Using Energy Shaping and Tracking to Generate Natural Limit Cycles in Mechanical Systems with Impacts

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If a system has both a periodic orbit and a plastic, dissipative impact, it must have some mechanism to regain the energy lost due to the impact. A well known example of this phenomenon is the passive compass gait walker, which reaches an energy equilibrium between the potential energy gained and the kinetic energy lost at each impact [1]. The idea of energy conservation is also important to another simple walking model, the spring loaded inverted pendulum (SLIP) [2]. It is a point mass that walks and runs on level ground using ideal springs. A fundamental difference between these two models is the nature of the impact dynamics. The SLIP model has elastic, lossless impacts while the compass gait biped has plastic, dissipative impacts. However, the COM of both models exhibit a conserved mechanical energy along their periodic orbits. This conservation can be used to drive the biped towards the associated orbit via control as in [3].

Recent research in [4], connected to this energy conservation idea, introduces generalized and constructive methods for the stabilization of periodic orbits for Hamiltonian systems using the method of interconnection and damping assignment passivity-based control (IDA-PBC). Two somewhat different methods are offered by the IDA-PBC techniques in [4]. The first method is to identify a Hamiltonian system endowed with a desired periodic orbit and set the control equal to the difference between the open-loop system and this desired system. This makes the closed-loop dynamics behave like the desired Hamiltonian system, and we refer to this technique as energy shaping. The second method is to use energy as an explicit control objective and drive the system to a constant energy level set associated with a desired periodic orbit. We refer to this technique as energy tracking.

The purpose of this presentation is to use energy shaping and tracking techniques to emulate "natural" (in the sense of [1]) or passive-like walking gaits in a mechanical system with impacts. First, we consider a bouncing ball system and present novel analytical results that use energy shaping and tracking to generate nearly globally stable hybrid limit cycles. We show that elastic impacts generate a compact set of marginally stable periodic orbits that are also energy level sets, so an energy tracking method can target and stabilize any one of these natural gaits/orbits in the set. Plastic impacts drive the system to a single stable limit cycle, so energy shaping must be used to generate new natural limit cycles which can then be robustified with energy tracking. We then consider two different biped walking models (the SLIP and the compass gait biped) and develop walking controllers based on our results for the ball system. We present simulation results that strongly suggest the impact type plays a similar role in the application of energy shaping and tracking techniques to these systems.

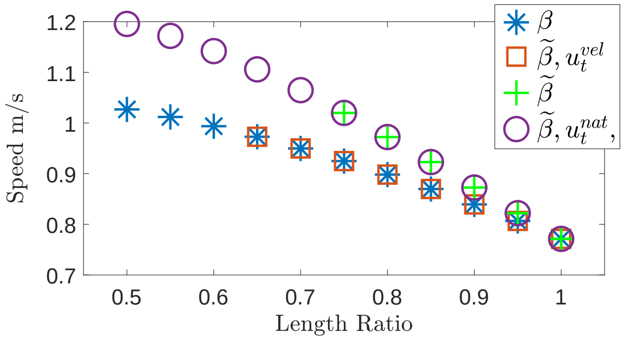

Fig. 1. Length ratio versus average speed for physical and virtual dynamics. β is the physical length ratio, $\widetilde{\beta}$ is the virtual length ratio, u_t^{vel} converges to a desired walking speed, u_t^{nat} converges to the energy equilibrium induced by the discrete dynamics.

An example of these results is given below, for the compass gait biped. In [5], it is shown that the dynamics of the compass gait biped can be normalized to depend on the mass ratio $\mu = \frac{m_h}{m_s}$ (m_h and m_s are the hip and shank masses respectively), and the length ratio β (which locates the position of m_s relative to m_h). Both of these parameters influence the average walking speed. We use energy shaping to virtually change these ratios to $\widetilde{\mu}$ and $\widetilde{\beta}$ during the continuous dynamics, and use the true ratios in the discrete impact dynamics. In addition, we use energy tracking controls u_t^{vel} and u_t^{nat}, separately, to achieve different walking speeds. The reference energy for u_t^{vel} is updated every step to achieve a desired walking speed, while for u_t^{nat} it is updated to achieve a "natural" limit cycle. The affect of this technique applied to the length ratio is given in Fig. 1. We plan to apply insights from this work on energy shaping and tracking controllers for a powered knee-ankle prosthesis.

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