# Contact-Implicit vs. Hybrid Trajectory Optimization: Performance Comparison

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## SUMMARY

Until recently, most trajectory optimization problems for legged robots have been formulated through scheduling the contact sequence using hybrid trajectory optimization. While this will result in a locally optimal trajectory, this optimality relies heavily upon the predefined contact mode transitions. These transitions can also be difficult to determine for complex problems. Contact-implicit trajectory optimization offers a solution to this problem, as this method allows the optimization to decide the contact mode throughout the trajectory. However, this method suffers from long computation times. This study will provide a comparison between contact-implicit and hybrid trajectory optimization for common configurations of a monopod, biped, and quadruped robot. It will specifically investigate convergence rate, computation time, and the optimality of the final trajectories, thus exploring selection criteria for using contact-implicit or hybrid methods.

#### I. MOTIVATION

Hybrid trajectory optimization encodes the motion planning problem in separate trajectories, stitched together using contact mode transitions. In the case of simple problems, such as walking, running, and bounding, this hybrid method results in optimized gaits for these motions, defined by a user specified contact sequence. However, if a more complicated trajectory is desired, the contact sequence and exact contact locations are difficult to determine. Initial guesses about these parameters have to be made and iterated upon. This often limits the possible trajectories that the robot could have taken to reach its goal, many of which could have been more optimal.

Recently, contact-implicit trajectory optimization has been used to solve these same problems [1]. In this method, the optimization is allowed to choose the robot's contact mode at every instance, allowing it to perform a wider variety of possible behaviors. This often results in complex dynamic behaviors, but can have very long computation times. For example, given a robot model and a goal, such as the monopod jumping 1.5 meters to the right shown in Figure 1, using the contact-implicit method allows the robot to reorient itself, jump, and then land and slide to the goal position. In order to determine the best optimization technique to use for a specific problem, a comparison must be performed between these two methods. Results from this study would allow researchers to select an optimization method that will best suit the needs of their robot model, and specifics of the task at hand, including the anticipated number of contacts and the complexity of the contact sequence and constraints.



Figure 1: One legged robot moving forward 1.5 meters, exhibiting a hopping behavior. Problem formulated through contact-implicit method.

### II. METHODOLOGY AND EXPERIMENTAL PLANS

The optimizations are set up in Python, using the Pyomo framework, and IPOPT as the solver. To improve solution accuracy for both the hybrid and contact-implicit methods this framework is currently being converted from trapezoidal collocation to orthogonal collocation [2], [3]. This comparison will be performed by using both hybrid and contact-implicit trajectory optimization methods to determine the optimal trajectory for a monopod, biped, and quadruped robot performing a number of behaviors. The behaviors will include both steady-state gaits like walking, running, and bounding, as well as transient behaviors like traversing a gap in the floor, maneuvering over a box, and climbing stairs. The hybrid method will be set up with a number of different possible contact sequences, and the two methods will be compared on their convergence rate, computation time (over the multiple possible sequences), and optimality of the final solution. This analysis will provide clarity on when to use each method and the situational benefits to each.

#### REFERENCES

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