A Fast Solver for Contact-implicit Trajectory Optimization

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Big Idea

Contact-implicit trajectory optimization generates motion plans for walking robots without requiring prespecified or fixed contact-mode sequences.

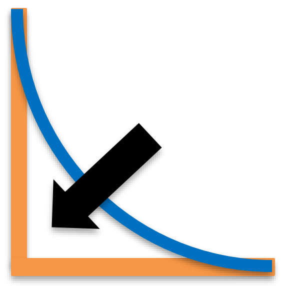
Existing non-convex solvers, such as Ipopt, poorly handle **complementarity constraints**, lack native support for **cone constraints**, and **fail to exploit structure** within the symmetric KKT system.

We developed a new solver to handle complex locomotion problems.

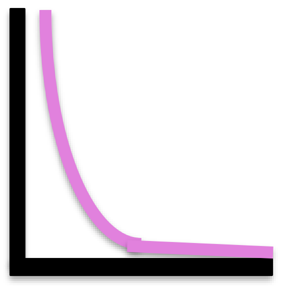
Augmented Lagrangian + Barrier

- hybrid formulation solves sequence of smooth problems that **converge to hard-contact solution**

$$
\begin{aligned}
\min_{z := (x,s,r)} \quad & f(x) - \mu \sum \log(z^{(i)} - z_L^{(i)}) \\
& + \lambda^T r + \frac{\rho}{2} r^T r \\
\text{s.t.} \quad & c_{\mathcal{I}}(x) - s = 0, \\
& c_{\mathcal{E}}(x) = 0, \\
& c_{\mathcal{A}}(x) - r = 0, \\
& (z \geq z_L)
\end{aligned}
$$



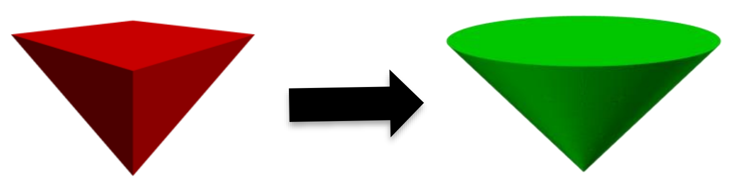
- augmented Lagrangian implicitly **relaxes and smooths complementarity constraints**

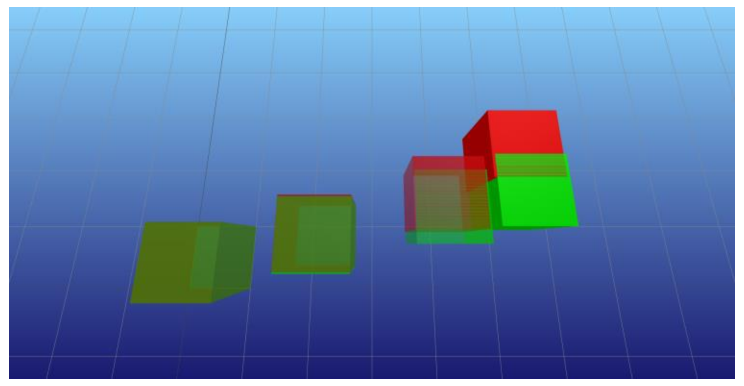


- barrier for inequality constraints and **friction cone**

Second-order Cone Constraints

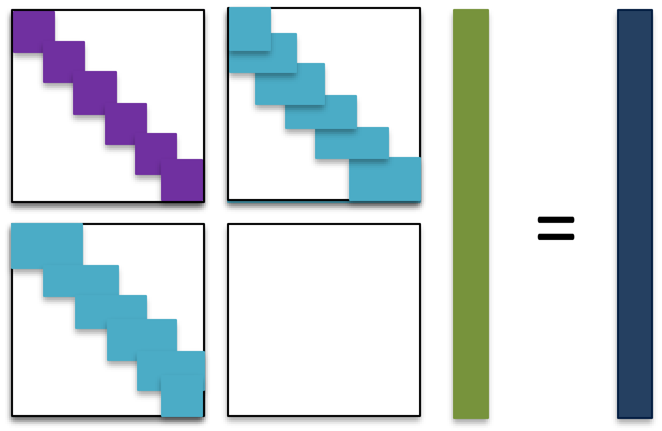
- **second-order friction cone** is squared and embedded in log barrier without approximation





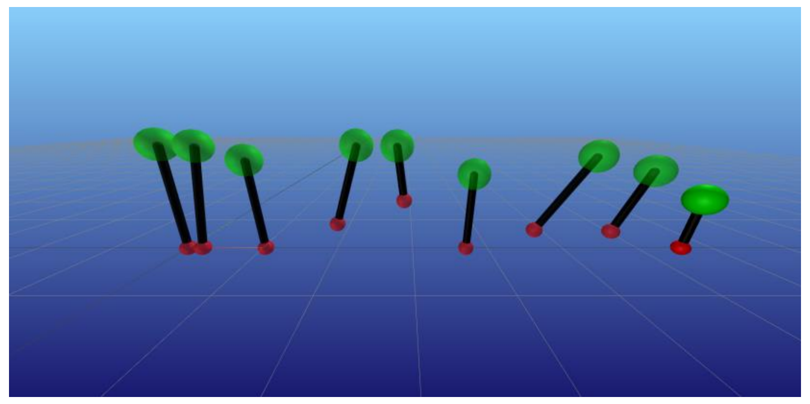
- decision variables are **reduced by at least 4 per contact**
- simulated sliding particle with **linearized friction cone** experiences drift compared to **second-order friction cone**

Problem Structure



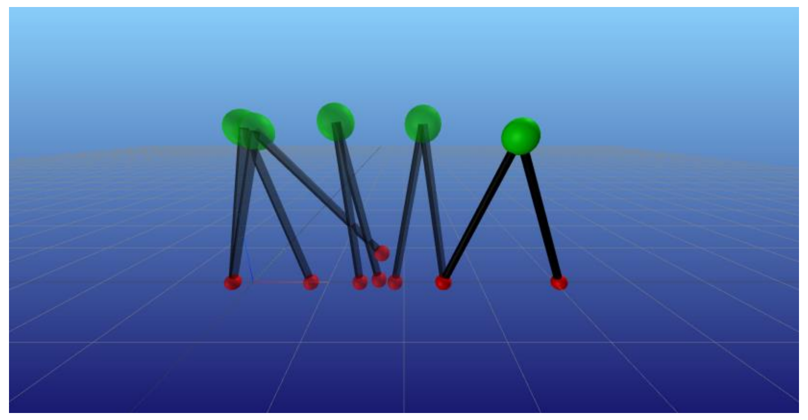
- smart linear algebra exploits banded structure in **Hessian of the Lagrangian** and **constraint Jacobian** to reduce complexity of linear solve
- introduction of slacks has limited effect on complexity

Raibert Hopper 2D



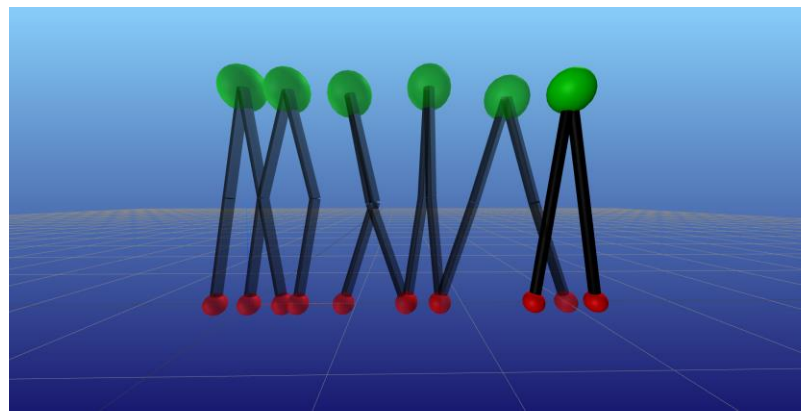
- horizon: T = 50
- states: n= 5, controls: m = 2

Compass 2D



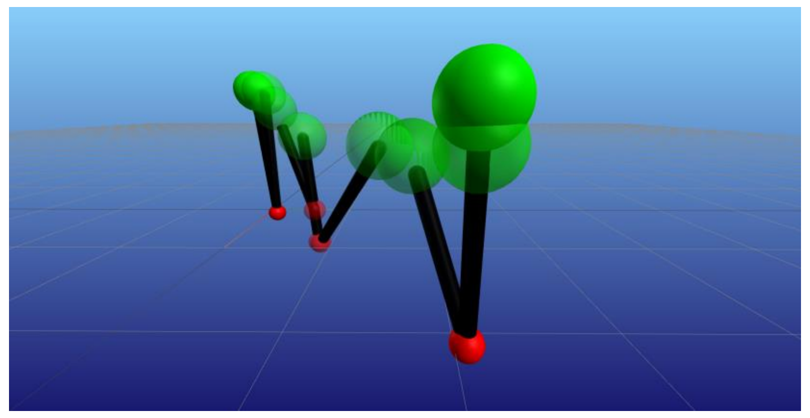
- horizon: T = 15
- states: n = 5, controls: m =2

Kneed Walker 2D



- horizon: T = 20
- states: n = 7, controls: m = 4

Hopper 3D



- horizon: T = 20
- states: n = 7, controls: m = 3