Calculating Gait Variability with a Markov Chain

Daniel S. Williams
Pennsylvania State University
Email: dmw5737@psu.edu

Anne E. Martin
Pennsylvania State University
Email: aem34@psu.edu

***Abstract*—Quantifying gait variability typically requires long simulations. This abstract introduces a novel method of directly calculating gait variability that yields similar results.**

I. Introduction

Gait parameters correlate with human falls [1], but require hundreds of steps to accurately calculate [2]. While brute force methods of calculating parameters by simulating hundreds of steps are easy to implements, they can be computationally expensive and must be repeated for every condition. We introduce a novel method to directly calculate gait parameters using a Markov chain; we demonstrate with mean and standard deviation (SD), but other measures may be calculated.

II. Methods

Bipedal gait can be analyzed as a Markov chain by generating a deterministic state-transition matrix T^d [3]. T^d describes a chain of steps (the gait) from the nominal step to the failure state given known perturbations at each step. Each state $p \in T^d$ includes the positions and velocities at heelstrike, and the trajectory for each transition was recorded. A threshold determined if a state was already in T^d, so a small range of discrete perturbations resulted in the same state transition. For these ranges, the trajectory from the median perturbation was used. Once a distribution of random perturbations is defined, a Markov chain T^s describing the perturbed gait is created from T^d:

$$T^s = \begin{pmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & 1 \end{pmatrix}, \tag{1}$$

where $\mathbf{0}$ is a zero vector, $\mathbf{Q} \in \mathbb{R}^{t\times t}$, and $\mathbf{R} \in \mathbb{R}^{1\times t}$ [4]. The first t rows and columns, i.e. states in the Markov chain, are transient and the last state is absorbing (i.e. once reached, the biped cannot recover). We then calculate $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$, where $n_{ij} \in \mathbf{N}$ is the expected number of times that the biped will be in state p_j if it starts in state p_i before reaching the absorbing state. Because the trajectory of each transition was recorded, step parameters for every transition are known. With this, we determine the parameters of the gait starting at the nominal step p_1. First, we calculate weighted parameters for step p_i where the weights are the elements $s_{ij} \in T^s$. Then, the overall weighted parameters for the gait is calculated using the first column $n_{1i} \in \mathbf{N}$ as the frequency weights. The overall SD is

$$\sigma^2 = \frac{\sum_{i=1}^{t} n_{1i} \left(\sigma_i^2 + (\mu_i - \mu)^2\right)}{\sum_{i=1}^{t} n_{1i}}, \tag{2}$$

This work was supported by the NSF under award 1727540.

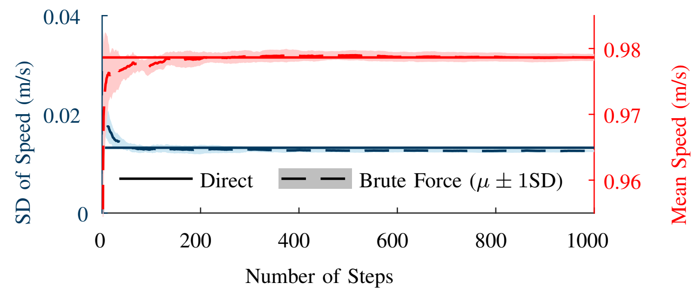

Fig. 1. Change in speed parameters with number of steps. The perturbation distribution was normal ($\mu = 0$, $\sigma = 0.033$ m/s).

where μ_i is the mean and σ_i is the SD with starting state p_i.

III. Results

For testing, a planar six link biped (leg length 1.16m, mass 84.7kg) was simulated. The set of perturbations for T^d was $[-0.2, 0.2]$m/s with 0.002m/s spacing. T^s was found using zero-mean normal distributions with SD of 0.0167, 0.033, and 0.067m/s. The mean and SD of speed was calculated using the direct method. To compare, the biped was simulated for 1000 steps 10 times for each perturbation distribution, and the parameters were calculated up to each step. The methods closely match once hundreds of steps are used with the brute force method (Fig. 1). The difference in means between methods were $< 0.5\%$. The difference in SD were $12\% - 0.80\%$, with larger perturbations resulting in less error. This may be because the sets of SD used to calculate the overall SD are not statistically independent, and Eq. 2 assumes statistical independence. Despite this, the values are similar, and the direct method offers more flexibility because T^d only needs to be found once for a given gait and a T^s can be found for every perturbation distribution.

References

[1] D. Hamacher, N. B. Singh, J. H. Van Dieen, M. O. Heller, and W. R. Taylor, "Kinematic measures for assessing gait stability in elderly individuals: A systematic review," *J. Roy. Soc. Interface*, vol. 8, no. 65, pp. 1682–1698, 2011.

[2] T. M. Owings and M. D. Grabiner, "Measuring step kinematic variability on an instrumented treadmill: How many steps are enough?" *J. Biomech.*, vol. 36, no. 8, pp. 1215–1218, 2003.

[3] C. O. Saglam and K. Byl, "Robust policies via meshing for metastable rough terrain walking," in *Robotic. Sci. Syst.*, 2014.

[4] C. M. Grinstead and J. L. Snell, *Introduction to Probability*. American Mathematical Soc., 2012.