On Highly Dynamic Behaviors of Quadrupedal Robots

Gabriel García[1], Robert Griffin[1], Jerry Pratt[1]

I. Introduction

Obtaining highly dynamic behaviors in robots is an extremely difficult task. In recent years, sophistication in mechanical design, improved algorithms, and high computational power allows new robots to perform natural gaits and more dynamic behaviors such as backflips [1,2]. Off-line optimization is necessary to obtain good performance in those difficult tasks. However, when a human performs a backflip or any parkour movements, the computation during those activities is updated online. One of the biggest challenges in robotics is to perform these awesome movements using online optimization with a good understanding of the dynamics of the robot. In this paper we present an approach to deal with complicated dynamic tasks that the world allows the robot to do. We aim to perform convex optimization to obtain highly dynamic behaviors from the control point of view.

II. Control Approach

The Centroidal Dynamics of any Multi-body System under multi-contact, point-foot placements holds:

$$\begin{bmatrix} \ddot{\mathbf{r}} = \sum_{i=1}^{n} \mathbf{f}_i + \mathbf{g} \\ \dot{\mathbf{L}} = \sum_{i=1}^{n} (\mathbf{r}_{pi} - \mathbf{r}) \times \mathbf{f}_i \end{bmatrix} \quad (1)$$

When the system has massless links but only one, the angular momentum reduces to

$$\mathbf{L} = \mathbf{I}\omega = \mathbf{R}\mathbf{I_B}\mathbf{R^T}\omega \quad (2)$$

where $\mathbf{I_B}$ represents the Inertia tensor of the body in a local frame, $\mathbf{R}$ is the rotation matrix from the local to the world coordinates of the link with mass, ω represents the angular velocity of that link. The dynamics of $\mathbf{R}$ hold:

$$\dot{\mathbf{R}} = [\omega]_\times \mathbf{R} \quad (3)$$

The control of the orientation matrix $\mathbf{R}$ is known as the *attitude control problem*. In [3], Euler angles are used to represent the rotation matrix $\mathbf{R} = \mathbf{R_z}(\psi)\mathbf{R_y}(\theta)\mathbf{R_x}(\varphi)$; where φ is the roll, θ is the pitch and ψ is the yaw. This consideration has a singularity in the dynamics when $\cos(\theta) = 0$, which will forbid the robot to perform extreme behaviors such as parkour wall-jumps or climbs. In this research, we use the well-known approach of the quaternion representation $\mathbf{q}_r$, which uses 4 variables and does not have a singularity. Let $\mathbf{q}_r$ be the quaternion representing the orientation of the robot with respect to a fixed coordinate system in the world frame. The dynamics of $\mathbf{q}_r$ are:

[1]Authors are with The Florida Institute for Human and Machine Cognition, Pensacola, FL, USA

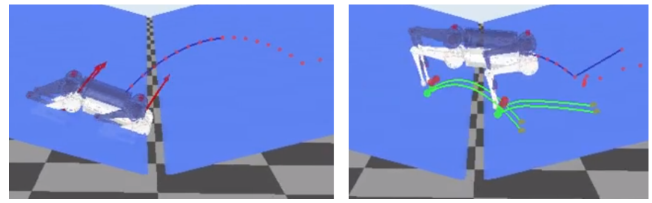

Fig. 1. Mini-Cheetah Sim performing a leap between two planes. (a) Initial forces and trajectory (b) Flight phase and future positions of the CoM according to the MPC

$$\dot{\mathbf{q}}_r = \frac{1}{2}\mathbf{q}_r \circ \omega = \mathbf{Q}(\mathbf{q}_r)\omega \quad (4)$$

where $\circ$ is the quaternion product and $\mathbf{Q}(\mathbf{q}_r) \in \mathbb{R}^{4\times3}$ is its matrix form representation. We do not simplify $\frac{d}{dt}(\mathbf{I}\omega) = \mathbf{I}\dot{\omega} + \omega \times (\mathbf{I}\omega) \approx \mathbf{I}\dot{\omega}$, as it is usually done. We will use the angular momentum $\mathbf{L}$ as our state variable instead of ω. We have:

$$\mathbf{L} = \mathbf{I}\omega \rightarrow \omega = \mathbf{I^{-1}}\mathbf{L} \quad (5)$$

We are using the angular momentum $\mathbf{L_2}$ around the origin of the world coordinates of the space instead of the angular momentum around the CoM $\mathbf{L}$. This linearizes the non-linearity of the second equation of (1) [4, 5].

$$\mathbf{L}_2 = \mathbf{L} + m\mathbf{r} \times \dot{\mathbf{r}} \quad (6)$$

Now we have the dynamics of the Single Rigid Body Model (SRBM), over which we can apply a time-varying linearization *only in the orientation* and apply classical QP strategies such as direct transcription to obtain forces which later can be tracked by a Whole-Body Controller:

$$\begin{bmatrix} \ddot{\mathbf{r}} = \sum_{i=1}^{n} \mathbf{f}_i + \mathbf{g} \\ \dot{\mathbf{L}}_2 = \sum_{i=1}^{n} \mathbf{r}_{pi} \times \mathbf{f}_i + m\mathbf{r} \times \mathbf{g} \\ \dot{\mathbf{q}}_r = \mathbf{Q}(\mathbf{q}_r)\mathbf{I^{-1}}(\mathbf{q_r})(\mathbf{L}_2 - m\mathbf{r} \times \dot{\mathbf{r}}) \end{bmatrix} \quad (7)$$

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