# Frequency Matching: Optimizing Bio-Inspired Robotic Legs with SLIP-like Dynamics

## Foundation

- SLIP-model [1,2,3]: dynamics of legged locomotion
- Rotational hip spring [4]: dynamically model the swing phase
- Embedded SLIP dynamics [5]: Robotic leg with decoupled polar task dynamics
  → constraints on **link parameters**

? How do we choose **leg properties:** mass, COM, length, stiffness?

This research is targeted to be used in future generations of the robotic quadruped DLR bert



**References**

[1] R. Blickhan, "The spring-mass model for running and hopping"

[2] H. Geyer, A. Seyfarth, and R. Blickhan, "Compliant leg behaviour explains basic dynamics of walking and running"

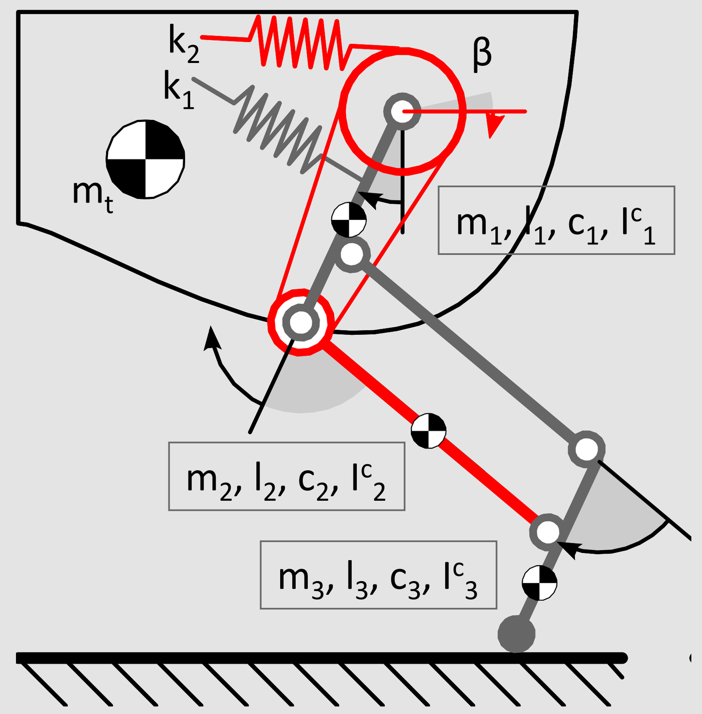
[3] H. M. Herr, G. T. Huang, and T. A. McMahon, "A model of scale effects in mammalian quadrupedal running"

[4] Z. Gan, Y. Yesilevskiy, P. Zaytsev, and C. D. Remy, "All common bipedal gaits emerge from a single passive model"

[5] D. Lakatos, W. Friedl, and A. Albu-Schäffer, "Eigenmodes of nonlinear dynamics: Definition, existence, and embodiment into legged robots with elastic elements"

[6] B. M. Kilbourne and L. C. Hoffman, "Scale effects between body size and limb design in quadrupedal mammals"

[7] C. T. Farley, J. Glasheen, and T. A. McMahon, "Running springs: Speed and animal size"


The geometric model of the three-segmented pantograph leg with its physical properties (adapted from [5]).

## Goal

- Design physical robotic legs
- SLIP-like dynamics
- Biologically plausible dimensions

## Method

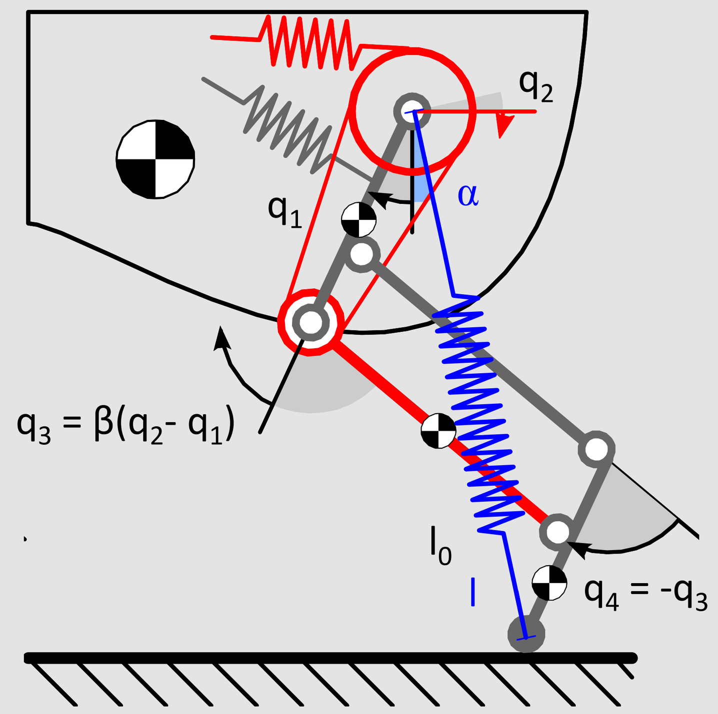
- Solve optimization problem (CMA-ES)
- Idea: demand swing frequency [4,6]
  $f_{\alpha,swing} = 1.38$ Hz

### Cost function

$cost = ½ * (f_{\alpha,swing} – f_{model})^2$

### Decision Variables

| | |
|---|---|
| Ratio $l_o$ / $l_{tot}$ | ∈ [0.6,0.98] |
| Ratio $l_3$ / $l_2$ | ∈ [0.1,0.9] |
| Ratio $c_1$ / $l_1$ | ∈ [0.15,0.5] |
| Ratio $c_2$ / $l_2$ | ∈ [0.15,0.5] |
| Ratio $c_3$ / $l_3$ | ∈ [0.15,0.5] |
| Shank Inertia | $I^c_2$ ∈ ]0,0.0004] |
| Foot Inertia | $I^c_3$ ∈ ]0,0.0004] |
| Pulley Ratio | β ∈ [0.2,1.9] |




The 2-DoF leg joint variables q = [$q_1$,$q_2$]. The knee and ankle angles $q_3$ and $q_4$ depend on q and the pulley ratio β. In blue the analogous SLIP model is given with the polar task coordinates z = [α,l].

### Constraints on link parameters

- Linear joint-to-task transformation
  $l_1 = l_2 - l_3$
- Leg COM on radial axis
  $m_3 = ((c_2 - l_1)m_2 - c_1 m_1)/(c_3 - l_3)$
- Task stiffness matrix decoupling
  $k_2 = \beta/(2-\beta) * k_1$
- Task inertia matrix decoupling in stance and swing phase
  $I^c_1 = ((2l_1 - 2c_2)m_2 + 2c_1 m_1 + 2c_3 m_3)\, l_3 - l_3^2 m_3 + (c_2 - l_1)^2 m_2 - c_1^2 m_1 + 2c_1 l_1 m_1 - c_3^2 m_3 + I^c_2 - I^c_3$

### Constraints on leg parameters

- Scale effects [6,7] between body weight and limb design of mammals
- Biologically plausible

| | |
|---|---|
| Robot mass | $m_{tot}$ = 4 kg |
| Leg mass | $m_{leg}$ = 0.363 kg |
| Leg COM | $c_{leg}$ = 0.068 m |
| Passive leg length | $l_o$ = 0.253 m |
| Leg stiffness | $k_{leg}$ = 1.81 kN/m |

## Results

- cost = 0.01
- $f_{model}$ = 1.53 Hz

| | |
|---|---|
| Trunk mass | $m_t$ = 0.637 kg |
| Thigh mass | $m_1$ = 0.225 kg |
| Shank mass | $m_2$ = 0.05 kg |
| Foot mass | $m_3$ = 0.088 kg |
| Passive length | $l_0$ = 0.253 m |
| Thigh length | $l_1$ = 0.052 m |
| Shank length | $l_2$ = 0.129 m |
| Foot length | $l_3$ = 0.077 m |
| Thigh COM | $c_1$ = 0.008 m |
| Shank COM | $c_2$ = 0.019 m |
| Foot COM | $c_3$ = 0.038 m |
| Thigh Inertia | $I^c_1 = 9.44*10^{-4}$ kg $m^2$ |
| Shank Inertia | $I^c_2 = 4.00*10^{-4}$ kg $m^2$ |
| Foot Inertia | $I^c_3 = 0.71*10^{-4}$ kg $m^2$ |
| Stiffness 1 | $k_1$ = 0.48 kN/m |
| Stiffness 2 | $k_2$ = 0.05 kN/m |
| Pulley Ratio | β = 0.2 |

## Discussion

- cost is acceptable
- $m_2$ goes to zero
- $k_1$, $k_2$ very small

## Further Work

Implement these legs in the quadrupedal robot DLR bert

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Anna Sesselmann (anna.sesselmann@dlr.de), Dominic Lakatos (dominic.lakatos@dlr.de), Alin Albu-Schäffer (alin.albu-schaeffer@dlr.de)

DLR Deutsches Zentrum für Luft- und Raumfahrt