A Fast Solver for Contact-Implicit Trajectory Optimization

Taylor Howell and Zachary Manchester
Stanford University, California, USA
{thowell, zacmanchester}@stanford.edu

I. INTRODUCTION

Contact-implicit trajectory optimization can generate physically accurate motion plans for robots that make and break contact with their environments without requiring prespecified or fixed contact-mode sequences [1]. This tool, in a model-predictive control framework, promises to produce dynamic and agile locomotion plans with interesting emergent behaviors, such a sliding. However, current implementations typically require minutes of compute time to produce trajectories lasting only a few seconds.

In order to substantially reduce solve times for this class of problems, we propose the development of a custom solver. This solver should: be well-suited for handling complementarity constraints such as interpenetration and friction modeled using maximum energy dissipation, exhibit superlinear convergence, and exploit problem structure. Preliminary work has found that custom interior-point methods exhibit these properties for simulating contact and we intend to extend these promising results to the trajectory optimization framework where an objective is simultaneously minimized.

II. BACKGROUND

Previous work on contact-implicit trajectory optimization has exclusively relied upon general purpose nonlinear programming (NLP) solvers such as SNOPT [2] or Ipopt [3] to solve problem formulations based on Stewart and Trinkle's time-stepping method for rigid-body contact [4]. While there have been successes using this approach, complementarity constraints such as those in,

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) \geq 0,\ h(x) \geq 0,\ g(x)h(x) = 0, \end{aligned} \tag{1}$$

are notoriously difficult to solve and their general numerical solution is an open problem in operations research [5].

To aid the solver, slack variables are typically introduced in an ad hoc manner to handle these constraints. Further, SNOPT usually operates in its "elastic mode," which introduces the constraints as penalties in the objective, whereas Ipopt will utilize a "feasibility restoration" mode where a secondary problem is solved, aiding convergence of the primary problem. Both approaches result in slow, sub-linear convergence rates.

A more principled approach, implemented as Ipopt-C [6], utilizes an additional central-path-like parameter to introduce complementary slackness to each complementarity constraint. The authors report superlinear convergence for this method.

III. APPROACH

In this work we develop a custom interior-point solver tailored for handling the interpenetration and friction complementarity constraints using an approach similar to that of Ipopt-C (i.e., having both constraint-level $s \rightarrow \infty$ and problem-level $t \rightarrow \infty$ complementary slackness parameters). Problem (1) can be reformulated as the barrier problem,

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) - \frac{1}{t}\log(g(x)) - \frac{1}{t}\log(h(x)) \\ \text{subject to} \quad & g(x)h(x) = \frac{1}{s}. \end{aligned} \tag{2}$$

Handling these constraints in a principled way, unlike previous approaches that attempt to fit the problem into a specific form for an NLP solver, should increase solver robustness and lead to superlinear convergence. Additionally, a custom linear solver will be developed to take advantage of the specific sparsity structure of the problem, leading to further speedups. Our preliminary results for simulating contact using custom interior-point methods has found superior performance compared to augmented Lagrangian and non-smooth, proximal algorithms. Early work suggests that this approach will also extend to the trajectory optimization framework.

REFERENCES

[1] M. Posa, C. Cantu, and R. Tedrake, "A direct method for trajectory optimization of rigid bodies through contact," en, *The International Journal of Robotics Research*, vol. 33, no. 1, pp. 69–81, Jan. 2014.

[2] P. E. Gill, W. Murray, and M. A. Saunders, "Snopt: An sqp algorithm for large-scale constrained optimization," *SIAM review*, vol. 47, no. 1, pp. 99–131, 2005.

[3] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," en, *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, Mar. 2006.

[4] D. E. Stewart and J. C. Trinkle, "An Implicit Time-Stepping Scheme for Rigid Body Dynamics with Inelastic Collisions and Coulomb Friction," en, *International Journal for Numerical Methods in Engineering*, vol. 39, no. 15, pp. 2673–2691, 1996.

[5] R. Fletcher and S. Leyffer, "Solving mathematical programs with complementary constraints as nonlinear programs," *Optimization Methods and Software*, vol. 19, Feb. 2004.

[6] A. U. Raghunathan and L. T. Biegler, "An Interior Point Method for Mathematical Programs with Complementarity Constraints (MPCCs)," en, *SIAM Journal on Optimization*, vol. 15, no. 3, pp. 720–750, Jan. 2005.