Contact-Aware Controller Design for Balancing

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Motivation

- Robots perform in complex, unstructured environments which involve physical interaction between the robot and the environment
- Tasks like locomotion are fundamentally based in making and breaking contact with the environment
- State-of-the-art control policies struggle to deal with the hybrid nature of multi-contact motion
- We propose a control framework which can close the loop on rich, tactile sensors
- The framework is non-combinatoric, enabling optimization algorithms to automatically synthesize provably stable control policies

Complementarity Systems

Continuous-time dynamics of rigid-body systems with contacts:

$$M(q)\dot{v} + C(q,v) = Bu + J(q)^T\lambda. \quad (1)$$

q – Generalized coordinates | v – Generalized velocities | λ – Contact forces

Describe contact forces using the complementarity framework:

$$\lambda \geq 0,\ \phi(q,\lambda) \geq 0,\ \phi(q,\lambda)^T\lambda = 0. \quad (2)$$

Linearize the smooth components ($M(q), C(q,v), J(q), \phi(q,\lambda)$):

$$\begin{aligned}\dot{x} &= Ax + Bu + D\lambda,\\ 0 \leq \lambda &\perp Ex + F\lambda + c \geq 0,\end{aligned} \quad (3)$$

where x is the state, u is the input and $\perp$ denotes orthogonality.

Contact-Aware Controller

We propose a controller of the form

$$u(x,\lambda) = Kx + L\lambda \quad (4)$$

- Feedback based on tactile sensing (e.g., ground reaction force)
- Controller switches based on active contacts (modes), even though the modes are not enumerated
- Can work under partial state observation

Non-smooth Lyapunov Function

Captures the non-smooth nature of the dynamics [2]:

$$V(x,\lambda) = x^T P x + 2x^T Q\lambda + \lambda^T R\lambda.$$

- Quadratic in terms of the pair (x,λ)
- Piecewise quadratic in x
- Directionally differentiable and Lipschitz continuous

Controller Design

Solve a bilinear matrix inequality (BMI) to simultaneously find a Lyapunov function and a policy

$$\begin{aligned}&\text{find} && V, K, L\\ &\text{subject to} && V(0,0) = 0,\\ &&& V(x,\lambda) > 0, \quad \text{for } (x,\lambda) \in \Gamma_{SOL}(E,F,c),\\ &&& V'(x;\dot{x}) \leq 0, \quad \text{for } (x,\lambda,\lambda(x;\dot{x})) \in \Gamma'_{SOL}(E,F,c,\dot{x}),\end{aligned}$$

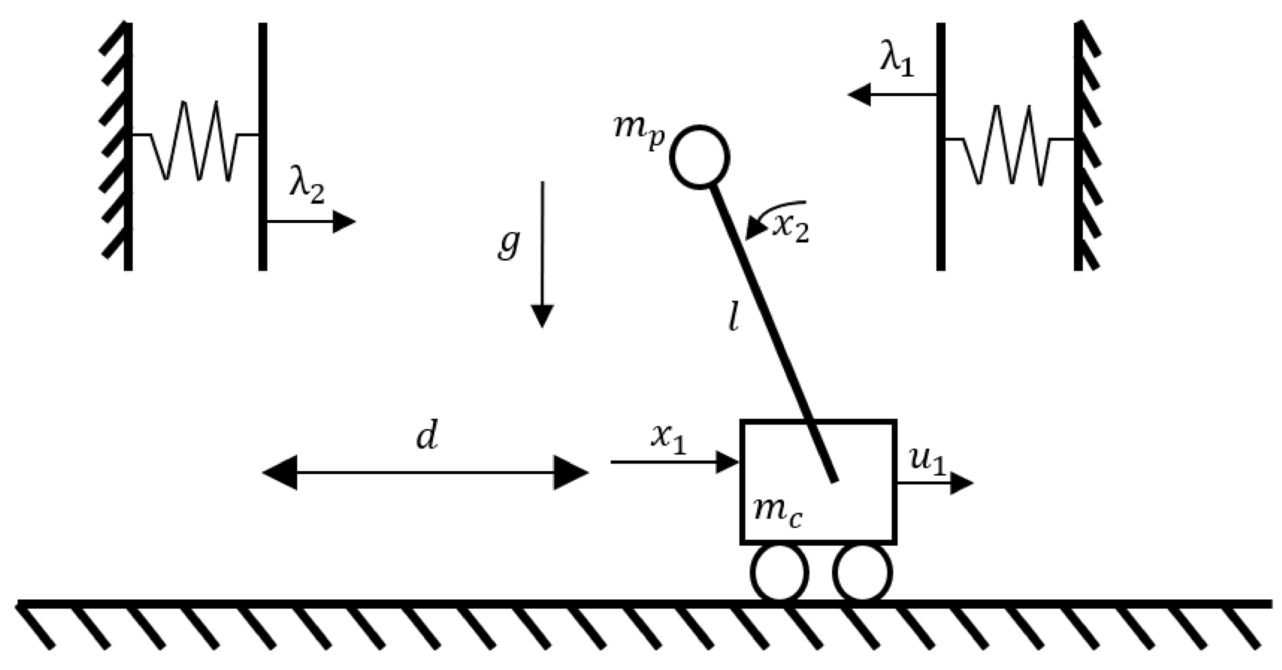
where Γ and Γ' are described as

$$\Gamma_{SOL}(E,F,c) = \{(x,\lambda) : \lambda \in SOL(Ex+c,F)\},$$

$$\Gamma'_{SOL}(E,F,c,d) = \{(x,\lambda,\lambda'(x;d)) : \lambda \in SOL(Ex+c,F)\}.$$

- No mode enumeration in controller design (avoids 2^m scaling)
- Potentially different functions $V_i(x), u_i(x)$ for each mode i
- Between common Lyapunov function [4] and purely hybrid design [3]

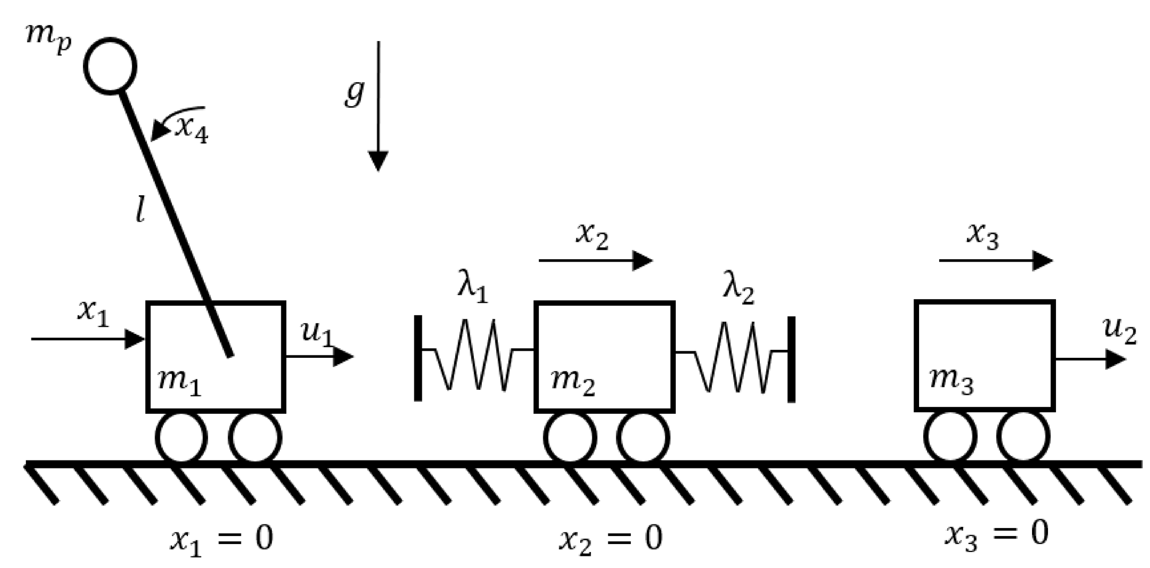
Cart-Pole with Soft Walls



x_1, x_2 – cart, pendulum positions | λ_1, λ_2 – Contact forces

- 100 trials where $(x_1(0), \dot{x}_1(0), \dot{x}_2(0)) \sim U[-1,1]$ and $x_2(0) = 0$
- Contact-aware policy successfully stabilized 100 trials
- LQR with $Q = 10I$ and $R = 1$ was successful 81 times out of 100 trials

Partial State Feedback



x_1, x_2, x_3 – Positions of carts | λ_1, λ_2 – Contact forces

- State information is not always available
- Position (x_2) and velocity ($\dot{x}_2$) of the middle cart is not observed
- Successfully stabilized using tactile feedback (measuring λ_1, λ_2)

Summary

Contributions

- An algorithm for synthesizing control policies that utilize both state and force feedback
- Algorithm exploits the complementarity structure and avoids enumeration (scalable to multi-contact)
- Stability guarantees of the design method

Ongoing

- Friction models
- Five-link robot (balancing) (can utilize partial state feedback)

Related work in [1].

References

[1] Alp Aydinoglu, Victor M Preciado, and Michael Posa. Contact-Aware Controller Design for Complementarity Systems. *arXiv preprint arXiv:1909.11221*, 2019.

[2] M Kanat Camlibel, Jong-Shi Pang, and Jinglai Shen. Lyapunov Stability of Complementarity and Extended Systems. *SIAM Journal on Optimization*, 2006.

[3] Tobia Marcucci and Russ Tedrake. Warm Start of Mixed-Integer Programs for Model Predictive Control of Hybrid Systems. 2019.

[4] Michael Posa, Mark Tobenkin, and Russ Tedrake. Stability Analysis and Control of Rigid-body Systems with Impacts and Friction. *IEEE Transactions on Automatic Control*, 61(6):1423–1437, 2015.