Contact-aware controller design for balancing

Alp Aydinoglu, William Yang and Michael Posa
University of Pennsylvania
alpayd@seas.upenn.edu.edu

I. INTRODUCTION

Robots often need to perform well in complex environments and accomplish tasks where it makes contact with its surroundings. Furthermore, for many tasks such as locomotion, the robot has to initiate contact with its environment *intentionally*. In order to perform such tasks efficiently, it is crucial to consider the multi-contact nature of the problem.

State-of-the-art controllers struggle to deal with the hybrid nature of multi-contact motion, especially for real-time control applications, but there are notable successes where the multi modal structure is relaxed. People have utilized the model predictive control framework where they approximate the hybrid dynamics [1], or fix the mode sequence a priori [2]. Also, previous work explored feedback design but does not incorporate the measurement of the ground reaction forces [3].

In our previous work, we have developed control policies that utilize the measurement of the tactile forces (equivalently the ground reaction forces) with certain performance guarantees [4]. Inspired by the works [3] [5], we have designed feedback gains that consider both the state of the system and the tactile measurements without *enumerating* the modes of the system. More specifically, we design controllers that work in every nearby mode.

In this work, the aim is to design a controller using the method that we have proposed in [4] for legged robot models with the goal of balancing the robot. We want to find a static feedback policy that can stabilize the legged robot independent of the mode that the robot is in (e.g., feet that come off the ground). We will examine the performance of the proposed method on legged robots, and discuss both the benefits and shortcomings of both the proposed method, and using ground reaction force measurements in the controller design.

II. METHODS

For the model of our legged robot, we consider the general non-linear model of rigid body systems with contact:

$$M(q)\dot{v} + C(q,v) = Bu + J(q)^T\lambda, \tag{1}$$

where $q \in R^p$ represents the generalized coordinates, $v \in R^p$ represents the generalized velocities, $\lambda \in R^m$ represents the contact forces, $M(q)$ is the inertia matrix, $C(q,v)$ represents the combined Coriolis and gravitational terms, B maps the control inputs u into joint coordinates and $J(q)$ is the projection matrix. The contact force λ is represented by:

$$\lambda \geq 0, \ \phi(q,\lambda) \geq 0, \ \phi(q,\lambda)^T\lambda = 0, \tag{2}$$

where the function $\phi : R^p \times R^m \to R^m$ is a gap function which relates the distance between robot and object with the contact force. We first linearize the smooth components ($M(q), C(q,v), J(q), \phi(q)$, etc.) of the dynamics and obtain the following linear complementarity model:

$$\begin{aligned} \dot{x} &= Ax + Bu + D\lambda, \\ 0 \leq \lambda &\perp Ex + F\lambda + c \geq 0, \end{aligned} \tag{3}$$

where $u \in R^k$ is the input vector, A determines the autonomous dynamics of the state vector x, B models the linear effect of the input on the state, D describes the linear effect of the contact forces on the state. Then using the method discussed in [4], we utilize a non-smooth Lyapunov function of the form $V(x,\lambda) = x^TPx + 2x^TQ\lambda + \lambda^TR\lambda$ where $P \in R^{n\times n}$, $Q \in R^{n\times m}$, and $R \in R^{m\times m}$. Observe that the function is non-smooth, continuous, and B-differentiable [5]. Also, V is a piecewise quadratic Lyapunov function in x where the function switches based on the active contacts, hence a different quadratic piece is assigned to every different mode. Then, we find a controller $u(x,\lambda) = Kx + L\lambda$ with its respective stability guarantee V based on the system in (3) solving a system of bilinear matrix inequalities. The controller utilizes the tactile feedback, hence is a piecewise continuous function of x where the structure changes depending on which modes are active. Then, we observe its performance on the non-linear model that is specified by the equations (1), (2).

III. RESULTS/DISCUSSION

The preliminary results show that the method works for simple robot models. It is important to use the tactile information in our controller design in an effective way in order to be successful in real-time applications. Exploring and understanding different ways to incorporate the information related to ground reaction forces into controller design is a direction which can shed light into importance of tactile information for legged locomotion and enable new design methods that can work in real time.

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