}$ and $\lambda_b = 100\ \mu\text{m}$. It should be noted however, that if t_c is relatively small, the iteration will diverge unless a good initial guess for t_w is given. For example, for $t_c = 20\ \mu\text{m}$, $n > 1$ will be obtained in the above step 1. Even beginning with $t_w = 2t_c$, a divergent result will be obtained. By beginning with $t_w = 3t_c$, the result $t_{wo} = 2.63t_c = 52.6\ \mu\text{m}$ is obtained for $t_c = 20\ \mu\text{m}$ and $\lambda_b = 100\ \mu\text{m}$.

To assist in the design of microchannels, the foregoing calculation was generalized to obtain the optimum fin thickness. The result is shown in Fig. 7, which depicts the variation of t_{wo}/t_c with the ratio of fluid channel width to width of microchannel substrate t_c/w. This shows that t_{wo}/t_c is almost 1 for $t_c/w = 0.1$ and $w/\lambda_b = 100$. The value of t_{wo}/t_c gradually increases when t_c/w decreases from 0.1 to 0.01. It is worth noting that t_{wo}/t_c increases rapidly when t_c/w approaches 0.001. The curve of $w/\lambda_b = 10$ shows a same trend as that of $w/\lambda_b = 100$ but shifts toward a higher value of t_c/w. Since the number of fluid channels for the microchannel substrate will be less than five when t_c/w is larger than 0.2, it is not practical to show any result for $t_c/w > 0.2$.

Conclusion

This paper studies how to effectively enhance heat transfer by fins in the microscale regime, despite the size effect-induced reduction of the thermal conductivity of microchannel and fin walls. This study resolves three basic issues. The first issue comes from the question: "Is there a upper limit for the heat transfer enhancement?" It is found that although choking is usually observed in fins with clusters at the tip, choking may occur even in the case of simple plate fins or pin fins in the microscale regime.

The second issue considered is how the size effect affects the heat transfer enhancement ratio by increasing the fin number while keeping the total fin base area fixed. For this case, the analysis shows that heat transfer is significantly enhanced if there is no size effect. The heat transfer enhancement ratio is relatively low and will rapidly reach a saturated value if there is a size effect. The heat transfer enhancement ratio for plate fins, however, is significantly higher than that of pin fins. The reason is that the thermal conductivity reduction due to the size effect for plate fins is smaller than that for pin fins. This result shows that, in the microscale regime, the shape effect should be considered in the design of micro heat exchangers as well as the size effect. Plate fins are more desirable than pin fins for heat transfer enhancement.

The third issue considered is how the size effect affects the fin thickness (or channel wall thickness in microchannel heat exchangers) optimization. The size effect on the thermal conductivity of plate fins is approximated by a power law of fin thickness ratio, $k_w = k_b(t_w/t_b)^n$. Then the optimum fin thickness can be obtained as

$$t_{wo} = (0.5 + 0.5n)t_c/(0.5 - 0.5n),$$

where t_c is the width of the fluid channel. The result, $t_{wo} = t_c$, which was found by Tuckerman and Pease (1981), corresponds to the case of $n = 0$ in the present result. For convenience in design applications, the variations of t_{wo}/t_c with t_c/w are established to calculate the optimum wall thickness in the size effect regime.

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