# Discrepancy Modeling in Bipedal Dynamics


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***Abstract*—Models of complex nonlinear systems are often necessarily simplified for tractability. However, such idealizations typically create discrepancies between experiment and simulation. The missing physics are often unique to an individual and cannot be easily represented by universal model corrections. Perturbation theory uses exact linear solutions and perturbation corrections to approximate nonlinear behavior. We use a sparse regression algorithm to recover first-order perturbation solutions, or discrepancy models, for a passive dynamic walking model. Discrepancy models allow idealized models to be updated with interpretable, system-specific corrections for more robust control and dynamical stability.**



***Index Terms*—discrepancy modeling, bipedal walking, sparse regression**


## I. Introduction

Models of bipedal locomotion are often idealized representations of the governing physics and are unable to capture the complete dynamics observed in experiment, i.e. simulation does not match experiment. The inability to represent such discrepancies can have significant impact on the stability of the locomotion model. Discrepancies can arise from a variety of factors that are often ignored in idealized models, including nonlinear frictional forces, musculotendon elasticity, parameter mismatches, and the complex dynamics of soft tissue mechanics. These discrepancies are typically unique to an individual or robotic system and must be learned and quantified as such. Taking inspiration from perturbation theory, we develop a mathematical architecture based upon sparse regression that discovers interpretable discrepancy models to close the gap between the leading order idealized model and experimental observations, giving a more robust and quantitatively accurate locomotion model for control and stable dynamics.

## II. Methods

The sparse identification of nonlinear dynamics (SINDy) algorithm [2] uses sparse regression to discover a parsimonious representation of nonlinear system dynamics from measurement data. An important assumption about the model structure is that there are only a few salient terms governing the dynamics; this assumption holds for systems when represented in an appropriate basis. As such, interpretability is promoted and overfitting is avoided. Recently, this framework has been used to identify discrepancy models between empirical data and model outputs of a system [3]. Discrepancy models can analogously be thought of as a first-order perturbation model. Our solvable problem ($A_0$) is an oscillatory 2-link pendulum [1] with collision constraints, and the first-order perturbation solution ($\tilde{A}$) includes small nonlinear contributions to the equations of motion $A \approx A_0 + \tilde{A}$ where $\tilde{A} \ll 1$.

Two models of simple dynamic walking were tested: passive and active. Two conditions were tested for each model: ideal (no noise) and noisy. Unit variance Gaussian noise was added to the system measurements in varying increments. For all conditions, a randomly generated polynomial discrepancy (max. order 5) was added to one of the system states. Kinematics were evaluated for stability (10+ steps) before using SINDy to recover the discrepancy model.

## III. Results and Discussion

In the ideal condition, SINDy recovered "first-order physics" for both the passive and active dynamic walking models with high fidelity. Adding noise to system measurements made recovery more difficult, as numerical differentiation amplifies noise.

Every system will have its own discrepancy model. Consider the construction of a robot: if two robots are constructed using the same build instructions, each will inevitably have a unique discrepancy model. Part tolerances could be inconsistent between robots, or one robot's joint could experience more friction that the other robot. Similarly, human physiology is subject-specific. For example, skeletal and musculotendon parameters are unique, resulting in unique profiles of muscular performance and production [5]. Humans also have heterogeneous quantities and distributions of soft tissue, often ignored in musculoskeletal models and simulations.

While this initial investigation used a simple toy model to investigate the use and accuracy of discrepancy modeling for recovering locomotion dynamics, including system-specific corrections to an idealized model can help close the gap between experiment and simulation.


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## References


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