## Neurosymbolic Learning

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Ziyang Li, University of PennsyIvania


Ziyang Li


Jiani Huang


Prof. Mayur Naik

## Joint Work With:

Jason Liu, Felix Zhu, Eric Zhao, William Dodds, Neelay Velingker, Rajeev Alur

The need for Neurosymbolic AI

## Two separate paradigms of programming

Classical Algorithms

Suited for exactly defined tasks on structured input domains

## Deep Learning

Suited for tasks which cannot be handprogrammed or have unstructured input

## Two separate paradigms of programming

Classical Algorithms

Suited for exactly defined tasks on structured input domains
e.g.

- Sort a list of numbers
- Find shortest path
- Solve boolean constraints


## Deep Learning

Suited for tasks which cannot be handprogrammed or have unstructured input

## Two separate paradigms of programming

Classical Algorithms

Suited for exactly defined tasks on structured input domains
e.g.

- Sort a list of numbers
- Find shortest path
- Solve boolean constraints


## Deep Learning

Suited for tasks which cannot be handprogrammed or have unstructured input
e.g.

- Detecting objects in image
- Parse natural language text
- Control in physical environment


## Neurosymbolic to combine both worlds...

## Classical Algorithms

Suited for exactly defined tasks on structured input domains

## Deep Learning

Suited for tasks which cannot be handprogrammed or have unstructured input

## symbolic $\oplus$ neural $=$ neurosymbolic

## Neurosymbolic Learning...

=
Machine learning with both neural and symbolic components

An Example of Neurosymbolic Learning

## Neurosymbolic Learning of addition(3, 5, 8)



Logic Program


Neural Model


A Neurosymbolic Program

## Neurosymbolic Learning of addition(3, 5, 8)




Logic Program


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## Neurosymbolic Learning of addition(3, 5, 8)



Logic Program


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## Neurosymbolic Learning of addition(3, 5, 8)



## Neurosymbolic Learning of addition(3, 5, 8)



## Neurosymbolic Learning of addition(3, 5, 8)

## rel addition(a + b) = digit_1(a) and digit_2(b) <br> Scallop (Li et. al., Datalog like syntax) <br> addition(r) :- digit_1(a), digit_2(b), r is a + b



DeepProblog (Manheave et. al., Prolog like syntax)

```
@ised.blackbox(a=IntRange(10), b=IntRange(10),
    output=IntRange(19))
    def addition(a, b): return a + b
```


ymbolic Program
ISED (Breslin et. al., Python like syntax)

## Neurosymbolic Learning of addition(3, 5, 8)

Neurosymbolic Frameworks aim to provide a programming interface for developers to write Neurosymbolic solutions.


A More Difficult Motivating Example: PacMan-Maze

## A Motivating Example: PacMan-Maze



Step 0


Step 4


State: 200x200 colored image
Action: Up, Down, Left, Right
(Environments are $5 \times 5$ grids randomized for each session)

## How to combine neural and symbolic components?



Step 0


Step 4


Step 7

State: $200 \times 200$ colored image Action: Up, Down, Left, Right
(Environments are $5 \times 5$ grids randomized for each session)

1. What is the neural component?

CNN that parses the image into entity positions
2. What does the symbolic program do?

Plan the optimal action to take at each state, given the possibly noisy entity positions

## How to combine neural and symbolic components?



Step 0


Step 4


Step 7

State: 200x200 colored image Action: Up, Down, Left, Right
(Environments are $5 \times 5$ grids randomized for each session)


## Results after combining neural and symbolic



Step 0


Step 4


Step 7

State: $200 \times 200$ colored image Action: Up, Down, Left, Right
(Environments are $5 \times 5$ grids randomized for each session)

|  | Neurosymbolic <br> (with Scallop) | DQN |
| :---: | :---: | :---: |
| Success rate <br> (reaches the goal <br> within 50 steps) | $99.4 \%$ | $84.9 \%$ |
| \# of Training <br> episodes <br> (to achieve the <br> success rate) | 50 | 50 K |

(Note: this is not entirely a fair comparison since our Scallop program encodes system dynamics and human knowledge)

## A Motivating Example: PacMan-Maze



Step 0


Step 4


Step 7

State: 200x200 colored image Action: Up, Down, Left, Right
(Environments are $5 \times 5$ grids randomized for each session)


## A Motivating Example: PacMan-Maze

```
class EntityExtractor(nn.Module):
    def __init__(self):
        super(EntityExtractor, self).__init__()
    self.conv1 = nn.Conv2d(...)
    self.conv2 = nn.Conv2d(...)
    self.fc1 = nn.Linear(in_features=288, out_features=256)
    self.fc2 = nn.Linear(in_features=256, out_features=4)
    self.relu = nn.ReLU()
    def forward(self, x):
    batch_size, _, _, _ = x.shape
    x = self.relu(self.conv1(x))
    x = self.relu(self.conv2(x))
    x = x.view(batch_size, -1)
    x = self.fc2(self.relu(self.fc1(x)))
    return torch.softmax(x, dim=1)
```


## A Motivating Example: PacMan-Maze

```
type Action = UP | RIGHT | DOWN | LEFT
type actor(x: i32, y: i32), goal(x: i32, y: i32), enemy(x: i32, y: i32)
rel safe_cell(x, y) = grid_cell(x, y) and not enemy(x, y)
rel edge( }x,y,x,y+1,UP)=\operatorname{safe_cell(x, y) and safe_cell(x, y + 1)
// Rules for RIGHT, DOWN, and LEFT edges are omitted for brevity
rel next_pos(p, q, a) = actor(x, y) and edge(x, y, p, q, a)
rel path(x, y, x, y) = next_pos(x, y, _)
rel path(x1, y1, x3, y3) = path(x1, y1, x2, y2) and edge(x2, y2, x3, y3, _)
rel next_action(a) = next_pos(p, q, a) and path(p, q, r, s) and goal(r, s)
```


## Demo - Training the Agent!



Marker on the top-left indicating NN prediction on what the cell represents
(Saturation indicates confidence)


- PacMan (Blue)

Goal (Green)

## Demo - Testing the Agent!



Marker on the top-left indicating NN prediction on what the cell represents
(Saturation indicates confidence)

$\square \quad$ Enemy (Red)
PacMan (Blue)
$\square \quad$ Goal (Green)

## Key Take Aways

Decomposing end-to-end neural solutions into separated perception + reasoning solutions...

1. Improves accuracy

Reasoning module is generalizable to combinatorically diverse situations
2. Learns faster

Neural models are good at recognizing patterns; and they are used only for it

## 3. Is explainable

Decomposition gives explicit meaning to intermediate information

## Differentiating Symbolic Programs

## Back to adding two MNIST digits...



## Using Neural Network to Classify One MNIST Digit



## Training Loop for MNIST Addition Task



## Training Loop for MNIST Addition Task



## Training Loop for MNIST Addition Task



## Training Loop for MNIST Addition Task



Page 34

## Training Loop for MNIST Addition Task



Page 35

## Training Loop for MNIST Addition Task



Page 36

## Training Loop for MNIST Addition Task



Intermediate Representation ( $r$ )


Page 38





## Question 1: Forward probability estimation



## Question 1: Forward probability estimation

| 0 | 0 | 0.02 : digit_1(0) | $\operatorname{Pr}($ digit $1=0)$ |
| :---: | :---: | :---: | :---: |
| $\bigcirc$ | 1 | 0.02 : digit_1(1) | $\operatorname{Pr}($ digit $1=1)$ |
| - | 2 | 0.87 : digit_1(2) | $\operatorname{Pr}($ digit $1=2)$ |
| $\bigcirc$ | 3 | 0.01 : digit_1(3) | $\operatorname{Pr}($ digit $1=3)$ |
| $\bigcirc$ | $4 \longrightarrow$ | 0.02 : digit_1(4) | $\operatorname{Pr}($ digit $1=4)$ |
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| $\bigcirc$ | 6 | 0.01: digit_1 (6) | $\operatorname{Pr}($ digit $1=6)$ |
| $\bigcirc$ | 7 | 0.02: digit_1(7) | $\operatorname{Pr}($ digit $1=7)$ |
| $\bigcirc$ | 8 | 0.01 : digit_1(8) | $\mathbf{P r}($ digit $1=8)$ |
| 0 | 9 | 0.01 : digit_1(9) | $\operatorname{Pr}($ digit $1=9)$ |
| 0 | 0 | 0.02: digit_2(0) | $\operatorname{Pr}($ digit $2=0)$ |
| $\bigcirc$ | 1 | 0.01: :digit_2(1) | $\operatorname{Pr}($ digit2 $=1)$ |
| $\bigcirc$ | 2 | 0.01 : digit_2(2) | $\operatorname{Pr}($ digit2 $=2)$ |
| $\bigcirc$ | 3 | 0.01 : digit_2(3) | $\operatorname{Pr}($ digit2 $=3)$ |
| 0 | 4 | 0.02 : digit_2(4) | $\operatorname{Pr}($ digit $2=4)$ |
| $\bullet$ | $\longrightarrow$ | 0.88 : digit_2(5) | $\operatorname{Pr}($ digit $2=5)$ |
| $\bigcirc$ | 6 | 0.01 : digit_2(6) | $\operatorname{Pr}($ digit2 = 6) |
| $\bigcirc$ | 7 | 0.02 : digit_2(7) | $\operatorname{Pr}($ digit $2=7)$ |
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## Question 1: Forward probability estimation

| O | 0 | 0.02: :digit_1(0) | $\operatorname{Pr}($ digit $1=0)$ |
| :---: | :---: | :---: | :---: |
| $\bigcirc$ | 1 | 0.02: digit_1(1) | $\mathbf{P r}($ digitl $=1)$ |
| - | 2 | 0.87 : :digit_1(2) | $\operatorname{Pr}($ digit $1=2)$ |
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| $\bigcirc$ | 7 | 0.02: :digit_1(7) | $\operatorname{Pr}($ digit $1=7)$ |
| $\bigcirc$ | 8 | 0.01: :digit_1(8) | $\mathbf{P r}($ digitl $=8)$ |
| 0 | 9 | 0.01: digit_1(9) | $\operatorname{Pr}($ digit $1=9)$ |
| O | 0 | 0.02: digit_2(0) | $\operatorname{Pr}($ digit2 = 0) |
| $\bigcirc$ | 1 | 0.01 : digit_2(1) | $\operatorname{Pr}($ digit $2=1)$ |
| $\bigcirc$ | 2 | $0.01:$ digit_2(2) | $\operatorname{Pr}($ digit2 $=2$ ) |
| $\bigcirc$ | 3 | 0.01 : :digit_2(3) | $\operatorname{Pr}($ digit2 $=3$ ) |
| $\bigcirc$ | 4 | 0.02 : digit_2(4) | $\operatorname{Pr}($ digit2 $=4$ ) |
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## Question 1: Forward probability estimation

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## Question 1: Forward probability estimation

| - | 0 | 0.02 : digit_1(0) | $\operatorname{Pr}($ digit $1=0)$ |
| :---: | :---: | :---: | :---: |
| $\bigcirc$ | 1 | 0.02 : digit_1(1) | $\operatorname{Pr}($ digit $1=1)$ |
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| 0 | 9 | 0.01 : digit_2(9) | $\operatorname{Pr}($ digit $2=9)$ |

$$
\begin{aligned}
& \operatorname{Pr}(\text { addition }=2) \\
= & \mathbf{P r}(\text { digit } 1=0) \times \operatorname{Pr}(\text { digit } 2=2)+ \\
& \operatorname{Pr}(\text { digit } 1=1) \times \operatorname{Pr}(\text { digit2 }=1)+ \\
& \operatorname{Pr}(\text { digit } 1=2) \times \operatorname{Pr}(\text { digit2 }=0)+
\end{aligned}
$$

## Question 1: Forward probability estimation



## Question 2: Backward gradient estimation



A Neurosymbolic Program





|  | $r$ |
| :---: | :---: |
| 0.02 : digit_1(0) | (0) 0 |
| 0.02 : digit_1 (1) | 0 |
| 0.87 : :digit_1(2) | - |
| 0.01 : digit_1(3) | 0 |
| 0.02 : digit_1 (4) | 0 |
| 0.01 : digit_1(5) | 0 |
| 0.01 : :digit_1 (6) | 0 |
| 0.02 : digit_1(7) | $\bigcirc$ |
| 0.01 : digit_1 (8) | $\bigcirc$ |
| 0.01: digit_1 (9) | O 9 |
| 0.02 : digit_2(0) | (0) 0 |
| 0.01 : digit_2(1) |  |
| 0.01 : digit_2(2) | O 2 |
| 0.01 : digit_2(3) | O 3 |
| 0.02 : digit_2(4) |  |
| 0.88 : digit_2(5) |  |
| 0.01: digit_2(6) |  |
| 0.02 : digit_2(7) |  |
| 0.01 : digit_2(8) |  |
| 0.01: digit_2(9) | O 9 |

$\square$

|  |  |
| :---: | :---: |
|  |  |
| $\ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square}$ | $\ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square} \ddot{\square}$ |
| ¢．ㅇ．0．ㅇ．0．0 | क b ob o b o b o b o b |
|  | ＋r．n－r． |
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| －¢ ¢－ |  |
| ○○○○○○○○○○ | ○○○○○○○○○○ |
| がのル」w＋－ | －いのル A W N |


$\square$

|  |  |
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|  |  |
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## The Questions Are Solved!



## The Questions Are Solved! Or are we...?



## The Questions Are Solved! Or are we...?

1. Scalability Issue

Doing exact probabilistic estimation is too time consuming! We need to speed it up!

Solution: approximation, heuristics, etc.
2. Expressiveness Issue

This framework is too primitive, need to support more operations than simply summations.

Solution: a programming language that supports negation, recursion, aggregations, etc.


A Neurosymbolic Program

## Scallop, A Neurosymbolic Programming Language

## Relational Programming and Scallop



Scallop, a Neurosymbolic Programming Language

## Scallop: A Neurosymbolic Programming Language

Neurosymbolic
Programming
Language based on
Datalog

## Scallop: A Neurosymbolic Programming Language

Neurosymbolic<br>Programming<br>Language based on<br>Datalog

Expressive Logic:

- Recursion
- Negation
- Aggregation


## Scallop: A Neurosymbolic Programming Language

Neurosymbolic
Programming
Language based on
Datalog

Expressive Logic:

- Recursion
- Negation
- Aggregation


## Rich Reasoning

Framework:

- Discrete
- Probabilistic
- Differentiable


## Scallop: A Neurosymbolic Programming Language



## Scallop: A Neurosymbolic Programming Language



Goal: Provide a unified language for AI developers

## Scallop has been applied to many ML tasks...




CLUTRR: Kinship Reasoning

## CLUTRR: Kinship Reasoning

## Context:

[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.

## Question:

What is the relationship between Ruth and
Sheila?

## CLUTRR: Kinship Reasoning

## Context:

[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.

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## CLUTRR: Kinship Reasoning

## Context:

[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.

## Question:

What is the relationship between Ruth and Sheila?


## Answer:

Sheilla is Ruth's niece.

## CLUTRR: Kinship Reasoning

## Context:

[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana].

However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.

```
0.951::context(DAUGHTER, "Cristina", "Sheila")
0.002::context(MOTHER, "Cristina", "Sheila")
0.004::context(FATHER, "Cristina", "Sheila")
...
0.001::context(NA, "Cristina", "Sheila")
0.942::context(DAUGHTER, "Christina", "Diana")
0.015::context(MOTHER, "Christina", "Diana")
...
0.002::context(NA, "Sheila", "Diana")
Symbolic Context
```

> For each pair of names, classify them into 21 types of kinship

Mugen: Text/Video Retrieval

## Mugen using Scallop...

Mugen is a dataset containing multi-modality data (video, text, audio, etc)
In this task, we consider the video-text-alignment. The model takes in video and text, and returns whether they are aligned or not.

Video timeline



Text description

Mugen climbs up on a ladder, and walks to the right and collects a few coins

## Mugen using Scallop...

Video timeline


## Text description

Mugen climbs up on a ladder, and walks to the right and collects a few coins



## Mugen using Scallop...



## Mugen using Scallop...

Video timeline
 detailed actions from the video, providing better interpretability and explainability
0.99: :action(FRAME0, "climb"); ...
0.85: :action(FRAME5, "walk");
0.85::mod(FRAME5, "collect-coin"); ...
0.01::action(FRAME10, "jump"); ...

## Mugen using Scallop...

## Training video- and text-retrieval models under constrastive learning

## Potential Text Descriptions...

Video timeline


Choice A: Mugen climbs up on a ladder, and walks to the right and collects a few coins

Choice B: Mugen collects 5 coins before jumping to kill an enemy

Choice C: Mugen jumps twice to the right and uses the key to open the door

20bn: Video Reasoning via Linear-Temporal Specifications

## Grounding predicates through actions with Scallop...

Given a real life video of people doing a pre-specified task, ground the details of the shown objects indicated in the video

Example:

- Task: attach("sticky note", "cabinet")
- Predicates:

- far("sticky note")
- touching("sticky note", "hand")
- visible("sticky note")
- ...

```
rel precond("attach", a, b) =
touching(a, "hand") and visible("hand") and visible(a)
rel postcond("attach", a, b) =
    not touching(a, "hand") and touching(a, b) and far(a)
```


## Grounding predicates through actions with Scallop...



```
approach(doorknob)
/ Pre-conditions
x Post-conditions
0.02 नclose(doorknob)
0 . 8 1 ~ f a r ( d o o r k n o b )
0.00 ᄀvisible(hand)
1.00 visible(doorknob)
```



```
rel precond("approach", x) =
    far(x) and visible(x)
rel postcond("approach", x) =
    close(x) and not far(x) and visible(x)
```

```
rel precond("attach", a, b) =
    touching(a, "hand") and visible("hand") and visible(a)
rel postcond("attach", a, b) =
    not touching(a, "hand") and touching(a, b) and far(a)
```


## Grounding predicates through actions with Scallop...

Video timeline


## Scallop Foundation Models

## Foundation Models



## Foundation Models: Recap



## Relational Knowledge Extraction with GPT

```
Context:
[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.
```


## Question:

What is the relationship between Ruth and Sheila?

## Relational Knowledge Extraction with GPT

## Context:

[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.


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[Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch.


## Relational Knowledge Extraction with GPT

```
@gpt_extract_relation(
    prompt="Please extract the kinship relationships from the context:",
    examples=[("Alice is Bob's mother", [("alice", "bob", "son"), ...]), ...])
type parse_relations(bound context: String, sub: String, obj: String, rela: String), ..
```


## Relational Knowledge Extraction with GPT

```
Context: [Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch. What is the relationship between Sheila and Ruth?
```

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## Relational Knowledge Extraction with GPT

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Context: [Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch. What is the relationship between Sheila and Ruth?
```

```
@gpt_extract_relation(
    prompt="Please extract the kinship relationships from the context:",
    examples=[("Alice is Bob's mother", [("alice", "bob", "son"), ...]), ...])
type parse_relations(bound context: String, sub: String, obj: String, rela: String), ...
```



## Relational Knowledge Extraction with GPT

```
Context: [Cristina] was afraid of heights just like her daughters, [Sheila] and [Diana]. However, [Diana]'s father, [Jonathan], loved heights and even went skydiving a few times. [Ruth] and her son, [Jeremy], went to the park, and had a wonderful time. [Jeremy] went to the bakery with his uncle [Jonathan] to pick up some bread for lunch. What is the relationship between Sheila and Ruth?
```

```
@gpt_extract_relation(
    prompt="Please extract the kinship relationships from the context:",
    examples=[("Alice is Bob's mother", [("alice", "bob", "son"), ...]), ...])
type parse_relations(bound context: String, sub: String, obj: String, rela: String), ...
rel kinship(p1,p2,rela) = context(ctx) and parse_relations(ctx,p1,p2,rela)
rel kinship(p1,p3,r3) = kinship(p1,p2,r1) and kinship(p2,p3,r2) and composition(r1,r2,r3)
rel answer(r) = question(p1,p2) and kinship(p1,p2,r)
```


## Domain Specific Language (DSL) in Scallop

Question: How many green
objects are there in the image?

```
Count(FilterColor(Scene(), "green"))
```


## Domain Specific Language (DSL) in Scallop

Question: How many green objects are there in the image?

```
Count(FilterColor(Scene(), "green"))
```

```
type Expr = Scene()
    | FilterColor(Expr, String)
    | Count(Expr)
    | Exists(Expr)
```


## Domain Specific Language (DSL) in Scallop

Syntax of a Query DSL:<br>type Expr = Scene() | FilterColor(Expr, String)<br>| Count(Expr) | Exists(Expr) | ...

Semantics:

```
type eval<T>(bound expr: Expr, free output: T)
rel eval<Object>(e, o) = case e is Scene() and object(o)
rel eval<Object>(e, o) = case e is FilterShape(e1, s) and eval<Object>(e1, o) and shape(o, s)
rel eval<Object>(e, o) = case e is FilterColor(e1, c) and eval<0bject>(e1, o) and color(o, c)
rel eval<usize>(e, n) = n := count(o: eval<Object>(e1, o) where e: case e is Count(e1))
rel eval<bool>(e, b) = b := exists(o: eval<Object>(e1, o) where e: case e is Exists(e1))
```


## Semantic Parsing with GPT

Question: How many green objects are there in the image?

Syntax of a query DSL:<br>type Expr = Scene() | FilterColor(Expr, String)<br>| Count(Expr) | Exists(Expr) | ...

## Semantic Parsing with GPT

## Question: How many green objects are there in the image?

```
    Syntax of a query DSL:
type Expr = Scene() | FilterColor(Expr, String)
    | Count(Expr) | Exists(Expr) | ...
```

@gpt_complete(prompt="The programmatic representation of \"\{\{question\}\}\" is \{\{answer\}\}",
examples=[("Is there a sphere?", "Exists(FilterShape(Scene(), \"sphere\"))")])
type semantic_parse(bound question: String, answer: Expr)

## Semantic Parsing with GPT

## Question: How many green objects are there in the image?

```
    Syntax of a query DSL:
type Expr = Scene() | FilterColor(Expr, String)
    | Count(Expr) | Exists(Expr) | ...
```


@gpt_complete(prompt="The programmatic representation of \"\{\{question\}\}\" is \{\{answer\}\}",
examples=[("Is there a sphere?", "Exists(FilterShape(Scene(), \"sphere\"))")])
type semantic_parse(bound question: String, answer: Expr)

| question | answer |
| :--- | :--- |
| How many green objects are there in the image? | Count(FilterColor(Scene(), "green")) |

## Image Classification as Probabilistic Relation

## Image Classification as Probabilistic Relation



```
@clip_classifier(["cat","dog"])
type cat_or_dog(
    bound img: Tensor,
    free label: String,
)
```


## Image Classification as Probabilistic Relation



```
@clip_classifier(["cat","dog"])
type cat_or_dog(
    bound img: Tensor,
    free label: String,
)
```

| prob | id | label |
| :--- | :--- | :--- |
| 0.00 | 0 | cat |
| 0.99 | 0 | dog |
| $\mathbf{0 . 9 8}$ | $\mathbf{1}$ | cat |
| 0.02 | 1 | dog |
| $\ldots$ | $\ldots$ | $\ldots$ |

## Image Segmentation as Probabilistic Relation

## Segment Anything

Research by Meta AI

```
@segment_anything
type image_segment(
    bound img: Tensor,
    free id: u32,
    free segment: Tensor,
)
```


## Image Segmentation as Probabilistic Relation



@segment_anything<br>type image_segment (<br>bound img: Tensor,<br>free id: u32,<br>free segment: Tensor,<br>)

## Image Segmentation as Probabilistic Relation



## Combining Foundation Models

## $\square$ <br> .

Question: How many green objects are there in the image?

```
@segment_anything
type image_segment(
    bound img: Tensor,
    free id: u32,
    free segment: Tensor)
@clip_classifier(["green", "red", ...])
type obj_color(
    bound object_segment: Tensor,
    free label: String)
@gpt_complete(prompt=
    "Please semantically parse the
    following question...")
type semantic_parse(
    bound question: String,
    free answer: Expr)
```

| prob | count |
| :--- | :--- |
| 0.00 | 0 |
| 0.03 | 1 |
| 0.02 | 2 |
| $\mathbf{0 . 9 1}$ | 3 |
| $\ldots$ | $\ldots$ |

## Domain Specific Language (DSL) in Scallop

Syntax of a Query DSL:<br>type Expr = Scene() | FilterColor(Expr, String)<br>| Count(Expr) | Exists(Expr) | ...

Semantics:

```
type eval<T>(bound expr: Expr, free output: T)
rel eval<Object>(e, o) = case e is Scene() and object(o)
rel eval<Object>(e, o) = case e is FilterShape(e1, s) and eval<Object>(e1, o) and shape(o, s)
rel eval<Object>(e, o) = case e is FilterColor(e1, c) and eval<0bject>(e1, o) and color(o, c)
rel eval<usize>(e, n) = n := count(o: eval<Object>(e1, o) where e: case e is Count(e1))
rel eval<bool>(e, b) = b := exists(o: eval<Object>(e1, o) where e: case e is Exists(e1))
```


## Scallop + LLM for Program Analysis

```
type input_program(program: String)
@gpt_extract_info(
    header="""Please point out the dataflow graph in the given Java program""",
    prompts=["What are the dataflow edges?",
                                    "What are the sources of user inputs?"
                            "What are the sinks that may result in vulnerabilities?"],
    examples=[(
        ["public int f(int c) { int i = 0; int out = 0; while (i < c) { out += i; } int j = 42 / out; return out; }"],
        [[("i", "out")], [("c",)], [("out", "int j = 42 / out;")]]
    )])
type gen_dataflow_edge(bound program: String, from: String, to: String),
        gen_source(bound program: String, source: String),
        gen_sink(bound program: String, sink: String, loc: String)
rel source(s) = input_program(pgm) and gen_source(pgm, s)
rel sink(s, l) = input_program(pgm) and gen_sink(pgm, s, l)
rel edge(a, b) = input_program(pgm) and gen_dataflow_edge(pgm, a, b)
rel path(a, b) = edge(a, b) or (path(a, c) and edge(c, b))
rel vul(loc) = source(src) and sink(snk, loc) and path(src, snk)
```


## Scallop + LLM for Program Analysis

## Sample Java File:

```
public void doPost(HttpServletRequest request, HttpServletResponse response)
throws ServletException, IOException {
    // some code
    response.setContentType("text/html;charset=UTF-8");
    javax.servlet.http.Cookie[] theCookies = request.getCookies();
    String param = "noCookieValueSupplied"
    if (theCookies != null) {
        for (javax.servlet.http.Cookie theCookie : theCookies) {
            if (theCookie.getName().equals("Cdsr92")) {
                param = java.net.URLDecoder.decode(theCookie.getValue(), "UTF-8");
                break;
            }
        }
    }
    String fileName = null;
    java.io.FileOutputStream fos = null;
    try {
        fileName = org.pck.bcks.helpers.Utils.TESTFILES_DIR + param;
        fos = new java.io.FileOutputStream(fileName, false);
        response.getWriter().println(ESAPI.encoder().encodeForHTML(fileName))
    } catch (Exception e) {
        // System.out.println("File exception caught and swallowed")
    } finally {
    // we tried
    }
}
```

Extracted dataflow and source/sink information

## edge:

| fileName | fos |
| :--- | :--- |
| param | fileName |
| request | rd |
| request | theCookies |
| request | userCookie |
| theCookies | param |

sink:
fos $\mid$ new java.io.FileOutputStream(fileName, false) |

## source:

```
request
theCookies
```

vul:
new java.io.FileOutputStream(fileName, false) |


Documentation | Downloads | Resources | Tutorials

## Questions

