Towards Intrinsic Reference Tracking for Legged Locomotion

Andrew M. Pace  
Electrical & Computer Engineering  
University of Washington  
Seattle, WA, USA  
apace2@uw.edu

Todd D. Murphey  
Mechanical Engineering  
Northwestern University  
Evanston, IL, USA  
t-murphey@northwestern.edu

Samuel A. Burden  
Electrical & Computer Engineering  
University of Washington  
Seattle, WA, USA  
sburden@uw.edu

Many existing techniques exist to develop complex behaviors for simulated robots. When these behaviors are run as is, i.e. in an open loop or feedforward fashion, the differences between reality and simulation causes many failures when implemented in real-world robots. From a controls perspective, these failures are to be expected due to the lack of a key component, feedback. The use of feedback inherently makes the replay of behaviors more robust, handling the mismatch between the real-world and the model. We therefore consider reference tracking for frictionless mechanical systems impacting the ground (unilateral constraints), a common model of legged-locomotion. Leveraging an intrinsic (Riemannian-geometry) perspective for such fully-actuated mechanical systems undergoing inelastic impacts, we derive a state feedback controller to decrease tracking error, and a (Luenberger-based) state observer that decreases estimation error.

We take the view that under a judicious mapping a switched system provides local description of certain trajectories for the above class of systems near points of impact. One such mapping [1], captures the discontinuity in velocity for one trajectory and allows for a similar description of trajectories in a local neighborhood. Additionally this mapping elicits a mechanical interpretation of the relation between the new extended dynamics on the unconstrained space $\tilde{C}$ and the original dynamics on the constrained space $C$ [2, §1.6].

Fig. 1 provides an overview of an example for a fully actuated bouncing ball undergoing inelastic impacts.

When we stitch together the two domains of the switched system with a piecewise-defined change of coordinates along the impact surface$^1$, the underlying distance function (Riemannian-metric) becomes continuous. Hence, with this change of coordinates, well established tracking [4] and estimation [5] techniques for mechanical systems on manifolds can be applied.

We intuit, but have not yet shown, local (asymptotic) tracking in $\tilde{C}$ is equivalent to local (asymptotic) tracking $\epsilon > 0$ time away from impacts in $C$. We hope to extend the result to include perfectly plastic impacts and non-fully actuated systems in the future.

REFERENCES


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$^1$Using a Fermi coordinates [3, Chpt. 5] approach.

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Fig. 1. An example of the switched system view for a bouncing ball with an inelastic impact. The blue line is the phase plot of the reference trajectory $r(t)$ and the yellow line an actual trajectory $q(t)$. The outlined circles are the initial condition and the filled in circles are at time $t_i^+$, the time immediately after the reference trajectory impacts the constraint. When the (blue) reference trajectory in $C$ impacts the constraint surface before (yellow) actual trajectory, a reset $R$ causes a large jump in velocity and hence a discontinuity in the error between $r$ and $q$ at $t_i$. Whereas in the unconstrained configuration space $\tilde{C}$, only a change in the underlying dynamics occurs. Hence the error between the unconstrained reference trajectory $\tilde{r}$ and the unconstrained actual trajectory $\tilde{q}$ remains continuous at $t_i$. As the mapping $P: \tilde{C} \rightarrow C$ locally maps trajectories in $\tilde{C}$ to feasible trajectories in $C$, local tracking is done in $\tilde{C}$ where the error is continuous.