Viability and Global Stability of a Task-Regulated Compass Walker

Navendu S. Patil
Department of Kinesiology
Pennsylvania State University
University Park, PA, USA
nsp129@psu.edu

Jonathan B. Dingwell
Department of Kinesiology
Pennsylvania State University
University Park, PA, USA
dingwell@psu.edu

Joseph P. Cusumano
Department of Engineering Science & Mechanics
Pennsylvania State University
University Park, PA, USA
jpc3@psu.edu

***Abstract*—Experiments of human treadmill walking motivate an important question: why humans prefer one task-level regulation strategy over another; perhaps to enhance the ability to reject large disturbances? Here, we study task-level regulation and global stability in a powered compass walker on a treadmill, with additional speed and position regulators. We identify the viable region in the walker's state space where it is possible to step indefinitely by push-off control, thus allowing meaningful comparison of regulation strategies. For treadmill walking, we find that speed regulation enlarges and regularizes the walker's basin of attraction much more than position regulation. Thus, our results extend the experimental finding that humans strongly prioritize regulating speed from one stride to the next, even as they minimize metabolic cost on average. We further explain the geometric structure of the viable region and the basins within it via the invariant manifolds of the walker's hybrid dynamics.**

***Index Terms*—viability, task-level regulation, basin of attraction, bipedal walking**

I. Introduction and Methods

It remains unclear how short-term stability considerations, along with long-term energy minimization, shape the way humans walk. While active neuromotor control keeps them upright, humans also need to choose from multiple regulation strategies to achieve one or more task goals, such as maintaining a desired speed or direction. On a treadmill, humans tightly regulate speed at successive strides, while allowing their absolute position to drift for several strides [1]. Why this is so, remains an open question. Here, we impose different task-level regulation strategies on a powered walker and compare its ability to reject large state disturbances. We also identify a nonviable set of states for which the walker cannot avoid failure, let alone regulate to achieve task goals, even with the best possible active control

We used a 2D compass walker [2] with push-off P applied just before heel strike. We added nonlinear, optimal, speed or position regulators by applying P at each step obtained by minimizing a quadratic speed or absolute position cost, respectively. We assessed the treadmill walker's global stability by constructing basins of attraction (BoAs) of its period-1 gaits. We also identified a set of states iteratively for which the walker violates the viability constraints (including $P \geqslant 0$).

NIH/NIA Grant # R01-AG049735

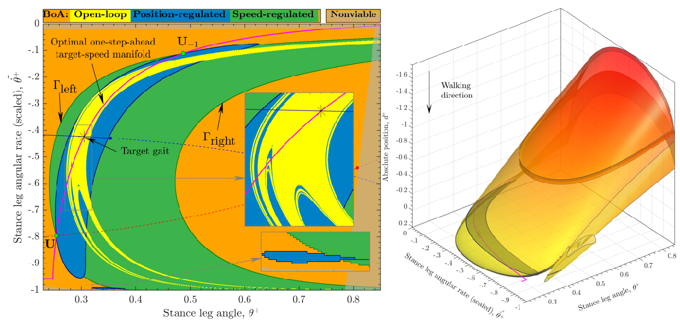

Fig. 1. (***left***) 2D slices of the BoAs of the open-loop and task-regulated walker. 2D Basin areas (% of viable area): 3.11% (open-loop), 6.54% (position-regulated; d^*-slice), and 36.30% (speed-regulated). Viable region is the state space area excluding states marked *Nonviable*. (***right***) 3D BoA of the position-regulated walker along with its 2D slice, $d^+ = 0$. Walker's absolute position d^+ on a treadmill is the 3rd state variable that was set to $d^* := 0$ for the 2D BoAs. Maximum slice area is 27.58% of viable area at $d^+ = -0.797$. 3D position-regulated BoA occupies $\approx 15\%$ viable volume and includes majority of states for which the walker starts in the back half of the treadmill ($d^+ < 0$).

II. Results and Discussion

The open-loop BoA (fixed P; Fig. 1 *left*) is quite thin with many disjoint boundaries. Both position (Fig. 1 *right*) and speed regulators yield connected BoAs which are much bigger than the open-loop BoA. Specifically, the speed regulation greatly improves the walker's global stability: its BoA occupies a maximum of 56% of viable area (speed ≈ 0.244). This is significant given that task-level regulators do not prioritize global stability *a priori*, unlike the "smartest possible" controls [3] that keep states within the viable region. We further explain the boundaries of the BoAs and the viable region via the invariant manifolds of the walker's noninvertible hybrid dynamics and also the optimality of the task-level regulation.

These simulations illustrate a previously unappreciated connection between task-level regulation strategies and global stability. Further, they suggest that humans might select, from among multiple options, strategies that most enhance motor robustness and, hence, decrease risk of fall.

References

[1] Dingwell et al. (2010). *PLoS Comput Biol*, **6**: e1000856.
[2] Kuo AD. (2001). *ASME J Biomech Eng*, **124**: 113–120.
[3] Zaytsev et al. (2018). *IEEE Trans Robot*, **34**(2): 336–352.