Motivation

- Robots perform in complex, unstructured environments which involve physical interaction between the robot and the environment.
- Tasks like locomotion are fundamentally based in making and breaking contact with the environment.
- State-of-the-art control policies struggle to deal with the hybrid nature of multi-contact motion.
- We propose a control framework which can close the loop on rich, tactile sensors.
- The framework is non-combinatoric, enabling optimization algorithms to automatically synthesize provably stable control policies.

Complementarity Systems

Continuous-time dynamics of rigid-body systems with contacts:

\[ M(q) \dot{v} + C(q, v) = Bu + J(q)\lambda, \quad (1) \]

where

- \( M(q) \) – Generalized coordinates
- \( C(q, v) \) – Generalized velocities
- \( \lambda \) – Contact forces

Describe contact forces using the complementarity framework:

\[ \lambda \geq 0, \quad \phi(q, \lambda) \geq 0, \quad \phi(q, \lambda)^T \lambda = 0. \quad (2) \]

Linearize the smooth components (\( M(q), C(q, v), J(q), \phi(q, \lambda) \)):

\[ \dot{x} = Ax + Bu + D\lambda, \quad 0 \leq \lambda \perp Ex + F\lambda + c \leq 0, \quad (3) \]

where \( x \) is the state, \( u \) is the input and \( \perp \) denotes orthogonality.

Contact-Aware Controller

We propose a controller of the form

\[ u(x, \lambda) = Kx + L\lambda \quad (4) \]

- Feedback based on tactile sensing (e.g., ground reaction force)
- Controller switches based on active contacts (modes), even though the modes are not enumerated.
- Can work under partial state observation.

Non-smooth Lyapunov Function

Captures the non-smooth nature of the dynamics [2]:

\[ V(x, \lambda) = x^T P x + 2 x^T Q \lambda + \lambda^T R \lambda, \]

- Quadratic in terms of the pair \((x, \lambda)\)
- Piecewise quadratic in \(x\)
- Directionally differentiable and Lipschitz continuous.

Controller Design

Solve a bilinear matrix inequality (BMI) to simultaneously find a Lyapunov function and a policy:

\[
\begin{align*}
\text{find} & \quad V, K, L \\
\text{subject to} & \quad V(0, 0) = 0, \\
& \quad V(x, \lambda) > 0, \quad \text{for} \quad (x, \lambda) \in \Gamma_{SOL}(E, F, c), \\
& \quad V'(x; \lambda) \leq 0, \quad \text{for} \quad (x, \lambda; x; \lambda) \in \overline{\Gamma}_{SOL}(E, F, c, \lambda),
\end{align*}
\]

where \( \Gamma \) and \( \Gamma' \) are described as

\[ \Gamma_{SOL}(E, F, c) = \{ (x, \lambda) : x \in SOL(Ex + c, F) \}, \]

\[ \overline{\Gamma}_{SOL}(E, F, c, \lambda) = \{ (x, \lambda, \lambda' : x; d) : x \in SOL(Ex + c, F) \}. \]

- No mode enumeration in controller design (avoids \( 2^m \) scaling).
- Potentially different functions \( V(x), u(x) \) for each mode \( i \).
- Between common Lyapunov function [4] and purely hybrid design [3].

Cart-Pole with Soft Walls

- 100 trials where \( (x_1(0), x_1(0), x_2(0)) \sim \mathcal{U}(-1, 1) \) and \( x_2(0) = 0 \)
- Contact-aware policy successfully stabilized 100 trials
- LQR with \( Q = 10I \) and \( R = 1 \) was successful 81 times out of 100 trials

Partial State Feedback

- State information is not always available.
- Position \( (x_2) \) and velocity \( (x_2) \) of the middle cart is not observed.
- Successfully stabilized using tactile feedback (measuring \( \lambda_1, \lambda_2 \)).

Summary

Contributions

- An algorithm for synthesizing control policies that utilize both state and force feedback.
- Algorithm exploits the complementarity structure and avoids enumeration (scalable to multi-contact).
- Stability guarantees of the design method.

Ongoing

- Friction models.
- Five-link robot (balancing) (can utilize partial state feedback).

Related work in [1].

References