Rapid Gait Optimization in CasADi

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I. Introduction and Approach

Passive and underactuated dynamic walking research began with planar models, but has been extended to 3D, where the control challenges are substantially more complex. Still, considerable research effort continues to limit biped locomotion to the plane because the easier analysis leads to faster simulations and optimizations. Recent tool development, though, has enabled rapid trajectory optimization for gait generation on complex 3D bipeds [1]. This abstract demonstrates that TOPIC[1]: a Trajectory Optimization Package In CasADi with a MATLAB interface provides a highly efficient framework for modeling constrained trajectory optimization problems for hybrid dynamical systems using advanced direct collocation methods. CasADi is an open-source software written in self-contained C++ for solving numerical optimization problems [2]. Nonlinear programming (NLP) solvers often require in-depth knowledge of optimization techniques, but TOPIC combines CasADi's building blocks for modeling optimization problems with unprecedented efficiency with MATLAB's interpreted language for user-friendly NLP implementation.

For high-dimensional systems, CasADi's state-of-the-art implementation of algorithmic differentiation for computing derivative information of the cost function and constraints is particularly advantageous. CasADi uses a sparse matrix representation of the necessary symbolic expressions that allows for efficient—both in terms of time and memory—computation of matrix multiplications and function evaluations. This capability is critical in creating a large, but efficient NLP formulation for complex biped robots, especially using direct collocation methods.

With the user's biped model provided as a Denavit-Hartenberg table, TOPIC derives the equations of motion using spatial vector arithmetic and dynamics algorithms and transcribes the trajectory optimization problem into an NLP using the selected collocation method. The MATLAB interface facilites gait-specific constraint declaration, and CasADi's full-featured front-end to numerical solvers provides seamless integration in IPOPT. Analogous to FROST [1], a MATLAB optimization toolbox that efficiently generated stable walking motions on full-order models of Atlas, DRC-HUBO, and Cassie, TOPIC unifies the motion planning and controller synthesis problems.

[1] https://github.com/fevrem/TOPIC

II. Results & Discussion

The performance of TOPIC and FROST were compared in building a 10-gait library for a 7-DOF planar five-link biped model spanning 0.15 to 1.05 m/s. TOPIC converged to nearly identical gaits (from the same initial conditions) on average 4 times faster than FROST (Fig. 1). Using conventional symbolic computer algebra in Mathematica, FROST (re)declare the virtual constraint parameters within each finite element of the trajectory in order to produce an efficient band structure in the Jacobian matrix of the constraints. In contrast, the sparse matrix representation of CasADi symbolics gives rise to a formulation that is highly efficient in nature using far less decision variables, leading to faster gait generation.

TOPIC was more recently employed on a spatial five-link biped model with 12 DOF and allowed gaits to be generated in less than 10 seconds. To the best of the authors' knowledge, this is the first application of CasADi's efficient backbone to gait and controller design on full-order biped models. Through CasADi's efficient NLP formulation, TOPIC makes online gait generation accessible and open source to the community.

References

[1] A. Hereid and A. D. Ames, "FROST*: Fast Robot Optimization and Simulation Toolkit," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems*, 2017, pp. 719–726.

[2] J. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl, "CasADi – A software framework for nonlinear optimization and optimal control," *Mathematical Programming Computation*, vol. 11, no. 1, pp. 1–36, 2019.

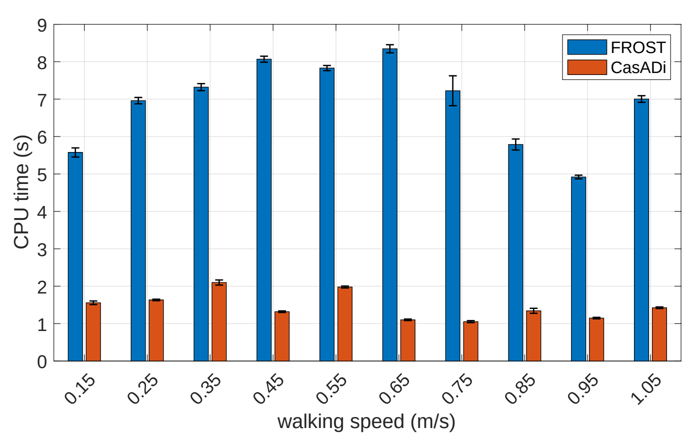

Fig. 1. CPU time to generate 10-gait library for 7-DOF planar five-link. Error bars represent standard deviation (10 gaits generated at each walking speed).