Variational-Based Optimal Control of Underactuated Balancing for Dynamic Quadrupeds

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***Abstract*—This paper presents a control strategy for quadruped balancing that enables postural control in underactuated contact configurations (e.g., when standing on two point feet). The proposed balance control framework combines constrained optimal control strategies with recent variational-based linearization approaches to solve the balancing problem for a common simplified quadruped model. The controller is implemented as a convex quadratic program that uses an unconstrained optimal control solution to approximate a friction-constrained optimal policy.**

***Index Terms*—Optimal control, linearization techniques, underactuated balancing, quadruped locomotion**

I. Background

Underactuated balancing has received considerable attention with prototype control models such as the cart pendulum or acrobot. Yet, when attempting to transition these solutions to balance in legged robots, technical challenges related to friction-limited contacts and the underlying manifold structure of the configuration space prevent straightforward application. Typical quadratic programming-based approaches to whole-body control [1], [2] are ill-suited for underactuated balancing because they fail to look beyond satisfaction of instantaneous desired dynamics. Model predictive control (MPC) based approaches are able to avoid undesired myopic behaviors [3], but challenges ranging from non-convexity to computational complexity pose an obstacle to real-time implementation. Linear MPC strategies are able to avoid non-convexity [4], but the resulting controllers are often only valid in restrictively small regions around the reference state.

II. Approach

This work proposes an alternative control algorithm that synthesizes optimal control and quadratic programming, as inspired by the approach demonstrated in [5]. We start by applying variational-based linearization (VBL) techniques [6] to a reduced-order model of the MIT Mini Cheetah quadruped and using that linear model to pose the balance control problem in linear quadratic form. The cost-to-go of the system is subsequently solved for using an unconstrained linear quadratic regulator. Rather than directly using this unconstrained linear optimal control policy, which may violate control contraints (e.g., friction cone constraints), the cost-to-go information is re-purposed to design an input-constrained control law that is able to consider the long-term consequences of present actions. Our VBL-QP control law, as it is referred to, captures the benefits of larger-scale MPC formulations while maintaining fast solve times and loop rates via a compact formulation.

III. Results

The VBL-QP controller enables perturbation-robust balance of the MIT Mini Cheetah on both four and two legs (Fig.1). Experiments for balancing in low friction regimes (80% reduction in friction coefficient) validate the unconstrained cost-to-go approximation. Furthermore, the region of attraction of the VBL is empirically shown to include nearly all of the robot's kinematic workspace.

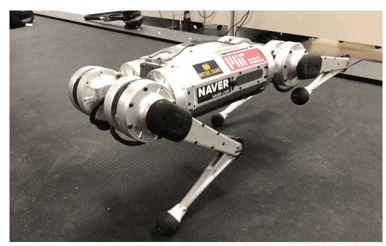

Fig. 1. The proposed control framework achieves two-leg balance of the MIT Mini Cheetah quadruped.

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