The Challenge

Accurately predicting locomotion using idealized model dynamics remains challenging. While first-order, physics-based models describing human or robotic systems generally capture salient characteristics of locomotion, physical systems are not ideal.

Discrepancies can arise due to:
- missing physics
- parameter mismatch

Discrepancies are typically unique to an individual or robotic system and must be learned and quantified as such. Further, the inability to represent the discrepancy between simulation and observation can have significant impact on model prediction and performance [1, 2].

We develop a mathematical architecture based upon sparse regression [1,3] that discovers interpretable, data-driven discrepancy models to augment first principle dynamics. This may aid in more accurate locomotion predictions and improve robust control and stability performance.

We used a simple dynamic walking model in an ideal simulation environment and hypothesized our simulation framework would recover small nonlinear discrepancies from the model’s locomotion dynamics.

Methods

**Goal:** apply a known discrepancy to an ideal system and use SINDy to recover the discrepancy dynamics

**Sparse Identification of Nonlinear Dynamics (SINDy)**

\[
\begin{align*}
X &= \begin{bmatrix} x(0) & x(1) & \ldots & x(t) & \ldots & x(t_{m}) \end{bmatrix}^{T} \in \mathbb{R}^{n \times m} \\
\hat{X} &= \begin{bmatrix} \hat{x}(0) & \hat{x}(1) & \ldots & \hat{x}(t) & \ldots & \hat{x}(t_{m}) \end{bmatrix}^{T} \in \mathbb{R}^{n \times m} \\
\Theta(X) &= \begin{bmatrix} \theta(0) & \theta(1) & \ldots & \theta(t) & \ldots & \theta(t_{m}) \end{bmatrix}^{T} \in \mathbb{R}^{n \times m} \\
\hat{X} &= \Theta(X) \Xi \\
\Xi &= \begin{bmatrix} \xi(0) & \xi(1) & \ldots \end{bmatrix} \in \mathbb{R}^{n \times m} \\
& \text{sparse coefficients of library terms}
\end{align*}
\]

**SINDy for Discrepancy Modeling**

\[
\begin{align*}
\hat{X}_0 &= f(x) \quad \text{ideal system dynamics} \\
\hat{X}_m &= f_m(x) \quad \text{passive dynamic walker} \\
\delta \hat{X} &= \hat{X}_0 - \hat{X}_m = f(x) - f_m(x) = g(x) \quad \text{discrepancy model} \\
\delta \hat{X} &= \Theta(X) \Xi \quad \text{SINDy algorithm}
\end{align*}
\]

**Results**

SINDy recovered discrepancy dynamics with high fidelity

- 88.6% of the discrepancies were correctly identified
- No discrepancy model was identified for the remaining 11.4% due to an inappropriately high sparsity regularization parameter

**Discussion**

This work is a critical step in integrating heterogeneity into musculoskeletal and robotic models to improve prediction accuracy and ensure robust performance.

- Hyperparameter tuning could improve discrepancy model identification capabilities
- Choosing potential library terms requires some level of system knowledge
- SINDy identifies discrepancies quite well in an ideal environment; noise may complicate model recovery due to noise amplification in numerical differentiation

**Next Steps**

**Theoretical**

- increase model complexity (actuation, knees)
- add contact modeling
- add noise filtering/identification

**Experimental**

- collect experimental data from a bipedal robot to validate the simulation framework

References


Acknowledgments

Special thanks to Dr. Amy Wu at Queen’s University for dynamic walking robot fabrication guidance. Research reported in this abstract was supported by the National Science Foundation under award NSF GRF DGE-1762114.