Energy Shaping and Tracking for Natural Limit Cycles in Hybrid Systems

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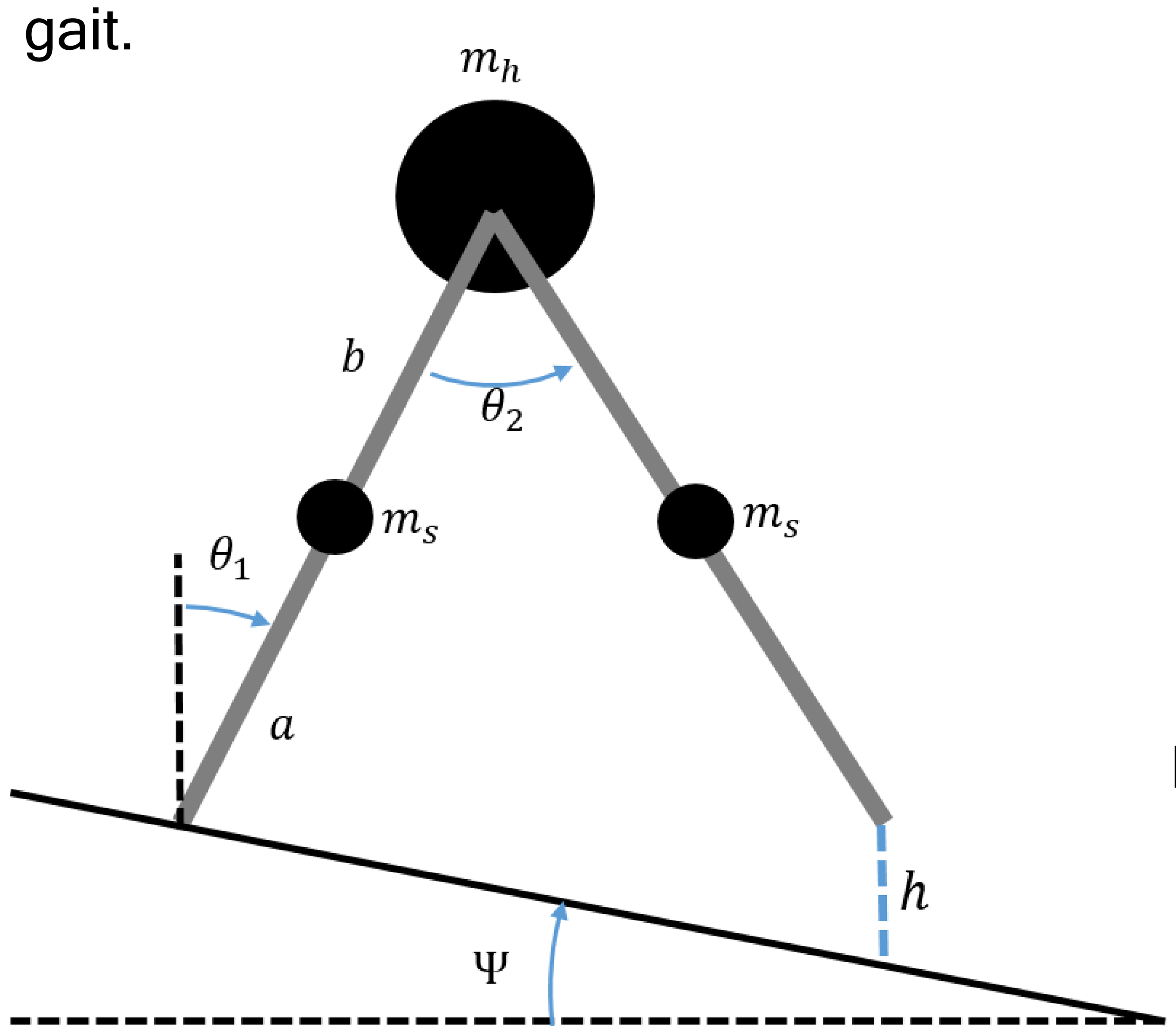
Energy Shaping and Tracking Control

- Mechanical Energy is conserved when the Compass Gait Biped walks with a passive gait.



Dynamics

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u$$

$$\begin{bmatrix} \dot{q}^+ \\ F_I \end{bmatrix} = R \begin{bmatrix} M & -A^\top \\ A & 0_{2x2} \end{bmatrix}^{-1} \begin{bmatrix} M \\ 0_{2x2} \end{bmatrix} \dot{q}^-$$

$$q^+ = Rq^-.$$

Energy
Kinetic + Potential

$$E = \frac{1}{2}\dot{q}^\top M \dot{q} + V(q)$$

Length Ratio

$$\beta = \frac{b}{a}$$

- Energy shaping is used to modify the structure of the dynamics. We create a new virtual length ratio and a new energy function, resulting in a new "natural", pseudo-passive gait and new conserved energy level.
- Impact dynamics depend on the mass/inertia properties and cannot be changed.
- Energy tracking is used to track the new conserved energy level E_{ref}, robustify the gait, and achieve changes in walking speed.

Energy Shaping Control

$$u_s = -M\widetilde{M}^{-1}(\widetilde{C}\dot{q} + \widetilde{G}) + C\dot{q} + G$$

Energy Tracking Control

$$u_t = \kappa M(\widetilde{E} - E_{ref})\dot{q}$$

New Dynamics

$$\widetilde{M}\ddot{q} + (\widetilde{C} + \kappa\widetilde{M}(\widetilde{E} - E_{ref}))\dot{q} + \widetilde{G} = 0$$

Similar to Van-der-Pol Rayleigh Harmonic Oscillator!

Reference Energy Adaptation, Step-to-Step:

Track an Average Walking Velocity

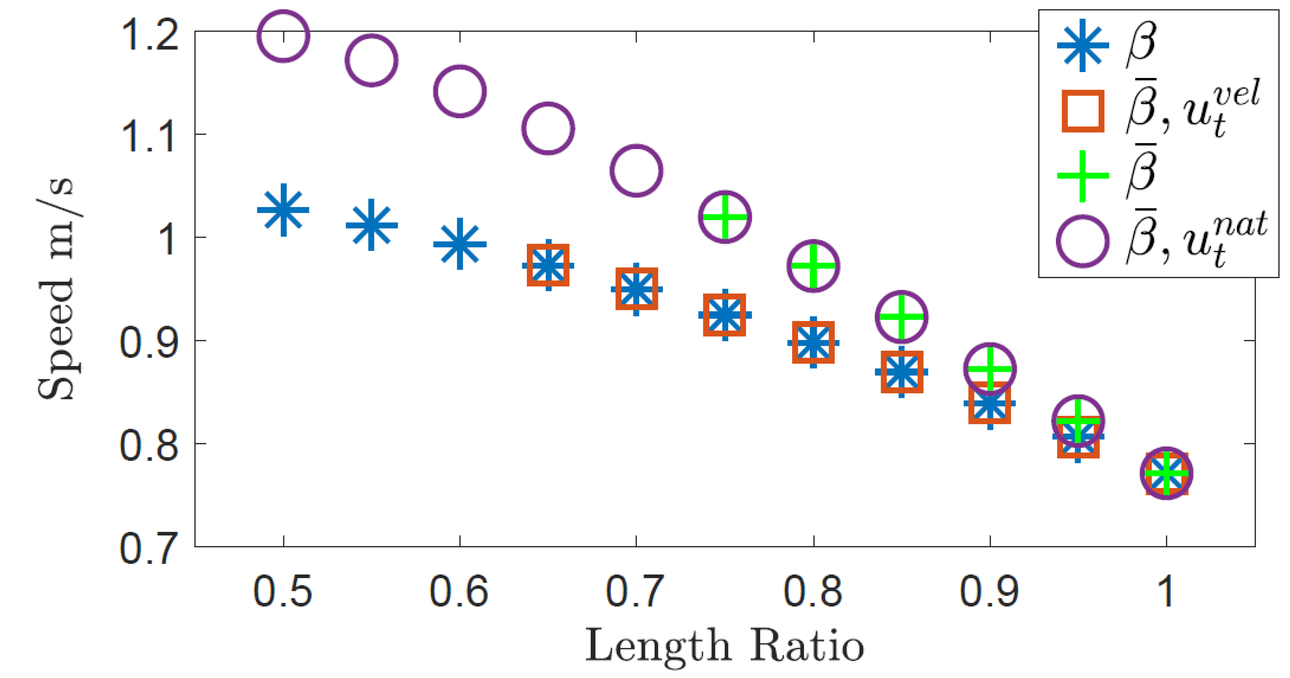
$$E_{ref_{i+1}} = E_{ref_i} + \lambda(v_{ref} - v_i)$$

Track a "Natural" Gait

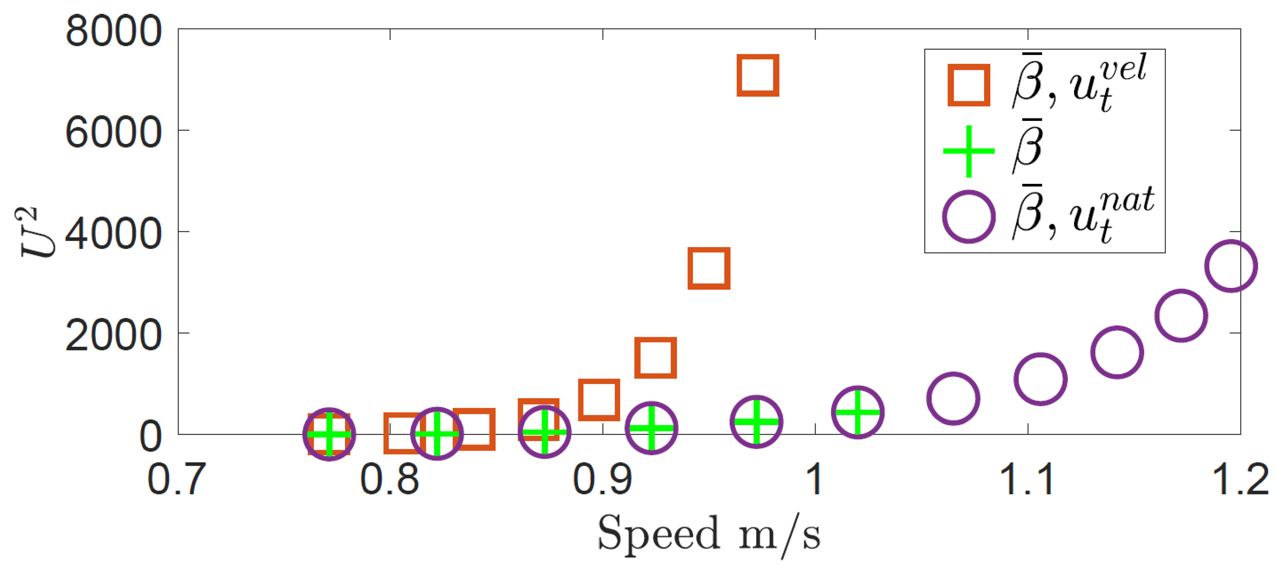
$$E_{ref_{i+1}} = E_{ref_i} + \lambda(E_i^+ - E_i^-)$$

Simulation Results

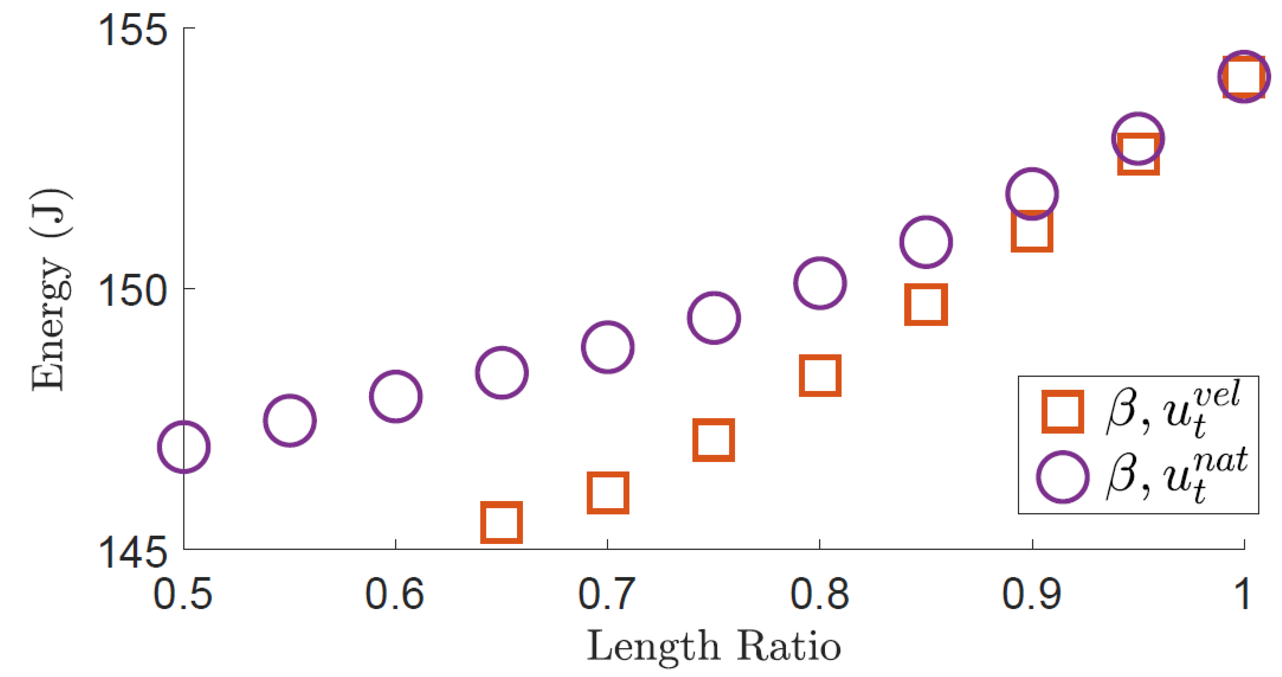
- Mismatch between virtual and physical length ratios, $\tilde{\beta}$ and β. This due to the impact dynamics.
- Energy tracking increases walking speed range, enables changes online.
- "Natural" gait requires less torque.



Length ratio versus average speed for physical and virtual dynamics.



Time integral of torque squared versus average speed for physical and virtual dynamics length ratio changes.



Energy versus length ratio for adaptive reference energy.

Future work

- Apply to wearable prosthesis or orthosis.
- Explore task variation with multiple virtual parameters.

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