Limit Cycle Control on Hopping Motion

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Motivation

- Hopping height control is one of the sub-tasks in legged-mobile robot
- From the natural behavior of a damped spring-mass system, we observed the decrease in the 2-norm of the state-space plot
- To control the hopping height, we control the system to travel on a limit cycle when it is in contact with the ground
- As the robot is under real-time feedback control on ground and can handle uncertainty in ground condition if there is any

Model and Hardware

The dynamic equation of the model is as follow:

$$m\ddot{x} + b\dot{x} + kx = F$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x}/\omega \end{bmatrix} = -\omega \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x}/\omega \end{bmatrix} - 2\zeta \begin{bmatrix} 0 \\ \dot{x}/\omega \end{bmatrix} + \begin{bmatrix} 0 \\ F/m\omega \end{bmatrix}$$

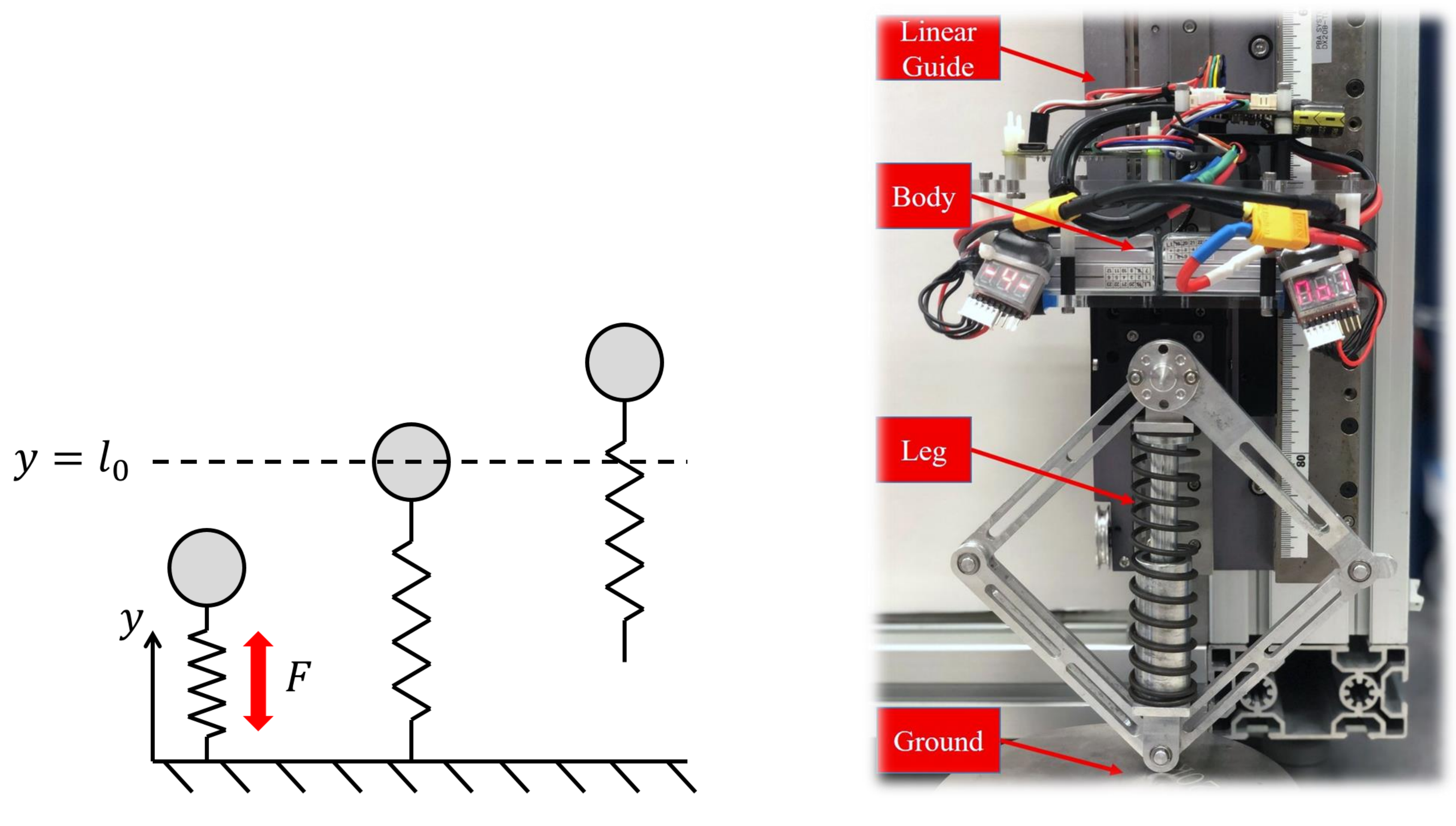

Fig. 1. (a) Model of vertical hopper. (b) Physical hardware of vertical hopper.

Control

The time derivative of the 2-norm of the state vector is as follow:

$$\|\dot{\boldsymbol{x}}\| = \frac{x_2}{\|\boldsymbol{x}\|\omega}\left(-2\zeta\omega^2 x_2 + \frac{F}{m}\right)$$

The proposed controller, Norm-regulation Control (*NRC*) is chosen as follow:

$$F = 2m\zeta\omega^2 x_2 + k_{NRC}\frac{m\omega\|\boldsymbol{x}\|}{x_2}\left(1 - \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|^*}\right)$$

And hence the time derivative becomes:

$$\|\dot{\boldsymbol{x}}\| = k_{NRC}\left(1 - \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|^*}\right)$$

$$\begin{cases} < 0 \ if \ \|\boldsymbol{x}\| > \|\boldsymbol{x}\|^* \\ = 0 \ if \ \|\boldsymbol{x}\| = \|\boldsymbol{x}\|^* \\ > 0 \ if \ \|\boldsymbol{x}\| < \|\boldsymbol{x}\|^* \end{cases}$$

As a result, if the norm is larger than the desired norm, the controller will pull the state inward; if the norm is smaller than the desired norm, the controller will push the state outward. Fig.2 shows the simulation result.

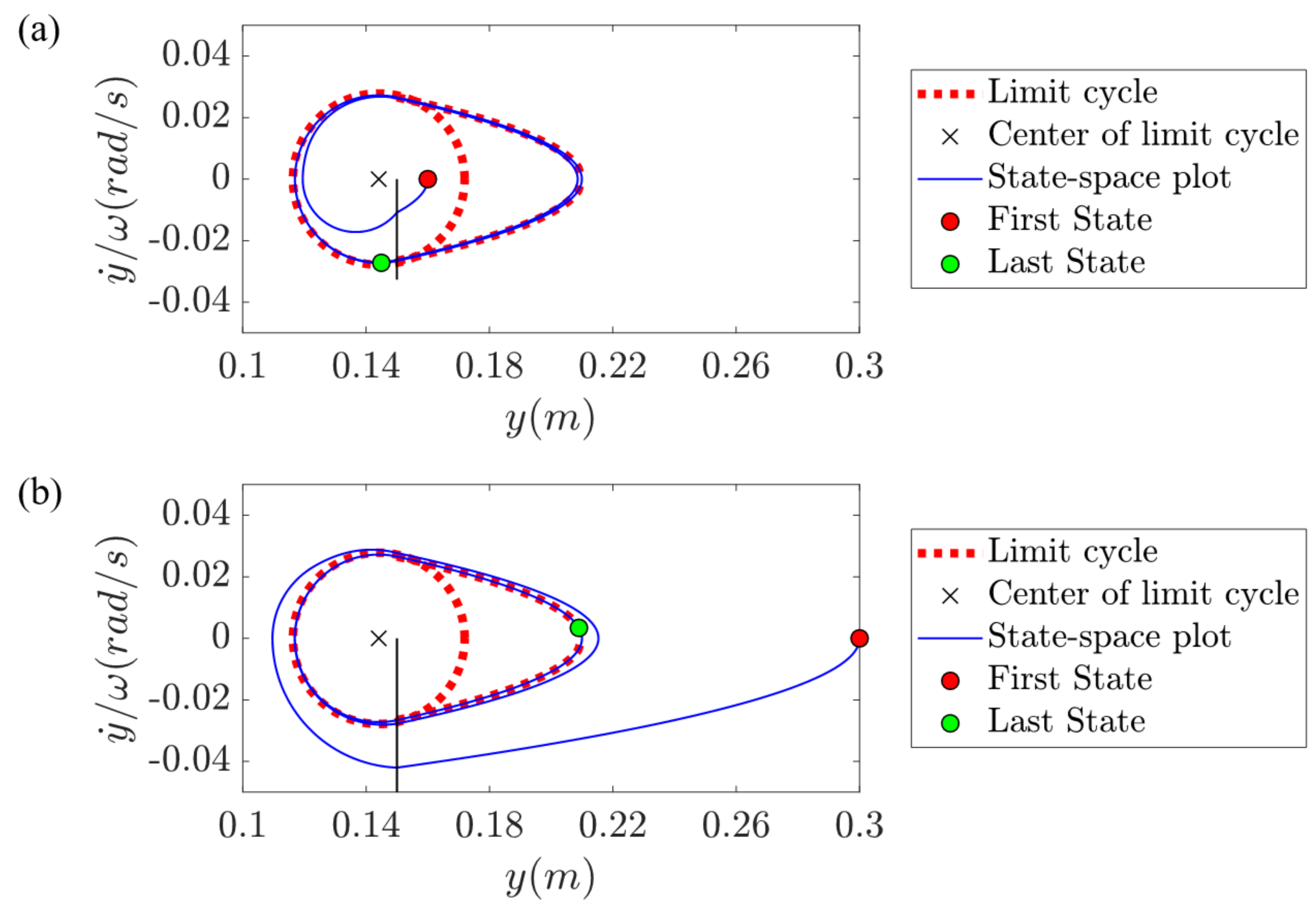

Fig. 2. (a) Outward convergence. (b) Inward convergence.

Results

Fig. 3 shows one of the experimental results. The state converges back to the desired limit cycle immediately after the collision. Note that there is only control when the system is on the ground, Hence the trajectory starts converging only when the state is on the left half plane of the state space plot.

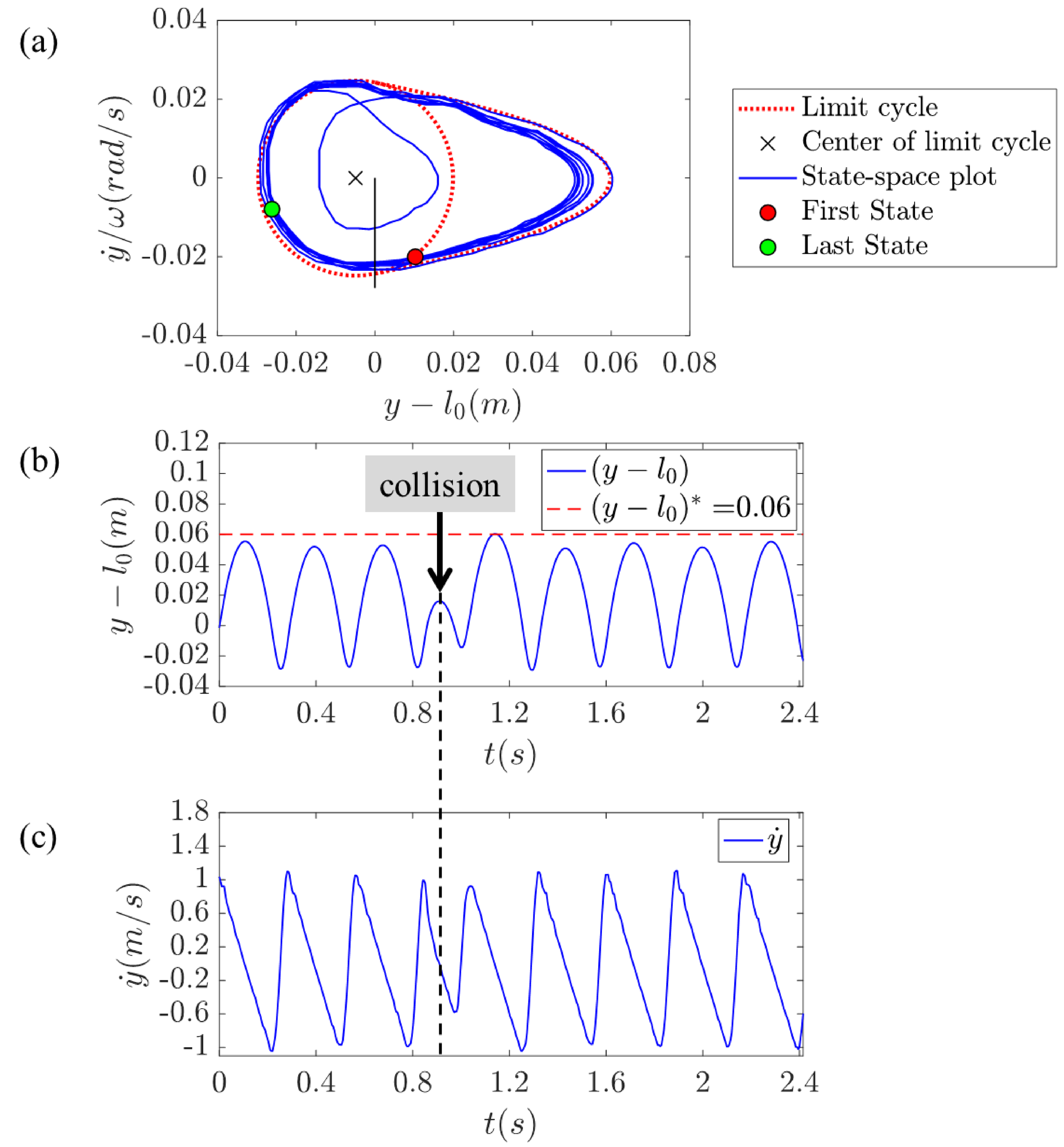

Fig. 3. Experimental result of hopping control with disturbance. (a) State-space plot of the vertical hopper under a single collision in flight phase. (b) Hopping height of the vertical hopper. (c) Velocity of the vertical hopper.

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