Big Idea

Contact-implicit trajectory optimization generates motion plans for walking robots without requiring prespecified or fixed contact-mode sequences. Existing non-convex solvers, such as Ipopt, poorly handle complementarity constraints, lack native support for cone constraints, and fail to exploit structure within the symmetric KKT system. We developed a new solver to handle complex locomotion problems.

Augmented Lagrangian + Barrier

- hybrid formulation solves sequence of smooth problems that converge to hard-contact solution

\[
\min_z \quad f(x) - \mu \sum \log(z^{(i)} - z_L^{(i)}) + \lambda_T^T r + \frac{\rho}{2} r^T r
\]

s.t.

- \( c_T(x) - s = 0 \),
- \( c_F(x) = 0 \),
- \( c_A(x) - r = 0 \),
- \( z \geq z_L \)

- augmented Lagrangian implicitly relaxes and smooths complementarity constraints
- barrier for inequality constraints and friction cone

Second-order Cone Constraints

- second-order friction cone is squared and embedded in log barrier without approximation
- decision variables are reduced by at least 4 per contact
- simulated sliding particle with linearized friction cone experiences drift compared to second-order friction cone

Problem Structure

- smart linear algebra exploits banded structure in Hessian of the Lagrangian and constraint Jacobian to reduce complexity of linear solve
- introduction of slacks has limited effect on complexity

Raibert Hopper 2D
- horizon: \( T = 50 \)
- states: \( n = 5 \), controls: \( m = 2 \)

Compass 2D
- horizon: \( T = 15 \)
- states: \( n = 5 \), controls: \( m = 2 \)

Kneed Walker 2D
- horizon: \( T = 20 \)
- states: \( n = 7 \), controls: \( m = 4 \)

Hopper 3D
- horizon: \( T = 20 \)
- states: \( n = 7 \), controls: \( m = 3 \)