## Fast Trajectory Generation for Quadrupedal Walking on Slopes

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Abstract-Hybrid Zero Dynamics (HZD) has been shown to be a successful framework for the design and control of provably stable gaits for bipedal and quadrupedal robots. Although the use of collocation methods has allowed HZD gait generation to converge quickly, quadrupedal dynamics pose computational challenges due to their large number of degrees of freedom and rich contact behaviors. To ameliorate these challenges, the problem of quadrupedal locomotion can be broken down into that of two coupled bipeds, resulting in noticeable improvements in computation time of gait generation. This manuscript systematically decomposes the dynamics of a quadrupedal robot --- Vision 60 --- into two bipedal robots to rapidly generate gaits and controllers for the quadrupedal robot walking up and down slopes, and on flat ground. By only varying the slope parameters  $(30^\circ, 0^\circ, -20^\circ)$  in the optimization generation process, we are able to produce these natural walking gaits on the quadrupedal robot in seconds. The results are validated in a third-party physics engine - RaiSim.

## I. METHODS

Inspired by the seminal work of Raibert [1], we develop a method of decomposing a quadruped into two bipeds to achieve walking on various ramp inclines. A model of the Vision 60 quadruped with 36 state variables and 12 control inputs is used for gait generation and simulation. In order to reduce the complexity of the gait generation problem, the quadruped is decomposed into two bipeds with a holonomic constraint and coupling wrench connecting them, as in [2]. Gaits are then generated for an individual biped for a 30 degree incline, level ground, and a 20 degree decline using an optimization algorithm that employs the HZD framework. The result is a set of 5<sup>th</sup>-order Bézier polynomials to dictate the position of each joint over a gait cycle. The gaits of the front biped that were optimized are then mirrored for the rear bipedal robot, as seen in Figure 1a, to obtain quadrupedal locomotion. These various gaits are then combined and

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entered into a gait library for the full-order quadrupedal system. In order to transition between ramps of different incline levels, a time-based interpolation method is used to smoothly blend the gait behaviours. Namely, the state variable, x, containing joint angle and velocity information is given by  $x = (1 - \eta)x_p + \eta x_n$  where  $x_p$  and  $x_n$  denote the previous and next gait, respectively, and  $\eta$  is scaled from zero to one over a predefined time interval. These gaits are then tested in the Raisim<sup>1</sup> simulation environment to verify their feasibility and stability.

## **II. RESULTS**

Gait generation for the Vision 60 quadruped takes on average 9.7 seconds on a Linux machine with an i7-6820HQ CPU @2.70 GHz and 16 GB RAM. The gaits are tracked by a time-based PD controller that provides actuation based on the difference between the actual and desired states. When tested in the simulation environment, the quadrupedal robot successfully ambulates over the various terrains as shown in Figure 1b and video [3], and is also able to overcome the transitions between different terrains. Notice the variety in behaviors over the different terrains, namely the difference in angle of the shank with respect to the floor. This diversity of behavior motivates the use of unique gaits conditioned on the terrain to ensure the stability of the robotic system. Additionally, the guaranteed optimality and fast generation of gaits further validates the use of the HZD framework for gait production.

## REFERENCES

- [1] M. Raibert and E. R. Tello, "Legged robots that balance," *IEEE Expert*, vol. 1, pp. 89–89, Nov 1986.
- [2] W.-L. Ma and A. D. Ames, "From bipedal walking to quadrupedal locomotion: Full-body dynamics decomposition for rapid gait generation," *arXiv preprint:1909.08560*, 2019.
- [3] Simulation of Vision 60 Traversing Various Slope Inclines, https: //youtu.be/\_xrWlMc7e0c.

<sup>1</sup>https://github.com/leggedrobotics/raisimLib



(a) Decomposition of the quadruped.



the quadruped. (b) HZD optimized gaits over various ramp inclines using the Raisim simulation software. Fig. 1: Methodology and results for gait generation on the Vision 60 quadruped.