Towards Intrinsic Reference Tracking for Legged Locomotion

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Many existing techniques exist to develop complex behaviors for simulated robots. When these behaviors are run as is, i.e. in an *open loop* or *feedforward* fashion, the differences between reality and simulation causes many failures when implemented in real-world robots. From a controls perspective, these failures are to be expected due to the lack of a key component, *feedback*. The use of feedback inherently makes the replay of behaviors more robust, handling the mismatch between the real-world and the model. We therefore consider reference tracking for frictionless mechanical systems impacting the ground (unilateral constraints), a common model of legged-locomotion. Leveraging an intrinsic (Riemannian-geometry) perspective for such fully-actuated mechanical systems undergoing inelastic impacts, we derive a state feedback controller to decrease tracking error, and a (Luenberger-based) state observer that decreases estimation error.

We take the view that under a judicious mapping a switched system provides local description of certain trajectories for the above class of systems near points of impact. One such mapping [1], captures the discontinuity in velocity for one trajectory and allows for a similar description of trajectories in a local neighborhood. Additionally this mapping elicits a mechanical interpretation of the relation between the new extended dynamics on the unconstrained space $\widetilde{C}$ and the original dynamics on the constrained space C [2, §1.6]. Fig. 1 provides an overview of an example for a fully actuated bouncing ball undergoing inelastic impacts.

When we stitch together the two domains of the switched system with a piecewise-defined change of coordinates along the impact surface[1], the underlying distance function (Riemannian-metric) becomes continuous. Hence, with this change of coordinates, well established tracking [4] and estimation [5] techniques for mechanical systems on manifolds can be applied.

We intuit, but have not yet shown, local (asymptotic) tracking in $\widetilde{C}$ is equivalent to local (asymptotic) tracking $\epsilon > 0$ time away from impacts in C. We hope to extend the result to include perfectly plastic impacts and non-fully actuated systems in the future.

References

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[4] F. Bullo and R. M. Murray, "Tracking for Fully Actuated Mechanical Systems: A Geometric Framework," vol. 35, Jan. 1999.
[5] N. Aghannan and P. Rouchon, "An Intrinsic Observer for a Class of Lagrangian Systems," vol. 48, June 2003.

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[1] Using a *Fermi coordinates* [3, Chpt. 5] approach.

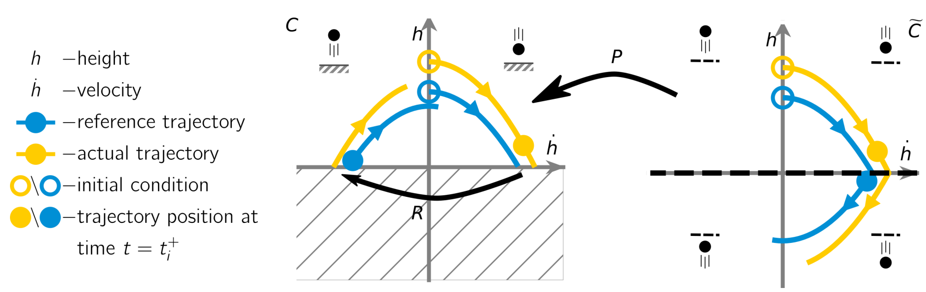

Fig. 1. An example of the switched system view for a bouncing ball with an inelastic impact. The blue line is the phase plot of the reference trajectory $r(t)$ and the yellow line an actual trajectory $q(t)$. The outlined circles are the initial condition and the filled in circles are at time t_i^+, the time immediately after the reference trajectory impacts the constraint. When the (blue) reference trajectory in C impacts the constraint surface before (yellow) actual trajectory, a reset R causes a large jump in velocity and hence a discontinuity in the error between r and q at t_i. Whereas in the unconstrained configuration space $\widetilde{C}$, only a change in the underlying dynamics occurs. Hence the error between the unconstrained reference trajectory $\tilde{r}$ and the unconstrained actual trajectory $\tilde{q}$ remains continuous at t_i. As the mapping $P\colon \widetilde{C} \to C$ locally maps trajectories in $\widetilde{C}$ to feasible trajectories in C, local tracking is done in $\widetilde{C}$ where the error is continuous.