Calculating Gait Variability with a Markov Chain

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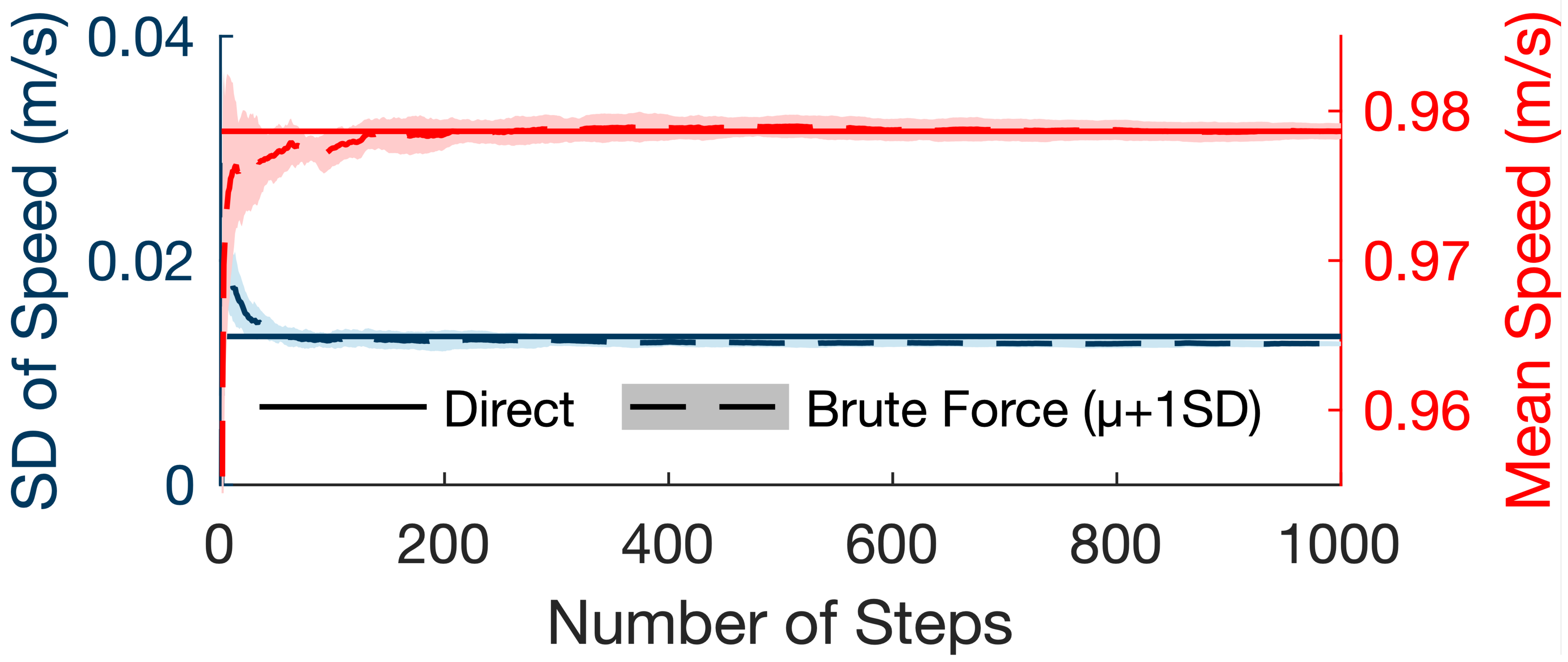
Introduction

- Gait parameters such as the mean and standard deviation of speed correlate with human falls [1].
- However, they can require hundreds of steps to accurately calculate using brute force methods [2].
- Further, brute force methods need to be repeated if testing conditions change.
- We introduce a novel method of directly calculating gait parameters using an absorbing Markov Chain.

Results

- We compared this novel method to a brute force method calculating walking speed.
  - The brute force method used 10 simulations of 1000 steps each.
- Testing was done using a six link, planar biped for three perturbation conditions.
- Brute force was inaccurate for low numbers of steps
- The difference in means between methods were < 0.5% when the brute force method was used for 1000 steps.
- The difference in SD were 0.80% to 12%.



Acknowledgements

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Methods

- Bipedal gait was represented as an absorbing Markov chain.
- First, a deterministic state-transition matrix T^d was created [3]:
  - The states of the Markov chain were the positions and velocities at heelstrike.
  - The transition from one state to another was dependent on the perturbation.
- This T^d can be used to analyze many testing conditions and only needs to be created once.
- By combining T^d and a specific distribution of random perturbations the absorbing Markov chain T^s was created.

$$T^s = \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{0} & 1 \end{pmatrix}$$

- T^s was used to find the expected number of times the biped will be in each state before falling.

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$$

where $n_{ij} \in \boldsymbol{N}$ is the expected number of times the biped will be in state p_j if it starts in state p_i.

- Weighted mean μ_i and standard deviation (SD) σ_i for each step that starts at state p_i were calculated using gait data saved when calculating T^d and $\boldsymbol{N}$.
- The overall parameters were found using

$$\mu = \frac{\sum_{i=1}^{t} n_{1i}\mu_i}{\sum_{i=1}^{t} n_{1i}}, \quad \sigma^2 = \frac{\sum_{i=1}^{t} n_{1i}(\sigma_i^2 + (\mu_i - \mu)^2)}{\sum_{i=1}^{t} n_{1i}}$$

References

- [1] Hamacher, D, et al. J. Roy. Soc. Interface 8, 1682-1698, 2011.
- [2] Owings, TM & Grabiner, MD. J. Biomech. 36, 1215-1218, 2003
- [3] Saglam, CO & Byl, K. Robotic. Sci. Syst., 2014.
- [4] Grinstead, CM & Snell, JL, Amer. Math. Soc., 2012