

# Robust optimal control for biomechanical simulations

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## I. INTRODUCTION

The robustness of human motor control to uncertain perturbations is remarkable but often neglected in biomechanical simulations, where open-loop control is typically assumed. Open-loop control prompts deterministic trajectory optimization with direct collocation as the method of choice to efficiently deal with the non-linear and stiff dynamics. The resulting control laws are generally not robust to even the smallest of perturbations. Existing approaches to movement simulation that generate control laws that are robust to modelled noise lack flexibility in the design of the control law and computational efficiency and have therefore only been applied to a small class of problems.

We therefore extended our optimal control framework for trajectory optimization to a generally applicable robust optimal control framework. Here, we show the potential of this method by predicting postural sway for healthy controls and vestibular loss subjects when perturbed by platform rotations in the presence of motor, proprioceptive and vestibular noise.

## II. METHODS

### A. Robust optimal control method

Consider a system with dynamics  $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{w})$ , with  $\mathbf{x}$  the states,  $\mathbf{u}$  the controls and  $\mathbf{w}$  a set of zero mean Gaussian disturbances (noise). To account for uncertainty, we augment the state-space by adding the state co-variance matrix  $\mathbf{P}$ . We describe the propagation of the co-variance matrix by the continuous Lyapunov equation [1]:  $\dot{\mathbf{P}} = \mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T + \mathbf{C}\Sigma_w\mathbf{C}^T$ , with  $\mathbf{A} = \partial f(\mathbf{x}, \mathbf{u}, 0)/\partial \mathbf{x}$ , and  $\mathbf{C}\Sigma_w\mathbf{C}^T$  the effect of noise with  $\mathbf{C} = \partial f(\mathbf{x}, \mathbf{u}, \mathbf{w})/\partial \mathbf{w}$  and  $\Sigma_w$  the noise co-variance matrix. In our robust optimal control framework, we solve for controls that minimize a task-related cost including state uncertainty (described by  $\mathbf{P}$ ) using a direct collocation approach.

### B. Postural control during platform rotations

Using optimal feedback control simulations with a musculoskeletal model that takes into account sensory and motor noise we simulated postural sway due to unpredictable platform rotations as applied by Peterka et al. [2]. Peterka identified sensory reweighting as a strategy used by healthy controls to minimize body sway with increasing rotations. As perturbation magnitude increased, healthy subjects relied more on vestibular and less on proprioceptive information to limit sway amplitude. Vestibular loss patients could not use this strategy resulting in increasing sway magnitude. We modeled this task using an inverted pendulum (ankle joint) driven by a Hill-type dorsi- and plantarflexor controlled by delayed (100ms) linear position and velocity feedback from the ankle and absolute angle to represent respectively proprioceptive and vestibular sensory information in addition to a baseline excitation. Gaussian noise is added to the muscle forces to model motor noise and to the vestibular and proprioceptive information to model sensory noise. We solved for feedback laws that minimized a trade-off between root mean square (RMS) sway amplitude and effort minimization for platform rotations with increasing magnitude. We evaluated the effect of platform amplitude and different trade-offs on sensory gains, sway and muscle co-contraction.

## III. RESULTS

Just as in experiments with healthy control subjects, the robust optimal control simulations predict sensory reweighting with increasing importance of the vestibular information with increasing platform disturbance (Fig. 1B). Robust optimal control simulations predict the same relation between RMS sway and disturbance magnitude for controls and vestibular loss subjects as observed in experiments (Fig. 1C). Simulated co-contraction index (Fig. 1C, CCI) increased when prioritizing stability over effort and increased with increasing platform disturbance but did hardly affect RMS sway. CCI was higher in vestibular loss subjects than in controls.

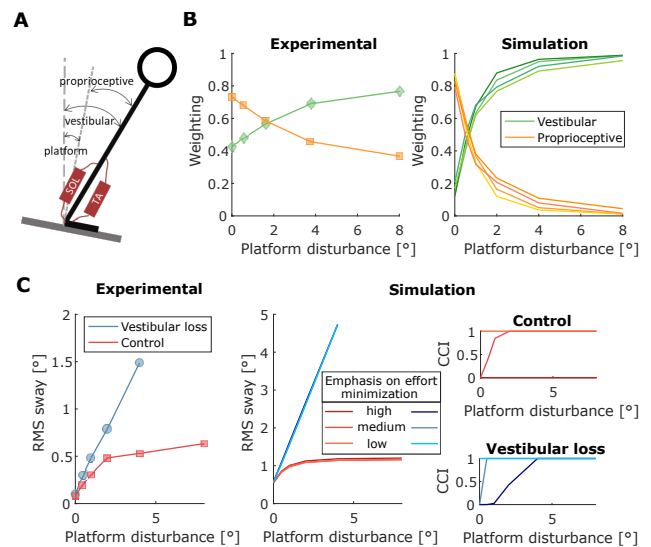


Figure 1 – (A) Model. (B) Sensory reweighting. Different shades of green and orange for the different relative gains (two muscles, two sensory inputs: angle and angular velocity). (C) Sway and muscle co-contraction. (Experimental data is adapted from [2] and [3]).

## IV. DISCUSSION

The proposed method yields control laws for non-linear musculoskeletal systems that are robust to modelled noise. Thereby, it allows addressing questions related to the influence of uncertainty on motor behavior and movement stability. The main advantages over existing methods are the compatibility with direct collocation formulations, which have greatly increased efficiency in movement simulation, and high flexibility in the design of the control law.

We demonstrate the method by simulating postural sway in healthy subjects and vestibular loss patients. We are currently using this method in combination with more complex musculoskeletal models to study how uncertainty influences movements such as gait.

## REFERENCES

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