Optimal Reduced-order Modeling of Bipedal Locomotion

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I. Summary

State-of-the-art approaches to legged locomotion are widely dependent on the use of reduced-order models such as the linear inverted pendulum [1] and the spring-loaded inverted pendulum [2], popular because their simplicity enables a wide array of tools for planning, control, and analysis. [3] However, they inevitably limit the ability to execute complex tasks or agile maneuvers. In our previous work, we automatically synthesized models that remain low-dimensional but retain the capabilities of the high-dimensional system. In our latest work, we extended the previous example of model optimization to a more complex robot, Cassie. We also implemented a naive motion planning algorithm with the optimal models as the first step towards real-time planning and control.

II. Reduced-order Model

Let q and u be the generalized position and input of the full-order model, and let y and τ be the generalized position and input of the reduced-order model. We define a reduced-order model μ by two things – an embedding function $r : q \mapsto y(q)$ and the second-order dynamics of the reduced-order model $g(y, \dot{y}, \tau)$. That is,

$$\mu \triangleq (r, g), \tag{1}$$

with $y = r(q)$ and $\ddot{y} = g(y, \dot{y}, \tau)$. Given a set of tasks, the model can be optimized through the algorithm shown in [4].

III. Planning with Optimal Models

With the optimized model, we could plan the motion in low dimensional space and track the trajectories in the full space with the techniques like operational space control [5]. As an example, let the task be walking l meters in n_s strides. Since the reduced-order model only captures the continuous dynamics, and perfect embedding of a reduced-order hybrid model is often impossible, we mix the reduced-order model with the discrete dynamics from the full-order model. This approach results in a low-dimensional trajectory optimization problem, a search for $y_j(t)$ and $\tau_j(t)$, with additional decision variables $x_{-,j}, x_{+,j}$, representing the pre- and post-impact full-order states. The index $j = 1, \ldots, n_s$ refers to the jth stride. Note that the reduced-order model trajectories are necessarily hybrid.

The above formulation preserves an exact representation of the hybrid dynamics but results in a significantly reduced optimization problem that can be used for real-time planning. Fig. 1 visualizes the pre-impact states in the case where the robot walks two meters with four strides, connected by the hybrid events and continuous low-dimensional trajectories $y_j(t)$.

We were able to retrieve $q(t)$ from $y_j(t)$ through inverse kinematics, which means that the optimal trajectories $y_j(t)$ are feasible for the robot. The resulting motion, shown in the accompanying video[2], looks qualitatively more efficient than those generated by the non-optimal model. As a part of our ongoing work, we are implementing the controller that tracks the planned trajectories.

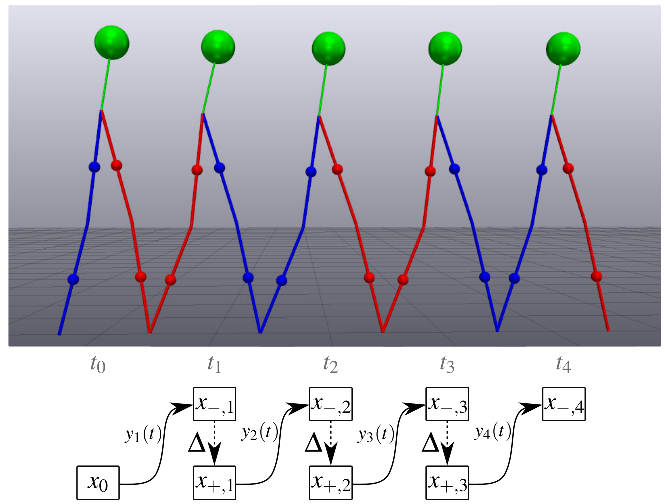

Fig. 1: Given a task of covering two meters in five steps, we rapidly plan a trajectory for the reduced-order model. Δ is the impact mapping of the full-order model.

References

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[2]https://youtu.be/SdY25aLWjnQ?t=92