Energy-efficient planning using approximate step-to-step dynamics for traversing stepping stones

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Introduction

A. Problem Definition

Navigating a series of stepping stones while optimizing an objective while taking into account the robot dynamics during planning phase.

B. Problem Solution

- A model predictive control framework that incorporates the terrain as a cost avoiding integer constraints
- Approximating the step-to-step dynamics with polynomials to enable fast calculations

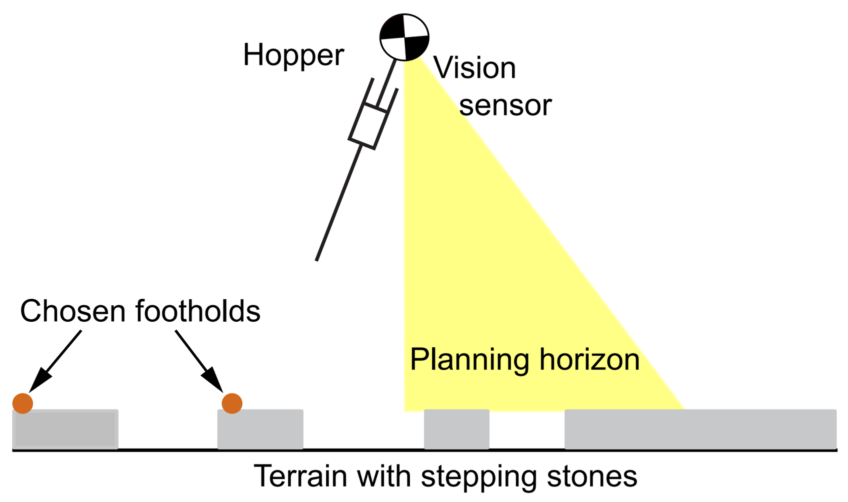

Fig. 1. Conceptualization of the problem: At mid-flight, the hopper plans the optimum strategy and the optimal steps for a planning horizon (yellow patch), then executes the optimum strategy for the first step until the next mid-flight. Then the hopper replans as before continuing the process until it finishes crossing the terrain.

Methods

A. Poincar´e map

Poincar´e map that relates robot state between steps:

$$\mathbf{z}_{i+1} = \mathbf{F}(\mathbf{z}_i, \mathbf{u}_i)$$

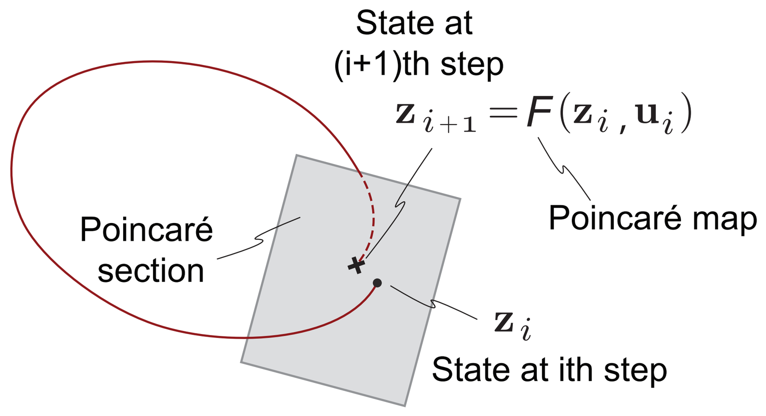

Fig. 2. Dynamical systems tools use for analysis

B. Approximation of the Poincar´e map

$$\mathbf{z}_{i+1} = \overline{\mathbf{F}}(\mathbf{z}_i, \mathbf{u}_i)$$

- Data generation: we choose a range of initial states on the Poincar'e section, $\mathbf{z_i}$, and a range of control actions in the step, $\mathbf{u_i}$.
- Data fitting: we use a suitable defined regression model to fit the data as shown by the gray plane in Fig. 2.

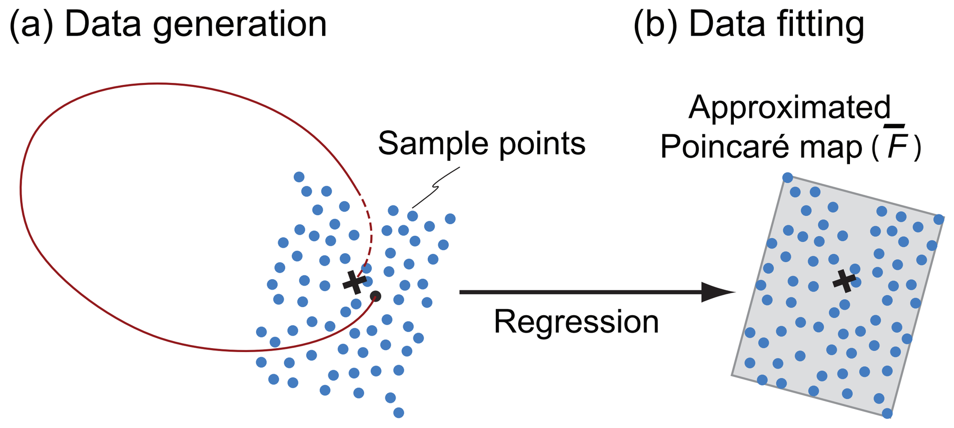

Fig. 3. Regression to approximate the Poincar´e map function $\bar{F}$.

C. Stepping stones terrain

- avoid specifying the stepping stones as a constraint, but as a terrain cost
- assign a high cost for stepping into the gaps between the stepping stones
- fit a cubic spline to the cost

This formulation is computationally efficient because it does not need to use branch-and-bounds as needed in mixed integer problems and also allows us to use nonlinear optimization solvers

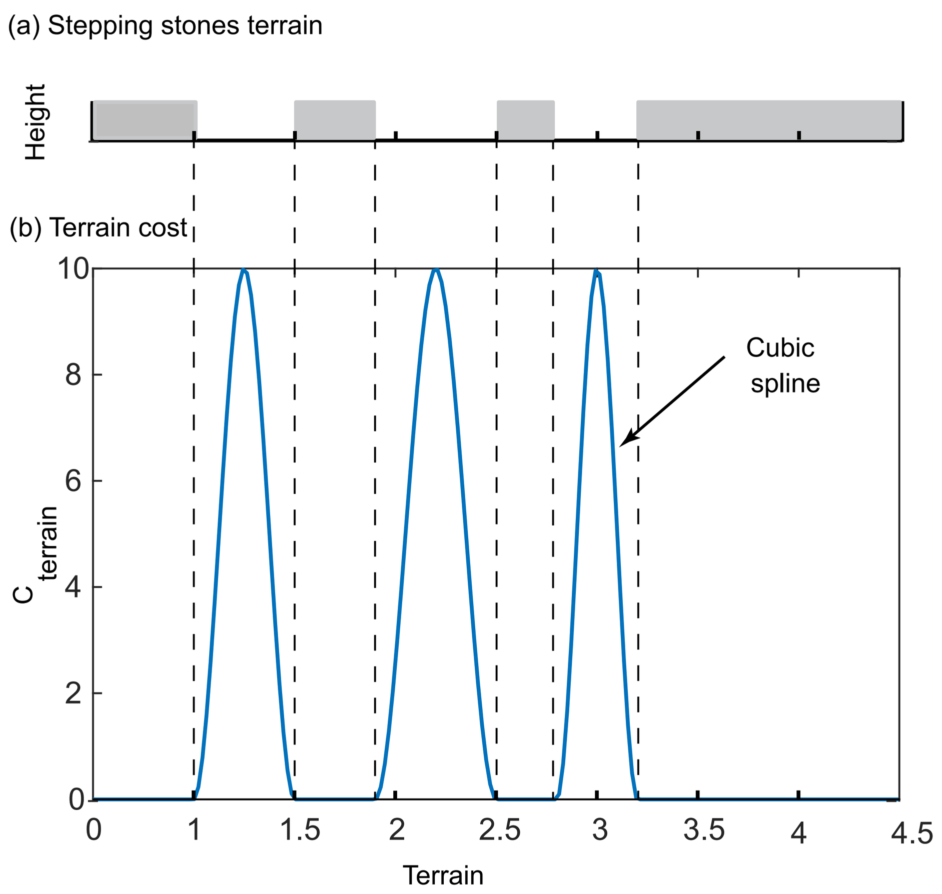

Fig. 4. Incorporating the terrain profile as a cost

D. Nonlinear MPC problem formulation

$$\min_{N,\mathbf{u}_i,x_{ci}} \frac{\sum_{i=1}^{i=N} E_i(\mathbf{u_i})}{mg \times x_N} + \sum_{i=1}^{i=N} C_{\text{terrain}}(x_{ci})$$

$$\begin{aligned} \text{subject to: } & \mathbf{z}_{i+1} = \mathbf{F}(\mathbf{z}_i, \mathbf{u}_i) \text{ (or } \overline{\mathbf{F}}) \\ & \mathbf{u}_{\min} < \mathbf{u}_i < \mathbf{u}_{\max} \\ & x_0 = 0; \\ & x_N = d_{\text{horizon}}; \\ & \dot{x}_0, y_0 \text{ are specified.} \end{aligned}$$

$\mathbf{u}_i = [\theta, P_b, P_t], \quad E_i = \int (|k(\ell - \ell_0)d\ell| + |P_t d\ell| + |P_b d\ell|)$

E. Model

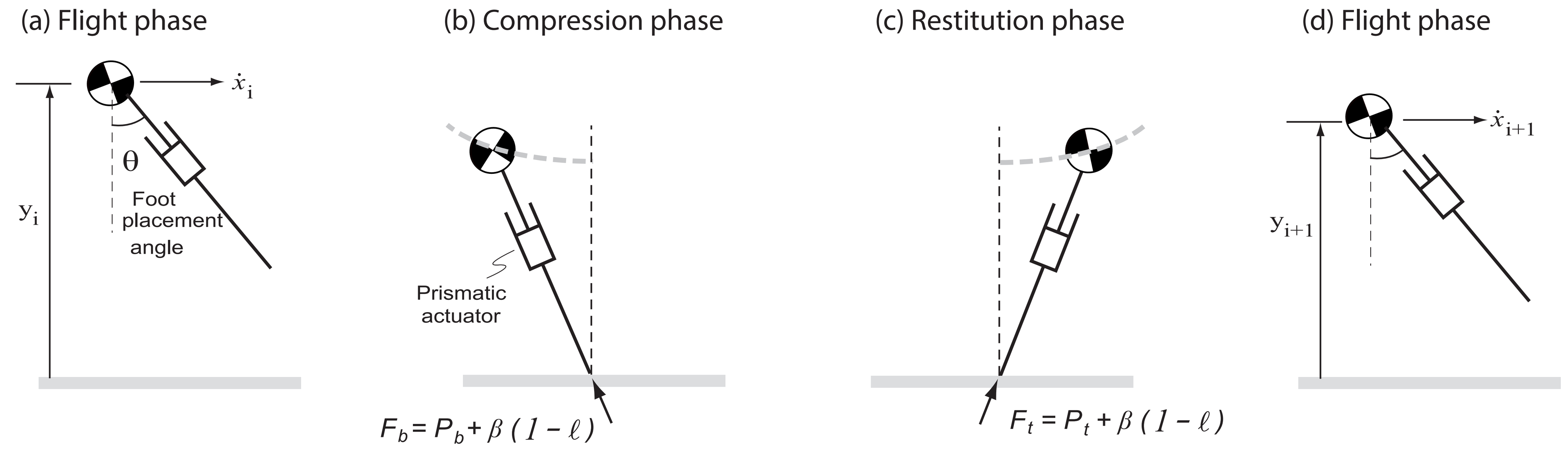

Fig. 5. A complete step for the hopping model

Results

(a) Baseline optimization: $x_N = d_{terrain} = 17.4$ m

(b) MPC with exact Poincar´e map $\mathbf{F}$: $x_N = d_{horizon} = 4$ m

(c) MPC with approximate Poincar´e map $\bar{\mathbf{F}}$: $x_N = d_{horizon} = 4$ m

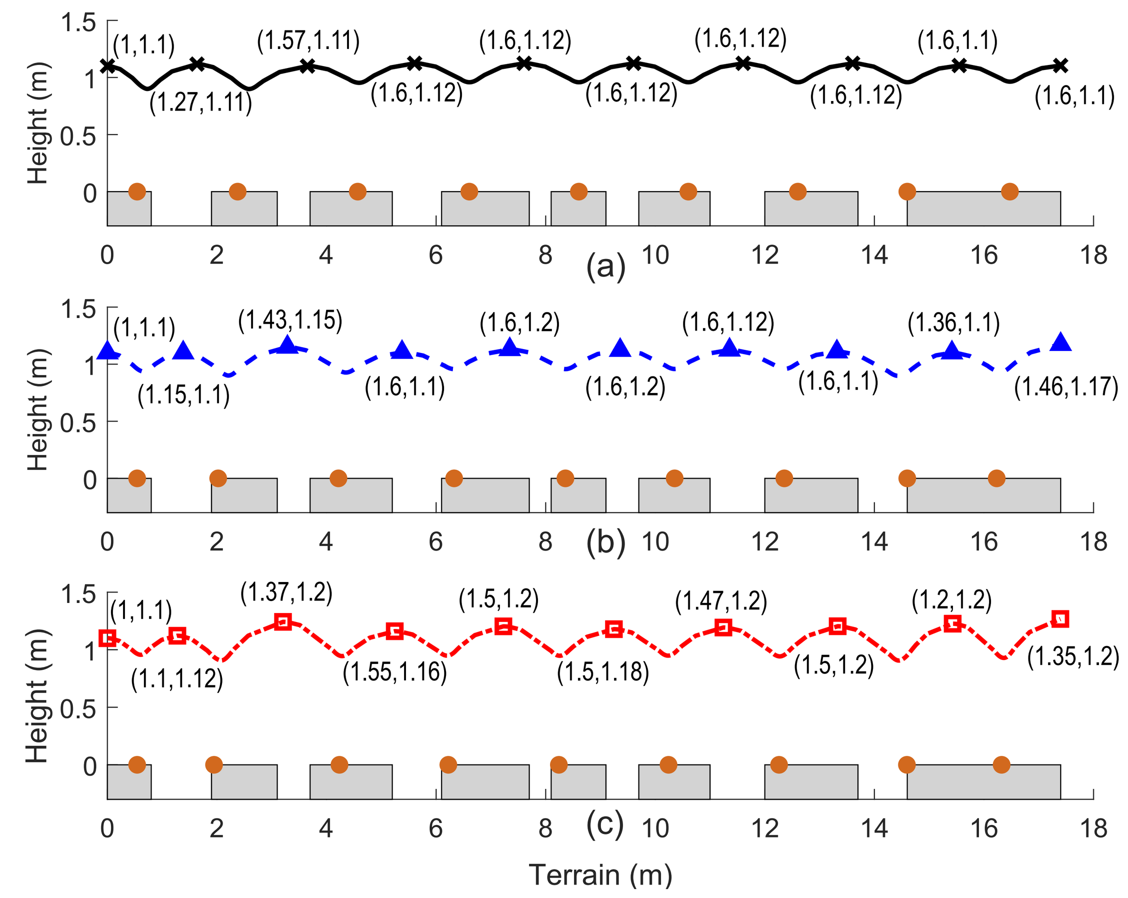

Fig. 6. Kinematic data for the stepping stones terrain: (a) Baseline with exact model (b) MPC with exact model, and (c) MPC with approximate model. The stepping stones terrain is shown as gray blocks. The foot placement location is shown as a brown dot on the gray blocks. The trajectory is shown for each of the optimization cases with the apex velocity and apex height at step i marked as $(\dot{x}_i, y_i)$.

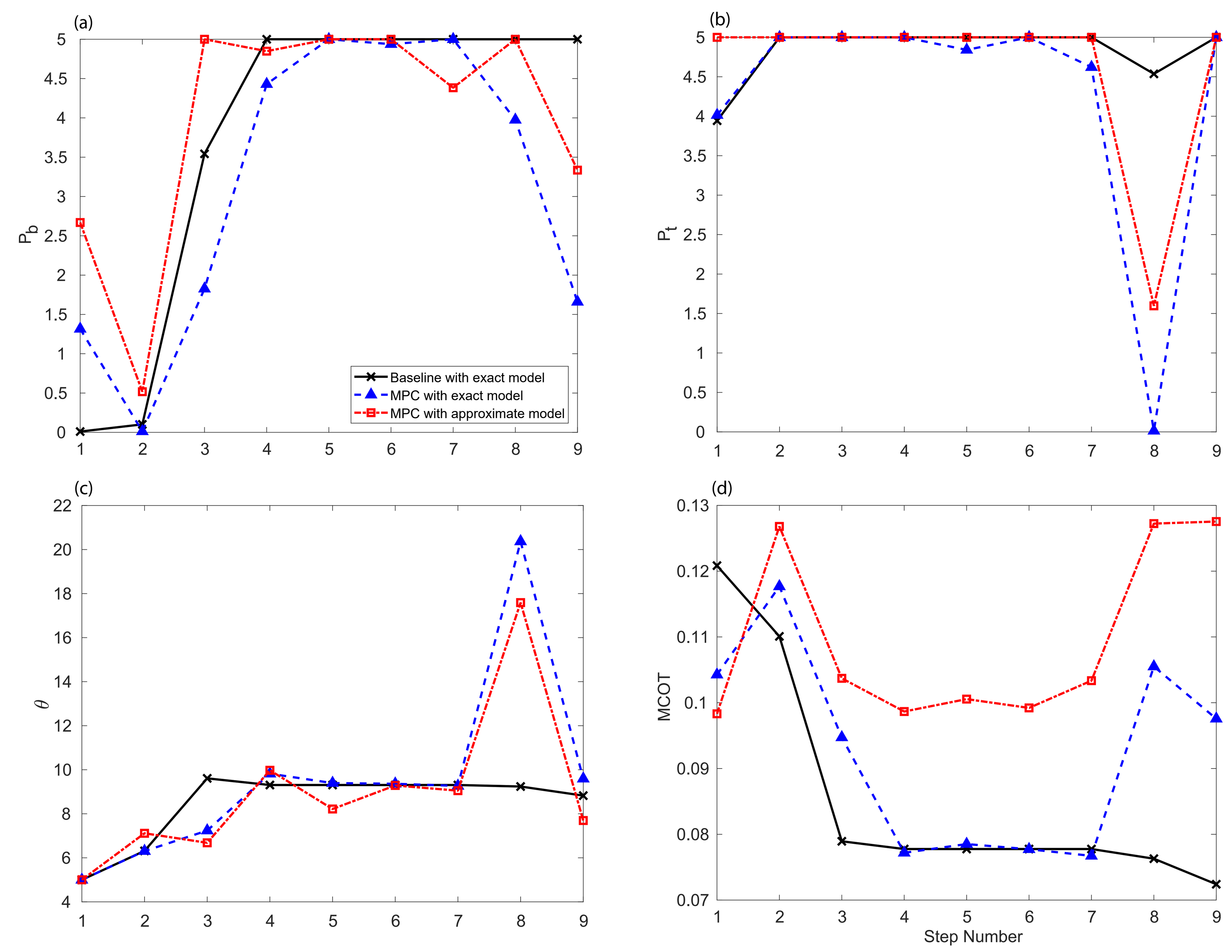

Fig. 7. Controls and energetics for the stepping stones terrain: (a) braking force P_b (b) thrust force P_t (c) foot placement angle θ (d) Mechanical Cost of Transport *MCOT*.