# CIS 7000 Lecture 20 Part 2

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# Saliency maps are popular for vision

Given model f: $R^n \rightarrow R$ and input x in $R^n$:

- $\alpha$ = AttributionMethod(f, x) in $R^n$

Simple "interpretation": if $\alpha_i$ is big, then $x_i$ is important!

- Useful when the end-user may not be ML specialists

… but what does "important" mean exactly?

# Feature attributions are "obvious" for simple models

Consider a linear model

$$f(x) = c_0 + c_1 x_1 + \cdots + c_n x_n$$

Clearly if the larger some $c_i$, the more $x_i$ will contribute to the score

- A natural feature attribution: $\alpha_i = c_i$ (alternatively, $\alpha_i = \mathrm{abs}(c_i)$)

… but what about for a quadratic model?

$$f(x) = c + b^\top x + x^\top A x = c + \sum_{i=1}^{n} b_i x_i + \sum_{1 \le i,j \le n} A_{ij} x_i x_j$$

It's less clear what score each feature $x_i$ should get

# "Fundamental dilemma" of feature attributions

Pro: feature attributions are "nice"

- Easy to understand: number big = feature important
- There's a lot of attribution methods

Cons:

- What does "important" mean?
- There's a lot of attribution methods
    - "This feature has Shapley value XXX", okay, so what?

# Idea: maybe we can measure the "quality" of FAs

If a feature is "important", then it should satisfy some properties.

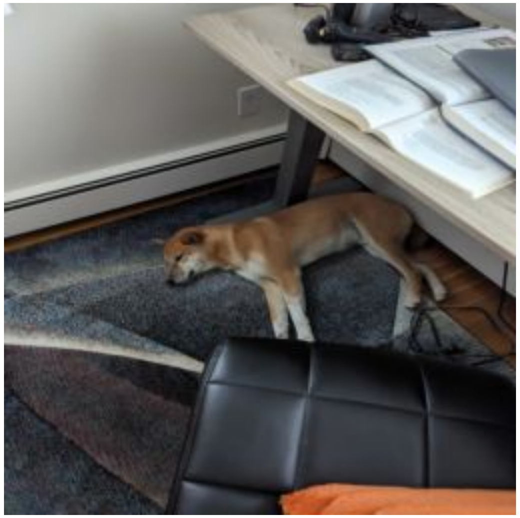
- … but what are these properties, and can we quantify them?

There is substantial work on developing metrics for feature attributions

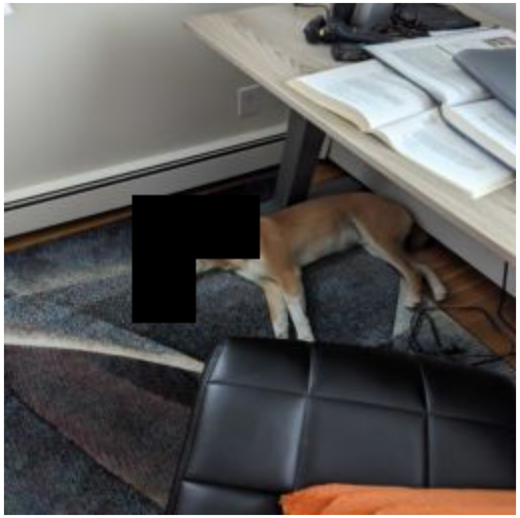
- There's a lot, we'll talk about a few

# Subtractive metrics

"If some feature is important, then removing it should decrease the score"



"Dog" (97%)

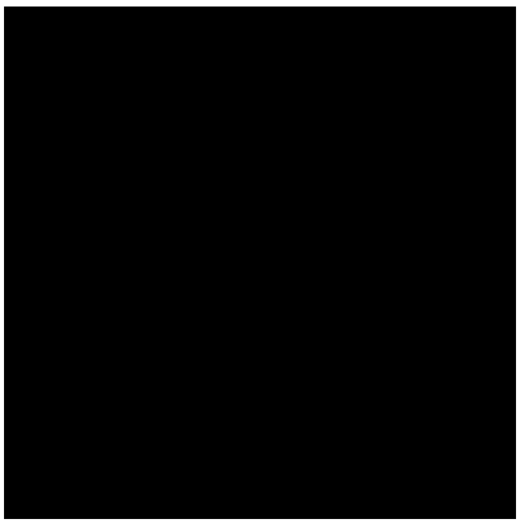


"Dog" (50%)

*I made up these numbers

# Additive metrics

"If a feature is important, then inserting it should increase the score"



"Dog" (<1%)

"Dog" (40%)

# Example of other metrics

Perturbation:

- How sensitive is your metric to perturbations?

Compactness:

- Is your explanation too "big"? (e.g., for feature selection)

Connectedness:

- Are two candidate explanations "connected" in some sense?

More here: https://arxiv.org/abs/2201.08164

# What mathematical properties should we expect?

Given a model, an input, an explanation method, and some metric …

… what formal (i.e., mathematical) properties should these things satisfy?

- e.g., "does the top-k% of features from this method guarantee a score decrease of q% wrt some metric, model class, and input family?"

In general? Hard to prove such statements

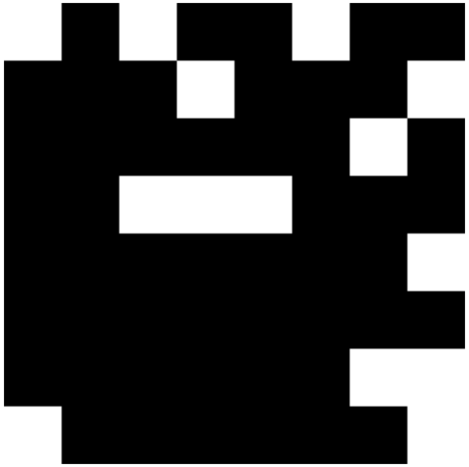
- Neural networks are **magic**

# What can we do from here?

Our work: under some conditions, one CAN guarantee formal properties

Special case: binary-valued feature attributions (i.e., feature selection)

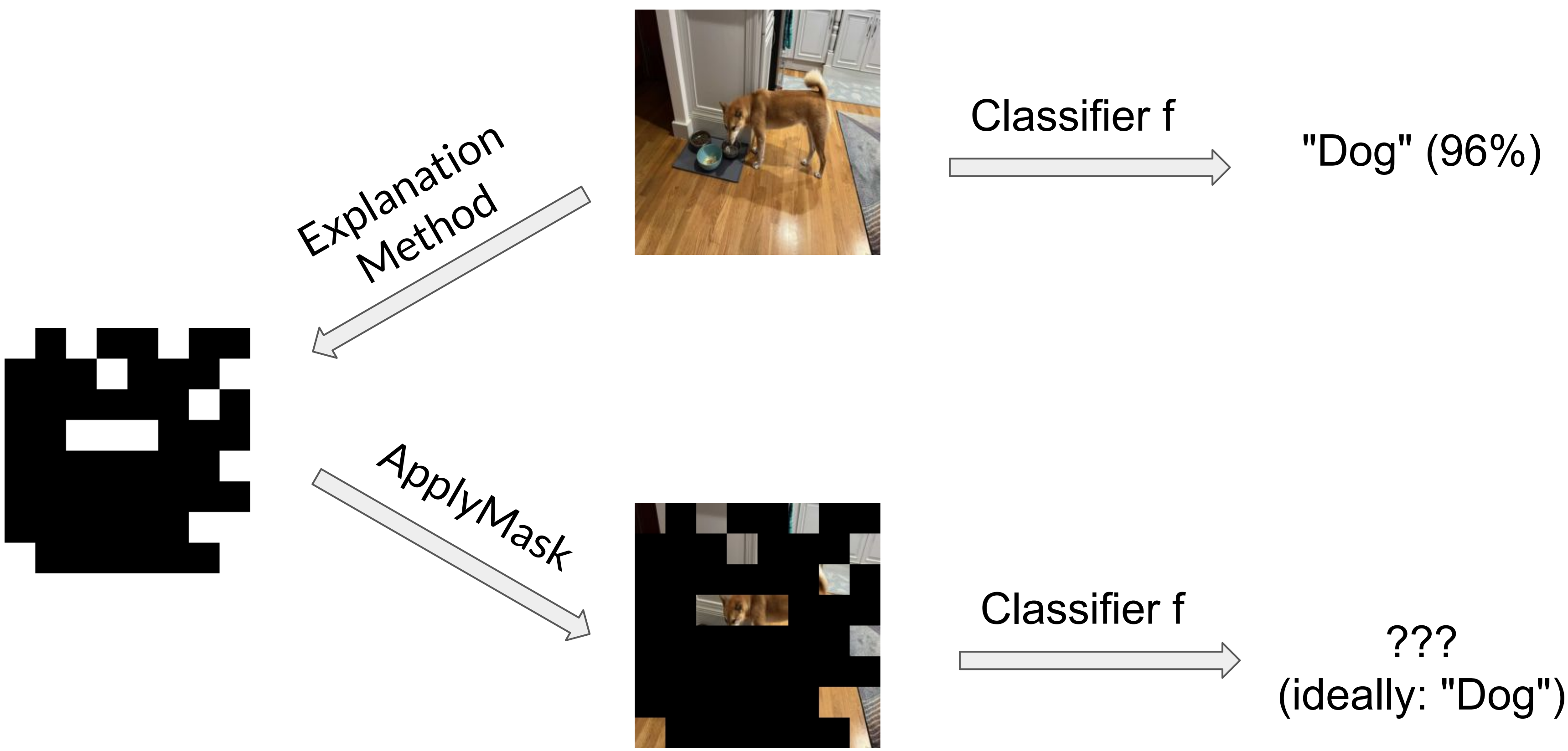
Input x in $R^n$



Attr $\alpha$ in $\{0,1\}^n$

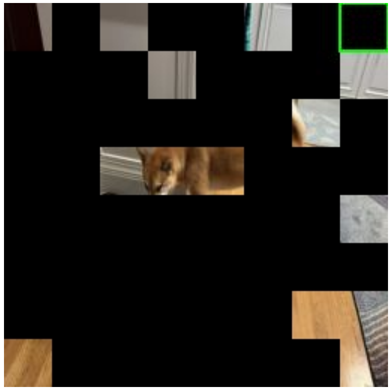
# I have an attribution, but how do I "evaluate" it?

In vision: we can use the original model

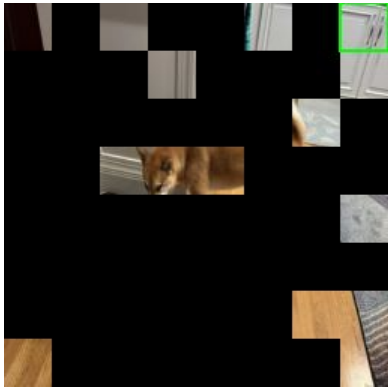


# What do we NOT want to happen?

"Dog"



"Dog"



"Cat"

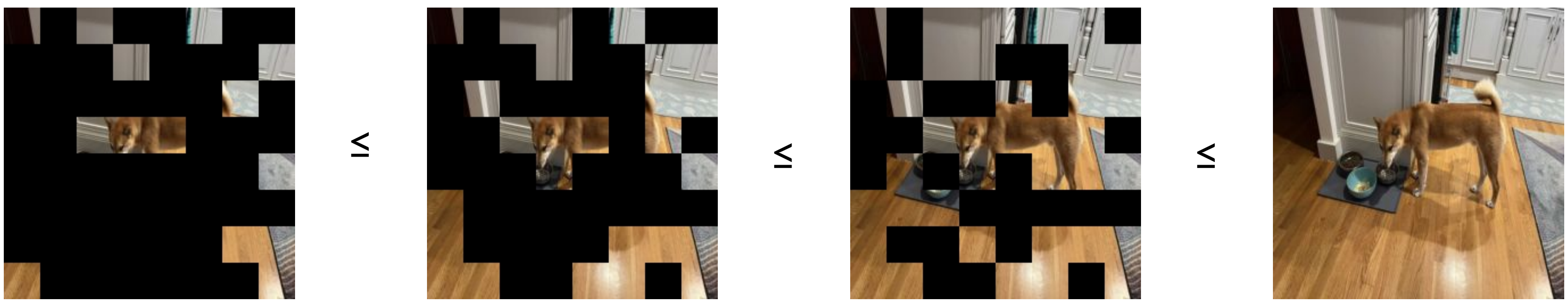
This is usually **NOT** desirable

Intuition: the original feature selection you gave me is not "convincing"

*I made up this example, but it can happen. Trust me, bro!

# Stability as a "desirable" property

Selected by your
favorite attribution method




"Dog" "Dog" "Dog" "Dog"

**Stability:** any superset of features induces the same prediction

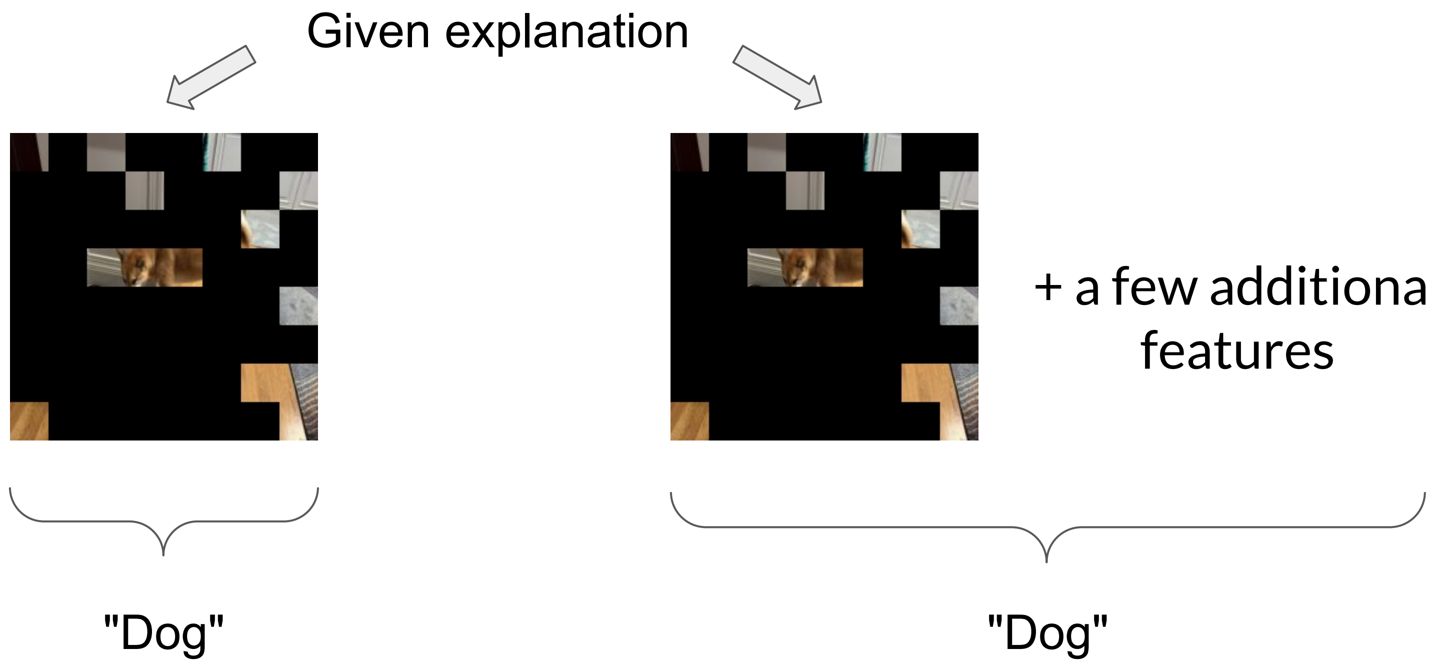
$$f(x \circ \alpha) \cong f(x \circ \alpha') \text{ for all } \alpha \leq \alpha', \text{ where } \alpha = \text{BinaryAttribution}(f,x)$$

# How can we achieve/guarantee stability!

You probably can't! (For Real Models™)

- There's $O(2^n)$ different $\alpha' \geq \alpha$ binary vectors to check

But we can maybe go for local approximations (Incremental Stability)



# The plan

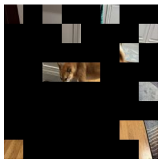
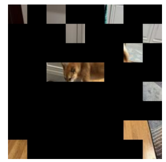
1. Incremental stability via Lipschitz smoothness

2. Achieve incremental stability with multiplicative smoothing (MUS)

"base classifier" h $\xrightarrow{\text{MuS}}$ Lipschitz-smooth classifier f

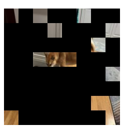
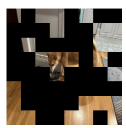
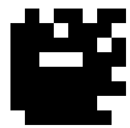
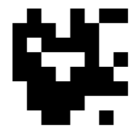
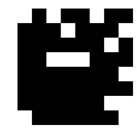
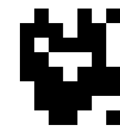
3. We can check if f is incrementally stable at some x in O(1) time.

# Step 1: incremental stability

f(  ) $\cong$ f(  $+\Delta$ )     for all small $\Delta$

Sufficient condition: Lipschitz wrt masking of features

- L1 norm on binary vectors = number of differences

f(  ) - f(  ) $\leq \lambda ||$  -  $||_1$   for all  , 

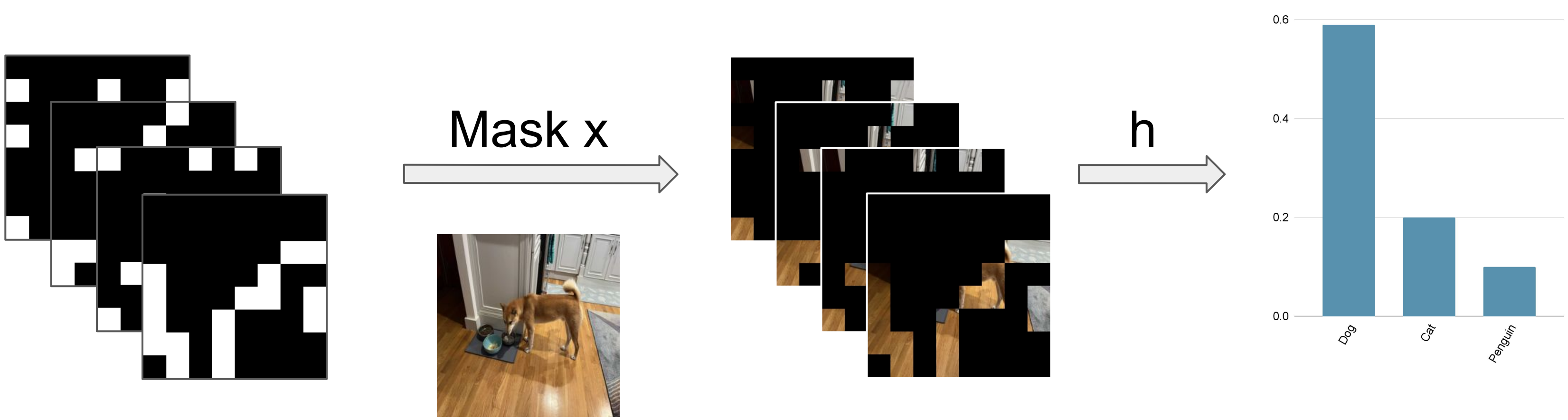
**Definition (Lipschitz wrt Feature Maskings).** The function f: $R^n \rightarrow [0,1]$ is $\lambda$-Lipschitz wrt the masking of features at x in $R^n$ if:

$$f(x \circ \alpha) - f(x \circ \alpha') \leq \lambda ||\alpha - \alpha'||_1 \quad \text{for all } \alpha, \alpha' \text{ in } \{0,1\}^n$$

# Step 2: Multiplicative Smoothing (MuS)

"base classifier" h $\xrightarrow{\text{MuS}}$ λ-Lipschitz-smooth f

$$\boxed{f(x) = \text{MuS}(h, x) = \text{avg}(h(x^{(1)}), \ldots, h(x^{(N)}))}$$




Sample $s^{(1)} \ldots s^{(N)}$
each $s^{(i)}_j \sim \text{Bern}(\lambda)$

$x^{(1)} \ldots x^{(N)}$ where
each $x^{(i)} = x \circ s^{(i)}$

$h(x^{(1)}) \ldots h(x^{(N)})$

# Step 2: The Math

Recall $f(x) = \text{MuS}(h, x)$ and let

$$g(x, \alpha) = \text{MuS}(h, x \circ \alpha) = \text{avg}(h(x \circ \alpha \circ s^{(1)}), \ldots, h(x \circ \alpha \circ s^{(N)}))$$

**Theorem (MuS).** Let D be any distribution on $\{0,1\}^n$ where each coordinate of $s \sim D$ is marginally distributed as $s_i \sim \text{Bern}(\lambda)$ and let

$$g(x, \alpha) = E_{s \sim D}\, h(x \circ \alpha \circ s), \qquad \text{for any } h: R^n \rightarrow [0,1],$$

then $g(x, \alpha)$ is $\lambda$-Lipschitz in $\alpha$ wrt the $L^1$ norm for all x.

Note: Lipschitz smoothness is a property of functions

D need NOT be coordinate-wise independent

- We just requires that each sample's coordinate marginally $\sim \text{Bern}(\lambda)$
- Allows for a deterministic evaluation with $N \ll 2^n$ samples
    - Recall that $\text{Bern}^n(\lambda)$ has $2^n$ unique values

# Step 3: provable incremental stability

Suppose $h: R^n \to [0,1]^m$ is a classifier

Let $f(x) = \text{MuS}(h, x)$ with parameter $\lambda$

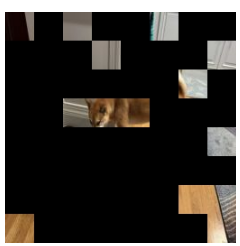
Let $\alpha = \text{BinaryAttribution}(f, x)$, such that $f(x) \cong f(x \circ \alpha)$

**Theorem (MuS).** Suppose that

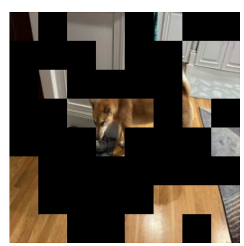
$$\text{Class1Prob}(f(x \circ \alpha)) - \text{Class2Prob}(f(x \circ \alpha)) \geq 2\lambda r,$$

then for any $\alpha' \geq \alpha$ with $||\alpha' - \alpha||_1 \leq r$, we have $f(x \circ \alpha') \cong f(x \circ \alpha)$.

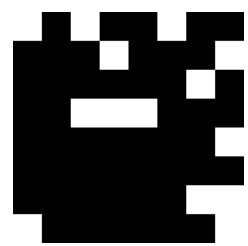
$x$



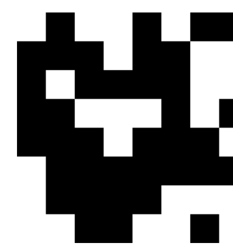
$x \circ \alpha$



$x \circ \alpha'$



$\alpha$



$\alpha'$

# Basic summary of MuS

Step 1: stability is hard, so we go for incremental stability

- Key idea: Lipschitz constants

Step 2: "randomized" smoothing

- $f(x) = \text{MuS}(h, x) = \text{avg}(h(x \circ s^{(1)}), \dots, h(x \circ s^{(N)}))$
- $f(x \circ \alpha) = \text{MuS}(h, x \circ \alpha) = \text{avg}(h(x \circ \alpha \circ s^{(1)}), \dots, h(x \circ \alpha \circ s^{(N)}))$

Step 3: Lipschitz constants → stability guarantees

# Experimental evaluations

E1: how good are the stability guarantees?

- How much incremental stability radius can we achieve?
    - for x in dataset with $\alpha$ = BinaryAttribution(f, x):
        - $r = [\text{Class1Prob}(f(x \circ \alpha)) - \text{Class2Prob}(f(x \circ \alpha))] / 2\lambda$

E2: what is the cost of smoothing?

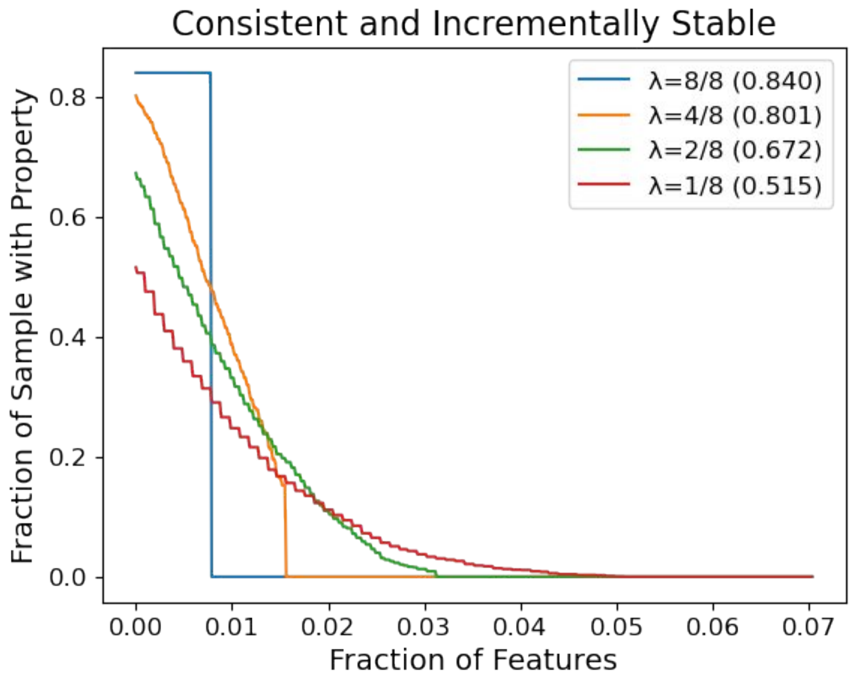
- Smoothing inherently requires us to inject noise
- Accuracy degradation of $f(x) = \text{MuS}(h, x)$

# E1: radius of incremental stabilities

Base classifier: h = Vision Transformer

BinaryAttribution: SHAP (top-25%)

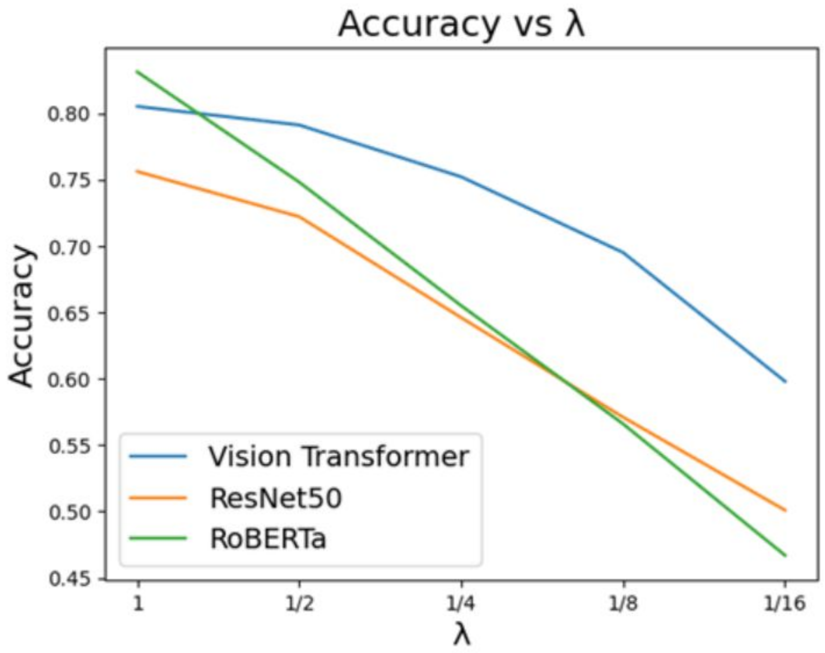
Dataset: N = 2000 samples from ImageNet



# E2: accuracy penalty of smoothing

Vision Dataset: ImageNet1K (N = 2000 samples)

Language Dataset: TweetEval (N = 2000 samples)



# Takeaways

1. We give a way to provably check for incremental stability
2. MuS: randomly drops features to these guarantees
    a. $\text{MuS}(h, x) = \text{avg}(h(x \circ s^{(1)}), \ldots, h(x \circ s^{(N)}))$
    b. $g(x, \alpha) = f(x \circ \alpha) = \text{MuS}(h, x \circ \alpha)$
    c. $g(x, \alpha)$ is $\lambda$-Lipschitz in $\alpha$ wrt the $L^1$ norm
3. Lipschitz smooth gives lower-bound on the incremental stability radius

arXiv: https://arxiv.org/abs/2307.05902

# Efficient Smoothing

Main challenge: MuS is defined in terms of an expected value

- $\text{Bern}^n(\lambda)$ has $N=2^n$ unique values (too many for the expected value!)
- MuS only requires that each coordinate is ~Bern(λ)
    - Do NOT need coordinate-wise independence
    - Algorithm below: N = q unique values
    - Main idea: use v as a pseudo-RNG seed, with 1-dim "randomness" $s_{\text{base}}$

**Proposition 3.4.** *Fix integer $q > 1$ and consider any vector $v \in \{0, 1/q, \ldots, (q-1)/q\}^n$ and scalar $\lambda \in \{1/q, \ldots, q/q\}$. Define $s \sim \mathcal{L}_{qv}(\lambda)$ to be a random vector in $\{0,1\}^n$ with coordinates given by*

$$s_i = \mathbb{I}[t_i \leq \lambda], \qquad t_i = v_i + s_{\text{base}} \mod 1, \qquad s_{\text{base}} \sim \mathcal{U}(\{1/q, \ldots, q/q\}) - 1/(2q).$$

*Then there are $q$ distinct values of $s$ and each coordinate is distributed as $s_i \sim \mathcal{B}(\lambda)$.*

*Proof.* First, observe that each of the $q$ distinct values of $s_{\text{base}}$ defines a unique value of $s$ since we have assumed $v$ and $\lambda$ to be fixed. Next, observe that each $t_i$ has $q$ unique values uniformly distributed as $t_i \sim \mathcal{U}(1/q, \ldots, q/q\}) - 1/(2q)$. Because $\lambda \in \{1/q, \ldots, q/q\}$ we therefore have $\Pr[t_i \leq \lambda] = \lambda$, which implies that $s_i \sim \mathcal{B}(\lambda)$. □