Trajectory Optimization over Uncertain Terrain using Stochastic Complementarity

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I. Summary

Robust trajectory optimization for legged locomotion without perfect terrain information is a significant yet underexplored problem in the dynamic walking field. A majority of current methods require precisely specifying terrain geometry and friction characteristics, which are often prone to uncertainty. In this work, we devise a trajectory optimization that explicitly incorporates parametric uncertainty in the terrain geometry and in the friction coefficient. We demonstrate in simulation that our method produces open-loop controls which are more robust to variations in the contact parameters; moreover, as the uncertainty decreases, our trajectories converge to those generated under known terrain parameters.This study represents a step towards reasoning about terrain uncertainty in the discipline of trajectory optimization.

II. Introduction

Designing safe, robust, and dynamic locomotion behaviors for bipedal robots poses a challenge to the field. Generated trajectories and controls ought to avoid a fall and enable the robot to progress towards the goal while handling unmodeled disturbances from the environment such as the terrain. Contact-implicit trajectory optimization has recently gained increasing attention given its ability to generate diverse locomotion behaviors [1], [2]; however, this approach relies on precise models of the system dynamics and the terrain characteristics, which are not always known exactly *a priori*. Model uncertainty has been approached in previous work by perturbing individual model parameters, resulting in an ensemble of reference trajectories. [3]. Here we explicitly study uncertainty resulting from the terrain and aim for the following contributions:

- Design risk-sensitive cost functions for uncertainty in the terrain geometry and friction coefficient.
- Demonstrate that our control trajectories are robust to terrain parameter variations when uncertainty is high.
- Prove that our method converges to the traditional method with known terrain parameters as uncertainty vanishes.

III. Methods

Our study assumed normal distributions over the terrain parameters and derived the corresponding stochastic complementarity problems (SCPs). These SCPs were converted into a risk-sensitive cost function using the Expected Residual Minimization (ERM) formulation [4]. We evaluated this approach on two toy examples: sliding a block over terrain with uncertain friction, and a single legged hopper traversing terrain with unknown height (see Figure 1). Trajectories generated under various levels of uncertainty were compared to one generated using the traditional non-robust contact-implicit method. We also tested the robustness of the open-loop controllers in simulations, where we perturbed the friction coefficient and the terrain geometry.

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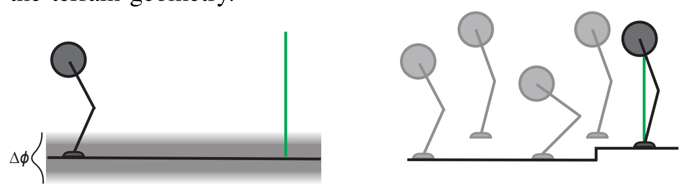

Fig. 1. Illustration of the single-legged hopper with uncertain terrain height, ϕ. (Left) During optimization, the hopper assumes a flat terrain with uncertain height in its plan to reach the goal (green). (Right) In simulation, the control must be robust to unmodeled step changes in terrain height.

IV. Results and Discussion

Under high uncertainty, our method produced more aggressive controls to achieve the goal position while also minimizing interaction duration with the terrain, i.e., minimizing the ERM cost. As the uncertainty decreased, however, these controls approached those generated by the traditional non-robust approach. In simulations, the trajectory roll-outs generated by our controls had less variation with respect to the parameter perturbations and achieved positions closer to the goal compared to the results from the traditional method.

Our study represents the first time parametric uncertainty in contact parameters has been incorporated in contact-implicit trajectory optimization. We realize that, while our controls are more energetic, the resulting trajectories are more robust to parameter variations in simulation than the traditional approach, and thus have the potential to be more effective in closed-loop control. Future work may build on our approach by incorporating more sources of uncertainty or by using data collected during real locomotion experiments to update the contact parameter distributions in an online fashion.

References

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