Towards Intrinsic Reference Tracking for Legged Locomotion

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Tracking a perfectly elastic bouncing ball through impacts

1. The bouncing ball is governed by two sets of dynamics, one being gravity and the other the reset that occurs at a height h of 0. The **reset law** $\boldsymbol{R}: \boldsymbol{TC} \to \boldsymbol{TC}$ maps velocity pre-impact velocity to post-impact velocity,

$$R(h, \dot{h}) = (h, -\dot{h}).$$

2. $\boldsymbol{P}: \tilde{\boldsymbol{C}} \to \boldsymbol{C}$ defines a map that takes trajectories generated in the extended space $\tilde{C}$ to trajectories that undergo impacts [1].

$$P(h) = \begin{cases} h, & h \geq 0 \\ -h, & h < 0. \end{cases}$$

3. Defining the dynamics on $\tilde{C}$ such that a trajectories in $\tilde{C}$ map to trajectories in C under P [2] yields a continuous switched system.

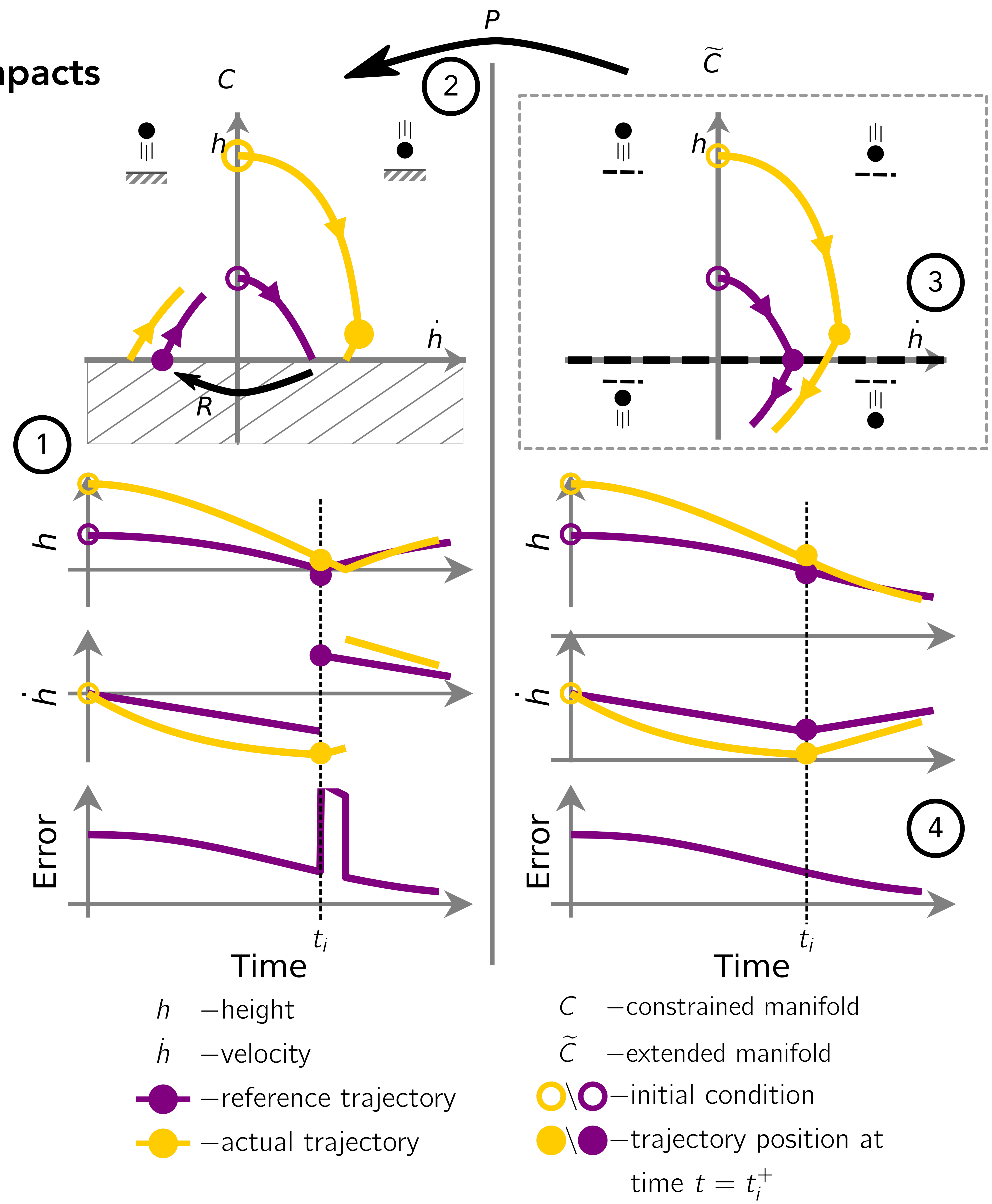
Let $\tilde{\boldsymbol{r}}(\boldsymbol{t})$ be the reference trajectory in $\tilde{C}$, then

$$\ddot{\tilde{\boldsymbol{r}}}(\boldsymbol{t}) = F\big(h(t), \dot{h}(t)\big) = \begin{cases} -g, & h(t) \geq 0 \\ g, & h(t) < 0. \end{cases}$$

4. Using results from geometric control [3], the reference trajectory can be tracked on $\tilde{C}$, implying tracking on C away from impacts.

Let $\tilde{\boldsymbol{q}}(\boldsymbol{t})$ be the actual trajectory in $\tilde{C}$, then

$$\begin{aligned} \ddot{\tilde{\boldsymbol{q}}}(\boldsymbol{t}) &= F_c\left(\tilde{\boldsymbol{q}}(\boldsymbol{t}), \dot{\tilde{\boldsymbol{q}}}(\boldsymbol{t}), \tilde{\boldsymbol{r}}(\boldsymbol{t}), \dot{\tilde{\boldsymbol{r}}}(\boldsymbol{t})\right) \\ &= F\left(\tilde{\boldsymbol{r}}(\boldsymbol{t}), \dot{\tilde{\boldsymbol{r}}}(\boldsymbol{t})\right) + K_p\big(\tilde{\boldsymbol{r}}(\boldsymbol{t}) - \tilde{\boldsymbol{q}}(\boldsymbol{t})\big) + K_d\left(\dot{\tilde{\boldsymbol{r}}}(\boldsymbol{t}) - \dot{\tilde{\boldsymbol{q}}}(\boldsymbol{t})\right). \end{aligned}$$



Current Findings

Locally, a mechanical system undergoing impacts is equivalent to a switched system. For perfectly elastic impacts, we develop a local reference tracking controller in the switched system $\tilde{C}$ and show tracking in $\tilde{C}$ is equivalent to tracking in C away from impacts. Our result reproduces the *mirror law* [4].

Additionally, we have shown for all inelastic collisions, a continuous intrinsic metric does not exist.

References

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Acknowledgements

This work is supported in part by U.S. Army Research Laboratory and the U.S. Army Research Office under contract/grant W911NF-16-1-0158.