

Lecture 16: Fairness Verification

CIS 7000: Trustworthy Machine Learning

Spring 2024

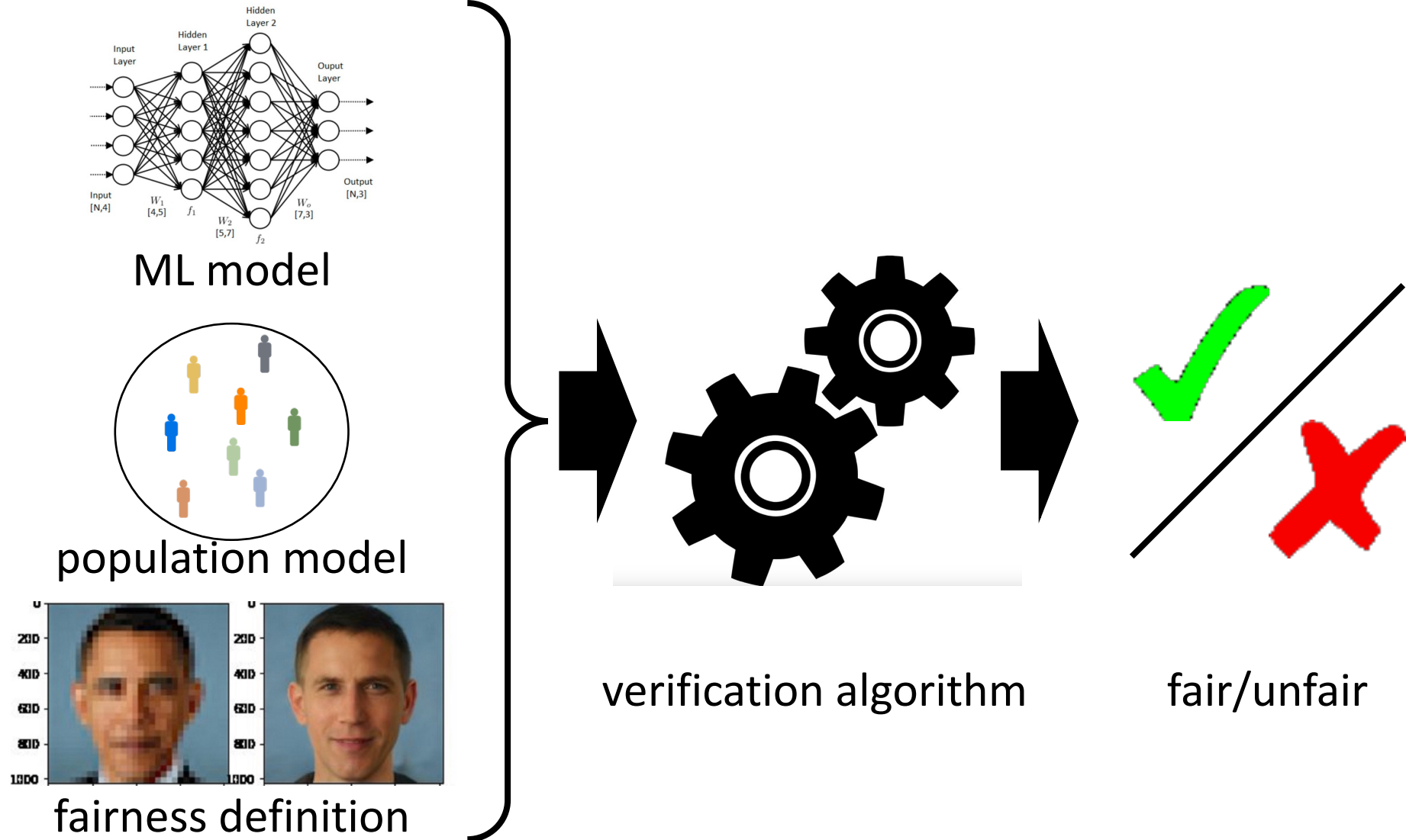
Agenda

- Fairness verification problem
- Symbolic fairness verification
- Statistical fairness verification

Fairness Verification

- **Goal:** Check if a given model satisfies a given fairness definition
- Ideally, the verification strategy should be flexible, and work on a broad family of fairness definitions
 - Focus on group fairness
- **Note:** Fairness is a **statistical property!**
 - Depends on data distribution $p(x, y)$
 - Therefore, we also need to specify $p(x, y)$, which we call the **population model**

Fairness Verification



Fairness

- **Problem Setup**

- Distribution $P_{\mathcal{V}}$ over individuals $v = (\tilde{v}, a) \in \mathcal{V}$ (called the **population model**)
- Sensitive attribute $a \in \{\text{majority, minority}\}$
- Binary classifier $f: \mathcal{V} \rightarrow \{0,1\}$, where 1 indicates a positive outcome

- **Fairness Properties:** Demographic parity, equality of opportunity, etc.

Demographic Parity

- Majority and minority members get positive outcomes at the same rate
- Let the **acceptance probability** for a be

$$\mu_a^* = \Pr_{v \sim \mathcal{V}}[f(v) = 1 \mid A = a]$$

- Then, f satisfies demographic parity if $Y_{\text{parity}}^* = 1$, where

$$Y_{\text{parity}}^* = 1 \left[\frac{\mu_{\text{minority}}^*}{\mu_{\text{majority}}^*} \geq c \right]$$

- The constant $c \in [0,1]$ is domain specific
- **Question:** Does $Y_{\text{parity}}^* = 1$?

Fairness Verification Problem

```
def population_model():  
    is_male ~ bernoulli(0.5)  
    col_rank ~ normal(25, 10)  
    if is_male:  
        years_exp ~ normal(15, 5)  
    else:  
        years_exp ~ normal(10, 5)  
    return col_rank, years_exp
```

```
def offer_job(col_rank, years_exp)  
    if col_rank <= 5:  
        return true  
    elif years_exp > 5:  
        return true  
    else:  
        return false
```

Question: Does OfferJob satisfy demographic parity?

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Fairness Verification

- **Goal:** Check if $Y_{\text{parity}}^* = 1$, where

$$Y_{\text{parity}}^* = 1 \left[\frac{\mu_{\text{minority}}^*}{\mu_{\text{majority}}^*} \geq 1 \right]$$

$$\mu_a^* = \Pr_{v \sim \mathcal{V}} [f(v) = 1 \mid A = a]$$

- **Step 1:** Compute approximation $\hat{\mu}_a \approx \mu_a^*$
- **Step 2:** Compute approximation $\hat{Y}_{\text{parity}} \approx Y_{\text{parity}}^*$

Fairness Verification Strategy

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def population_model():  
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Question: What is $\Pr[\text{OfferJob}(\text{PopulationModel}()) \mid \text{IsMale} = \text{True}]$?

Fairness Verification Strategy

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Question: What is $\Pr[\text{OfferJob}]$?

Fairness Verification Strategy

Pr[OfferJob]

$$= \int \text{OfferJob}(a, r, e) \cdot p_{\text{IsMale}}(a) \cdot p_{\text{ColRank}}(r) \cdot p_{\text{YearsExp}}(e) \cdot da \cdot dr \cdot de$$

$$\text{OfferJob} = (\text{ColRank} \leq 5 \vee \text{YearsExp} > 5)$$

$$= (\text{ColRank} \leq 5) \vee (\text{ColRank} > 5 \wedge \text{IsMale} \wedge \text{YearsExpLarge} > 5)$$

$$\vee (\text{ColRank} > 5 \wedge \neg \text{IsMale} \wedge \text{YearsExpSmall} > 5)$$

Pr[OfferJob]

$$= \Pr[\text{ColRank} \leq 5] + \Pr[\text{ColRank} > 5 \wedge \text{IsMale} \wedge \text{YearsExpLarge} > 5]$$

$$+ \Pr[\text{ColRank} > 5 \wedge \neg \text{IsMale} \wedge \text{YearsExpSmall} > 5]$$

$$= \Pr[\text{ColRank} \leq 5] + \Pr[\text{ColRank} \leq 5] \cdot \Pr[\text{IsMale}] \cdot \Pr[\text{YearsExpLarge} > 5]$$

$$+ \Pr[\text{ColRank} \leq 5] \cdot \Pr[\neg \text{IsMale}] \cdot \Pr[\text{YearsExpSmall} > 5]$$

$$= N(5; 25, 10) + (1 - N(5; 25, 10)) \cdot 0.5 \cdot (1 - N(5; 15, 5))$$

$$+ (1 - N(5; 25, 10)) \cdot 0.5 \cdot (1 - N(5; 10, 5))$$

```
def population_model():
```

```
    is_male ~ bernoulli(0.5)
```

```
    col_rank ~ normal(25, 10)
```

```
    if is_male:
```

```
        years_exp ~ normal(15, 5)
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```
    else:
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    return col_rank, years_exp
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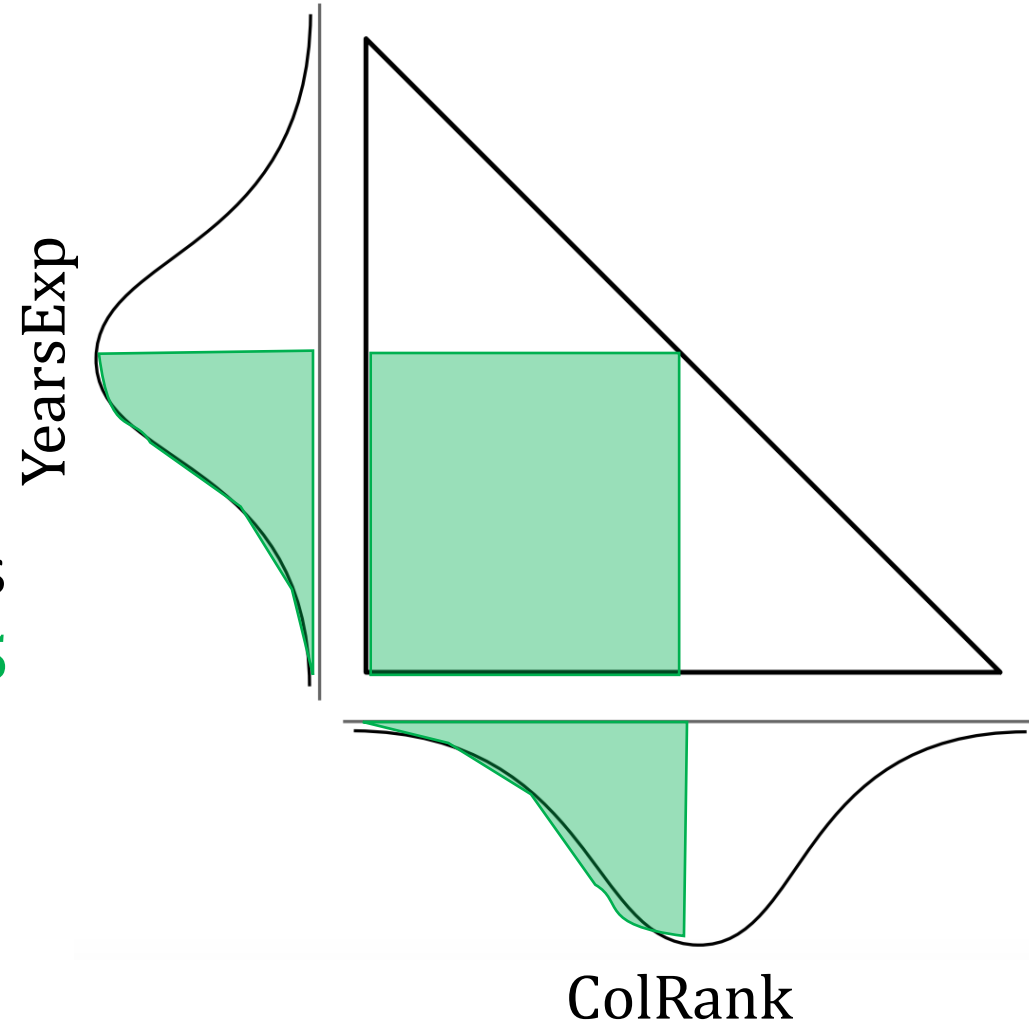
Fairness Verification Strategy

- **Alternative example**

- OfferJob = (ColRank + YearsExp) \leq 10
- Assume ColRank $\sim N(25,10)$ and YearsExp $\sim N(15,5)$
- **Goal:** Compute Pr[OfferJob]

- **Idea:** Break OfferJob into hyperrectangles

- $R_1 = 0 \leq \text{ColRank} \leq 5 \wedge 0 \leq \text{YearsExp} \leq 5$
- $\Pr[R_1] = (N(5; 25,10) - N(0; 25,10)) \cdot (N(5; 15,5) - N(0; 15,5))$



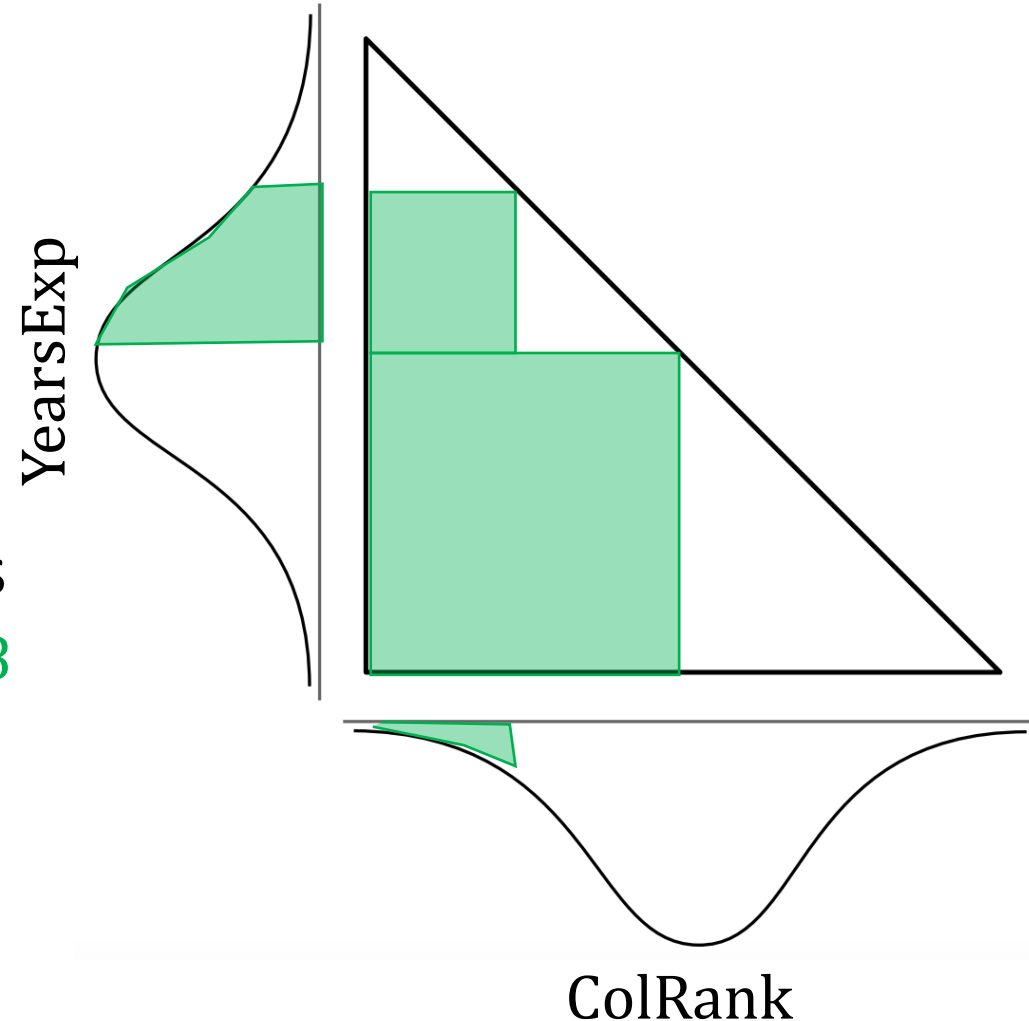
Fairness Verification Strategy

- **Alternative example**

- OfferJob = (ColRank + YearsExp) \leq 10
- Assume ColRank $\sim N(25,10)$ and YearsExp $\sim N(15,5)$
- **Goal:** Compute Pr[OfferJob]

- **Idea:** Break OfferJob into hyperrectangles

- $R_2 = 0 \leq \text{ColRank} \leq 2 \wedge 5 \leq \text{YearsExp} \leq 8$
- $\Pr[R_2] = (N(2; 25,10) - N(0; 25,10)) \cdot (N(8; 15,5) - N(5; 15,5))$



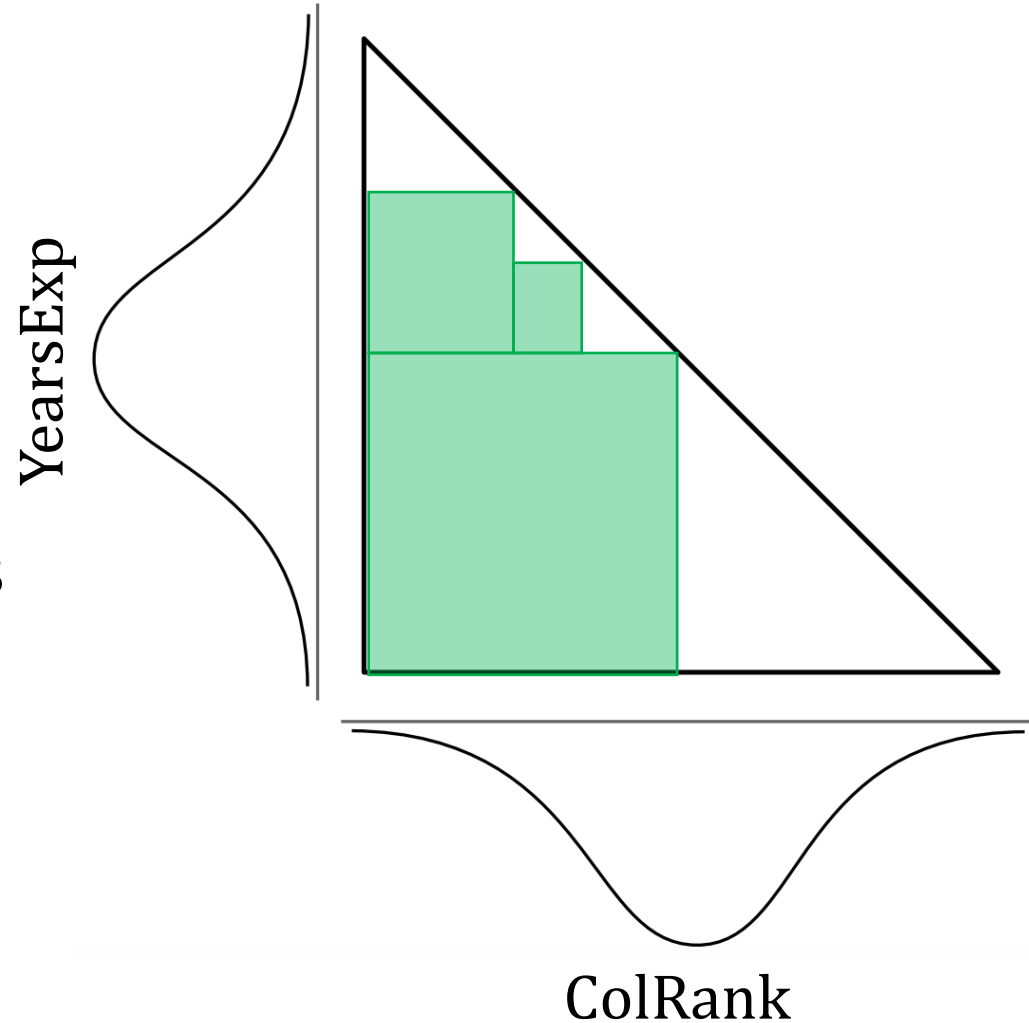
Fairness Verification Strategy

- **Alternative example**

- OfferJob = (ColRank + YearsExp) \leq 10
- Assume ColRank $\sim N(25,10)$ and YearsExp $\sim N(15,5)$
- **Goal:** Compute $\Pr[\text{OfferJob}]$

- **Idea:** Break OfferJob into hyperrectangles

- $\Pr[\text{OfferJob}] = \Pr[R_1] + \Pr[R_2] + \dots$



Fairness Verification Algorithm

for $t \in \{1, 2, \dots\}$:

for $i \in \{1, \dots, k\}$

 compute rectangle $R_{i,t}$ for ϕ_i

 compute estimate $\hat{\mu}_a \approx \mu_a^*$ using $R_{i,t}$

 compute estimate $\hat{Y} \approx Y_{\text{parity}}^*$ using $\hat{\mu}_a$

if converged: **return** \hat{Y}

Fairness Verification Algorithm

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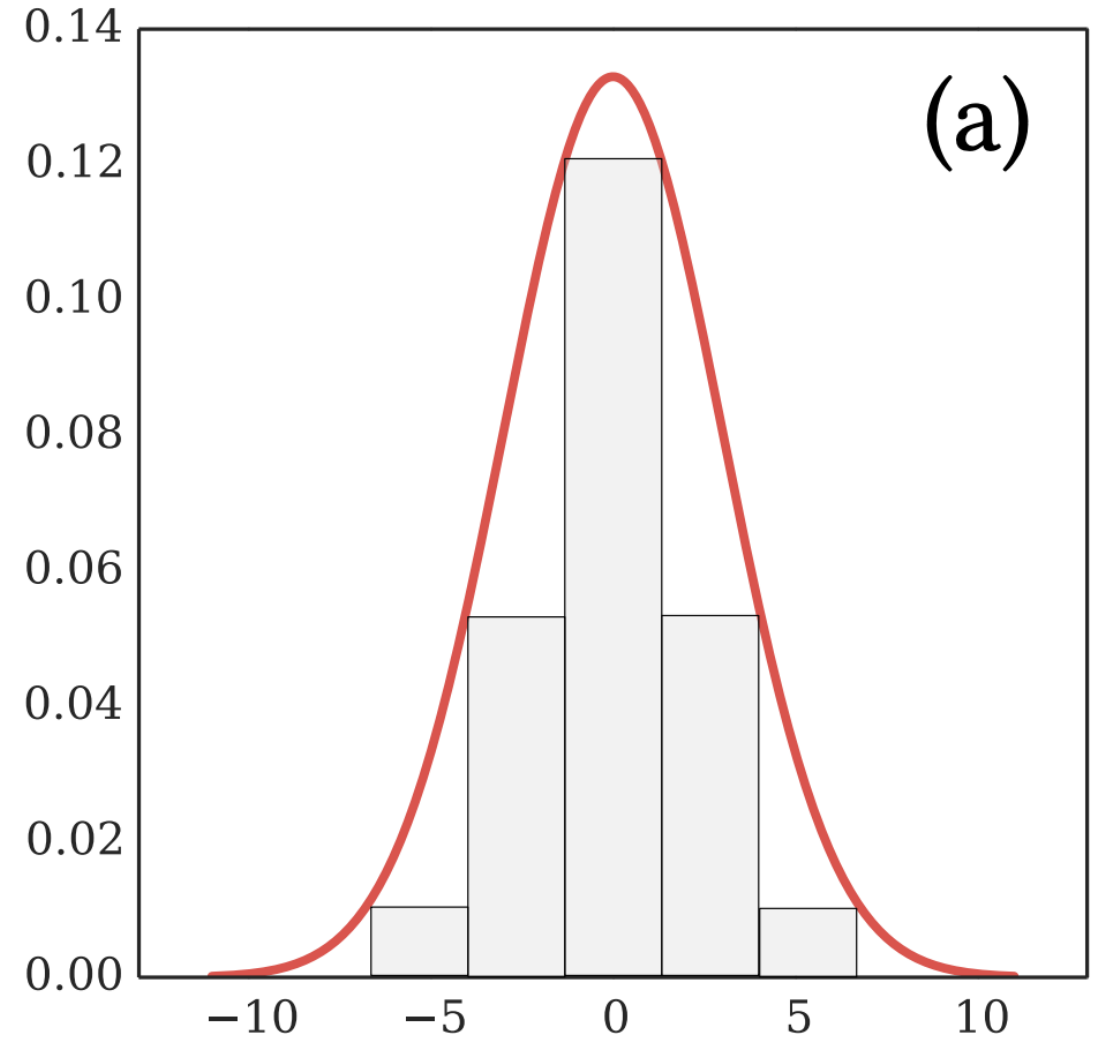
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Hyperrectangle Decomposition

- Compute the hyperrectangle with the largest probability:

$$\arg \max_R \widehat{\Pr}[R]$$

- We use a piecewise constant approximation of PDF to do so
- Then, computing the largest hyperrectangle can be expressed as an MaxSMT problem



Fairness Verification Algorithm

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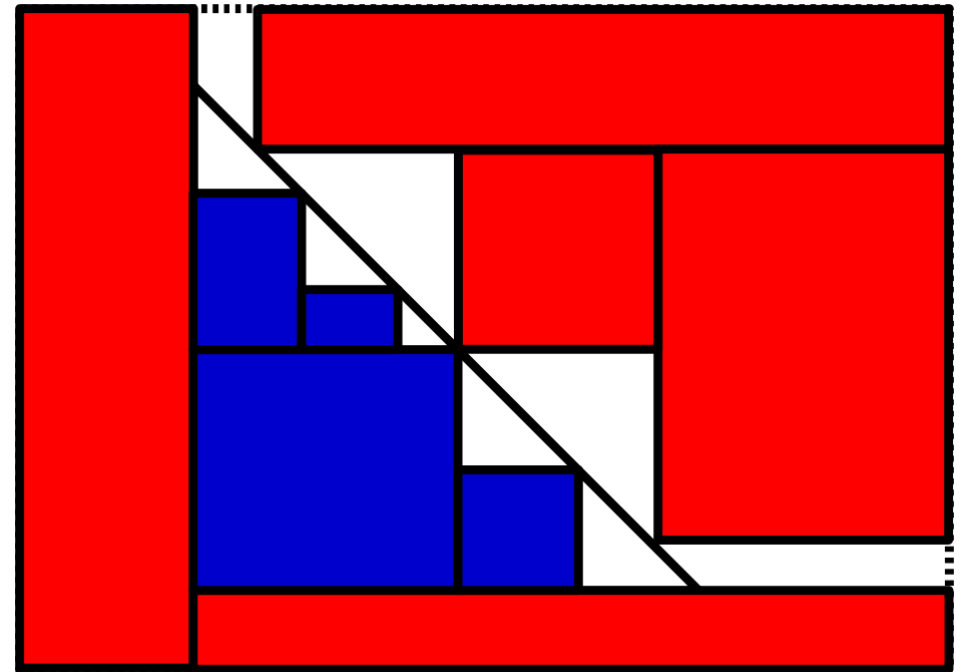
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Fairness Verification Algorithm

- **Question:** How to know when we can stop computing rectangles?
 - Keep upper and lower bounds
 - Stop computing rectangles once we accept or reject fairness
- **Note:** Assumes fairness does not “barely” hold:

$$\frac{\mu_{\text{minority}}^*}{\mu_{\text{majority}}^*} \neq 1 - c$$



Fairness Verification Algorithm

for $t \in \{1, 2, \dots\}$:

for $i \in \{1, \dots, k\}$

 compute rectangle $R_{i,t}$ for ϕ_i and $R'_{i,t}$ for $\neg\phi_i$

 compute estimate $\hat{\mu}_a \leq \mu_a^* \leq \hat{\mu}'_a$ using $R_{i,t}, R'_{i,t}$

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compute estimate $\hat{Y} \approx Y_{\text{parity}}^*$ using $\hat{\mu}_a, \hat{\mu}'_a$

if converged: return \hat{Y}

Upper/Lower Bounds on Fairness

- We need to determine if $\hat{Y} = Y_{\text{parity}}^*$, where

$$Y_{\text{parity}}^* = 1 \left[\frac{\mu_{\text{minority}}^*}{\mu_{\text{majority}}^*} \geq c \right] \quad \text{and} \quad \hat{Y} = 1 \left[\frac{\hat{\mu}_{\text{minority}}}{\hat{\mu}_{\text{majority}}} \geq c \right]$$

- **Strategy:** Abstract interpretation!
 - We have bounds on $|\hat{\mu}_{\text{minority}} - \mu_{\text{minority}}^*|$ and $|\hat{\mu}_{\text{majority}} - \mu_{\text{majority}}^*|$
 - Use abstract interpretation to obtain \hat{Y}
 - **Problem:** What are abstract semantics for Booleans and inequalities?

Upper/Lower Bounds on Fairness

- **Abstract domain for Booleans:** true, false, or uncertainty
 - $\gamma(\text{uncertain}) = \{\text{true}, \text{false}\}$
 - Called “three-valued logic”
- **Abstract transformers:** For $f_c(z) = 1(z \geq c)$, we have

$$\hat{f}_c((z_{\min}, z_{\max})) = \begin{cases} \text{true} & \text{if } z_{\min} \geq c \\ \text{false} & \text{if } z_{\max} < c \\ \text{uncertain} & \text{otherwise} \end{cases}$$

Upper/Lower Bounds on Fairness

$$\frac{\mu_Z : (E, \varepsilon, \delta) \in \Gamma}{\Gamma \vdash \mu_Z : (E, \varepsilon, \delta)} \text{ (random variable)} \quad \frac{c \in \mathbb{R}}{\Gamma \vdash (c, 0, 0)} \text{ (constant)} \quad \frac{\Gamma \vdash X : (E, \varepsilon, \delta), \Gamma \vdash X' : (E', \varepsilon', \delta')}{\Gamma \vdash X + X' : (E + E', \varepsilon + \varepsilon', \delta + \delta')} \text{ (sum)}$$

$$\frac{\Gamma \vdash X : (E, \varepsilon, \delta)}{\Gamma \vdash -X : (-E, \varepsilon, \delta)} \text{ (negative)} \quad \frac{\Gamma \vdash X : (E, \varepsilon, \delta), |E| > \varepsilon}{\Gamma \vdash X^{-1} : (E^{-1}, \frac{\varepsilon}{|E| \cdot (|E| - \varepsilon)}, \delta)} \text{ (inverse)}$$

$$\frac{\Gamma \vdash X : (E, \varepsilon, \delta), \Gamma \vdash X' : (E', \varepsilon', \delta')}{\Gamma \vdash X \cdot X' : (E \cdot E', |E| \cdot \varepsilon' + |E'| \cdot \varepsilon + \varepsilon \cdot \varepsilon', \delta + \delta')} \text{ (product)}$$

$$\frac{\Gamma \vdash X : (E, \varepsilon, \delta), E - \varepsilon \geq 0}{\Gamma \vdash X \geq 0 : (\text{true}, \delta)} \text{ (inequality true)} \quad \frac{\Gamma \vdash X : (E, \varepsilon, \delta), E + \varepsilon < 0}{\Gamma \vdash X \geq 0 : (\text{false}, \delta)} \text{ (inequality false)}$$

$$\frac{\Gamma \vdash Y : (I, \gamma), \Gamma \vdash Y' : (I', \gamma')}{\Gamma \vdash Y \wedge Y' : (I \wedge I', \gamma + \gamma')} \text{ (and)} \quad \frac{\Gamma \vdash Y : (I, \gamma), \Gamma \vdash Y' : (I', \gamma')}{\Gamma \vdash Y \vee Y' : (I \vee I', \gamma + \gamma')} \text{ (or)} \quad \frac{\Gamma \vdash Y : (I, \gamma)}{\Gamma \vdash \neg Y : (\neg I, \gamma)} \text{ (not)}$$

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if $|\gamma(\hat{Y})| = 1$: **return** \hat{Y}

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Agenda

- Fairness verification problem
- Symbolic fairness verification
- Statistical fairness verification

Shortcomings of Symbolic Verification

- Scales poorly to large models
 - Neural networks now have billions of parameters!
- Fairness is a statistical property
- Can we use a statistical approach to verify fairness?

Statistical Verification

- Use random sampling to check correctness, and use statistical tools to bound probability of false negatives

$$\Pr_{X^{(1)}, \dots, X^{(n)} \sim P_{\mathcal{X}}} [f \text{ correct} \mid \mathcal{A}(f; X^{(1)}, \dots, X^{(n)}) = \text{correct}] \geq 1 - \delta$$

- Guarantee of symbolic verification is equivalent to $\delta = 0$
- For statistical verification, some chance of error is inevitable ($\delta > 0$), but we can make δ as small as desired with sufficiently many samples

Statistical Verification for Fairness

- Given samples $v_a^{(1)}, \dots, v_a^{(n)} \sim P_{\mathcal{V}} \mid A = a$ (for each a)
 - Obtained via rejection sampling
- The estimated acceptance probability is

$$\hat{\mu}_a = \frac{1}{n} \sum_{i=1}^n \mathbb{1} \left[f \left(v_a^{(i)} \right) = 1 \right]$$

- We can bound $|\hat{\mu}_a - \mu_a^*|$ using **Hoeffding's inequality**

Hoeffding's Inequality

- Let $b_1, \dots, b_n \sim_{\text{i.i.d.}} \text{Bernoulli}(\mu)$ be samples
- Let $\hat{\mu} = n^{-1} \sum_{k=1}^n b_k$ be the empirical mean
- Then, with probability at least $1 - \delta$, we have

$$|\hat{\mu} - \mu| \leq \sqrt{\frac{\log(2/\delta)}{2n}}$$

Hoeffding's Inequality

- Apply Hoeffding's inequality to $\mu_a^* = \Pr[f(V) = 1 \mid A = a]$
- Ensures that $\hat{\mu}_a$ is “good estimate” of μ_a^* with high probability:

$$\Pr \left[|\hat{\mu}_a - \mu_a^*| \leq \sqrt{\frac{\log(2/\delta)}{2n}} \right] \geq 1 - \delta$$

Algorithm

for $i \in \{1, 2, \dots, n\}$:

 sample individual $v_a^{(i)} \sim P_{\mathcal{V}} \mid A = a$ (for each a)

 obtain high-probability bound $\hat{\mu}_a \leq \mu_a^* \leq \hat{\mu}'_a$ using Hoeffding's inequality

 compute estimate $Y_{\text{parity}}^* \in \gamma(\hat{Y})$ using $\hat{\mu}_a, \hat{\mu}'_a$

return \hat{Y}

Adaptive Concentration Inequalities

- **Key Shortcoming**

- In Hoeffding's inequality, number of samples n must be chosen beforehand
- Algorithm may not converge (i.e., $\hat{Y} = \text{uncertain}$)
- In practice, we often have a fixed test set!

- **Idea:** Try increasing values of n until one works

- **Problem:** Need a union bound!

- **Solution:** Use a concentration inequality that allows us to iteratively take more samples

Statistical Verification

- Use an *adaptive* variant of Hoeffding's, which lets us incrementally increase n and still maintain the guarantee
- **Adaptive Concentration:** With probability $\geq 1 - \delta$, we have

$$\forall n . \Pr \left[\left| \hat{\mu}_a^{(n)} - \mu_a^* \right| \leq \epsilon(n, \delta) \right]$$

- Here, $\hat{\mu}_a^{(n)} = n^{-1} \sum_{i=1}^n \mathbf{1} \left[f \left(v_a^{(i)} \right) = 1 \right]$

Adaptive Concentration Inequalities

THEOREM 4.1. *Given a Bernoulli random variable Z with distribution P_Z , let $\{Z_i \sim P_Z\}_{i \in \mathbb{N}}$ be i.i.d. samples of Z , let*

$$\hat{\mu}_Z^{(n)} = \frac{1}{n} \sum_{i=1}^n Z_i,$$

let J be a random variable on $\mathbb{N} \cup \{\infty\}$ such that $\Pr[J < \infty] = 1$, and let

$$\varepsilon(\delta, n) = \sqrt{\frac{\frac{3}{5} \cdot \log(\log_{11/10} n + 1) + \frac{5}{9} \cdot \log(24/\delta)}{n}}. \quad (10)$$

Then, given any $\delta \in \mathbb{R}_+$, we have

$$\Pr[|\hat{\mu}_Z^{(J)} - \mu_Z| \leq \varepsilon(\delta, J)] \geq 1 - \delta.$$

Algorithm

for $i \in \{1, 2, \dots\}$:

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obtain high-probability bound $\hat{\mu}_a \leq \mu_a^* \leq \hat{\mu}'_a$

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if $|\gamma(\hat{Y})| = 1$: **return** \hat{Y}

Theoretical Guarantees

- **Probabilistic correctness**

$$\Pr_{X^{(1)}, \dots, X^{(n)} \sim P_{\mathcal{X}}} [f \text{ correct} \mid \mathcal{A}(f; X^{(1)}, \dots, X^{(n)}) = \text{correct}] \geq 1 - \delta$$

$$\Pr_{X^{(1)}, \dots, X^{(n)} \sim P_{\mathcal{X}}} [f \text{ incorrect} \mid \mathcal{A}(f; X^{(1)}, \dots, X^{(n)}) = \text{incorrect}] \geq 1 - \delta$$

- **Probabilistic termination**

- Assume fairness does not “just barely” hold
- Then, with probability 1, terminates after finitely many steps

Value of Verification

- Concentration inequalities can give you provable guarantees for statistical properties

- What is the value of verification over directly using the estimate

$$\hat{Y}_{\text{parity}} = 1 \left[\frac{\hat{\mu}_{\text{minority}}}{\hat{\mu}_{\text{majority}}} \geq c \right]?$$

- Verification quantifies uncertainty in our estimate of fairness
- Do not mis-report fair or unfair due to too few samples

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