Limit Cycle Control On Hopping Motion

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I. Introduction

Hopping is considered one of the primitive motion behaviors when studying legged robot. In order to hop, the robot has to know how to pump energy into itself to maintain certain height to prevent it from stumbling. Due to the intermittent between the stance and flight phase dynamics, even though such a simple system like vertical hopping, the analysis of such systems has long been recognized as complex and interesting dynamical control problem [1]. In this paper, we present a continuous norm-regulation-based limit cycle control (NRC) for vertical hoppers. Our approach provides continuous real-time norm regulation during the stance phase, leading to faster convergent rate and larger disturbance rejection capability as compared to the conventional impulsive or continuous stance phase control approaches (called phase-locked controller) [2]. Simulations and experiments were conducted to demonstrate the effectiveness of the proposed controller.

II. Norm-Regulation Limit Cycle Control (NRC)

The vertical hopper is a discrete system, which is described by two different dynamic equations,
stance phase ($x_1 \leq 0$):

$$\dot{\boldsymbol{x}} = -\omega \boldsymbol{J}\boldsymbol{x} + (-2\beta\omega x_2 + f/\omega)\boldsymbol{e}_2, \tag{1}$$

flight phase ($x_1 > 0$):

$$\ddot{x}_1 = -g, \tag{2}$$

where $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\boldsymbol{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $\boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. m, k are mass and spring constant. g represents the gravitational acceleration while F denotes the force generated by an actuator. $\omega = \sqrt{\frac{k}{m}}$ is the natural frequency of the system. $\beta := \frac{b}{2m\omega}$, where b denotes the damping coefficient of the damper. Our goal is to design a controller that drives the system (Eq. 1) to the desired limit cycle defined in stance phase, and to prove that such controller can bring the stability to the overall discrete system (Eqs. 1 and 2) by using the Poincaré map analysis (Fig.1a). Detailed proof can be found in [3].

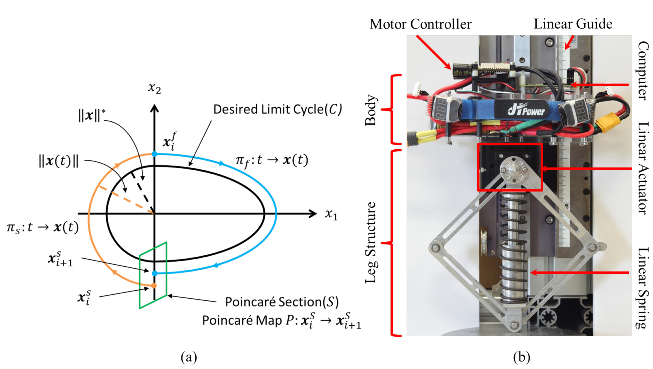

Fig. 1. (a) Conceptual diagram of the phase plot of the vertical hopping system using NRC. (b) Physical hardware of a vertical hopper.

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Consider the proposed controller,

$$f = 2\beta\omega^2 x_2 - k_R(2\|\boldsymbol{x}\|\omega)/(x_2+\epsilon)\left(1 - (\|\boldsymbol{x}\|^*)/(\|\boldsymbol{x}\|)\right), \tag{3}$$

where $k_R > 0$ is adjustable, and ϵ is a small number which carries same sign as x_2, i.e. $\epsilon = |\epsilon| sign(x_2)$. Substituting the controller into (1) follow by some calculation yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{x}\| = -k_R \frac{x_2}{x_2+\epsilon}\left(1 - \frac{\|\boldsymbol{x}\|^*}{\|\boldsymbol{x}\|}\right). \tag{4}$$

Hence, $\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{x}\| < 0$ if $\|\boldsymbol{x}\| > \|\boldsymbol{x}\|^*$, $\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{x}\| = 0$ if $\|\boldsymbol{x}\| = \|\boldsymbol{x}\|^*$, and $\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{x}\| > 0$ if $\|\boldsymbol{x}\| < \|\boldsymbol{x}\|^*$. Furthermore, the Lyapunov function,

$$V = \left(1 - \frac{\|\boldsymbol{x}\|^*}{\|\boldsymbol{x}\|}\right)^2 \tag{5}$$

is positive definite when $\|\boldsymbol{x}\| \neq 0$, and $V = 0$ when $\|\boldsymbol{x}\| = \|\boldsymbol{x}\|^*$. And the time derivative,

$$\dot{V} = -2k_R \frac{x_2}{x_2+\epsilon}\frac{\|\boldsymbol{x}\|^*}{\|\boldsymbol{x}\|^2}\left(1 - \frac{\|\boldsymbol{x}\|^*}{\|\boldsymbol{x}\|}\right)^2 \tag{6}$$

is negative definite when $\|\boldsymbol{x}\| \neq 0$, and $\dot{V} = 0$ when $\|\boldsymbol{x}\| = \|\boldsymbol{x}\|^*$. This shows analytically the stability of the system under NRC.

III. Simulation and Experiment

The performance of hopping with the use of the existing controller and NRC are compared in simulation as shown in Fig. 2. And the effectiveness of the use of NRC on physical hardware is shown in Fig. 3. The results confirm the proposed NRC ensures fast convergent rate in both simulation, and most importantly on the real system.

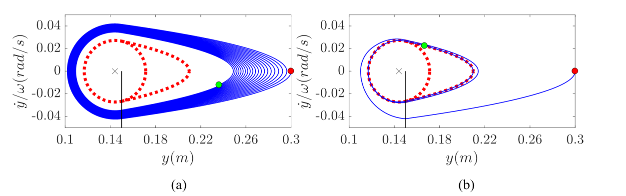

Fig. 2. Simulation result of limit cycle control using (a) Existing controller [2] and (b) NRC. y, l_0 denote the height of the mass and the natural length of the spring respectively. The red points denote the initial state, the green points denote the final state, and the red dotted lines denote the desired limit cycle.

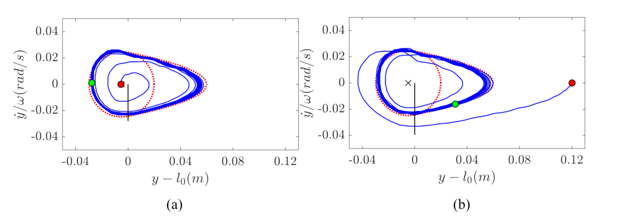

Fig. 3. Experimental result of limit cycle control converging (a) outward and (b) inward.

References

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