A throughput optimal transmission strategy for broadcast networks

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We will present a throughput optimum distributed scheduling strategy for broadcast networks. The throughput obtained by this distributed scheduling is equal to that obtained by an optimum centralized scheduling.

A broadcast network is one where all nodes are in the transmission range of each other. Consider a broadcast network with N terminals and a slotted time. The arrival process for terminal i is i.i.d. with average a_i, and first moment p_i^2. Also, $\sum_{i=1}^N a_i < 1$, and $\sum_{i=1}^N p_i^2$ finite. We denote a_i as the arrival rate for terminal i. Choose a time interval T such that $T > \frac{N}{1-\sum_{i=1}^N a_i}$. Note that for choosing T one needs only an upper bound on the sum of the arrival rates, and not their exact values. Furthermore, a terminal can receive at most K_i packets in a slot, where K_i is finite. The time axis is divided in cycles of duration T each. At the beginning of every cycle, every terminal broadcasts a packet with its queue length information in a predetermined sequence, i.e., these broadcast packets do not collide. Thus, a total of N slots are consumed in this broadcast every cycle. Terminals transmit packets in order of their queue lengths in the remainder of the cycle, i.e., the terminal with the largest queue length transmits packets till it has 0 queue length. Next, the terminal with the second largest queue length transmits packet, etc. Every terminal includes it current queue length in the header of its data packet. Thus, the terminals can decide the transmission order in a distributed manner, and there is no collision.

The scheduling policy is throughput optimal in the sense that it is stable for all i.i.d. arrival processes with $\sum_{i=1}^N a_i < 1$. A system is said to be "stable" if the average queue length is finite. Clearly, a system can not be stable under any scheduling policy if the total arrival rate exceeds 1. Note that the queue length process at the start of each cycle is a discrete time markov chain. We will prove that this markov chain has a steady state probability distribution with finite average queue lengths. Since cycle duration T is finite, a terminal can have only a bounded number of arrivals in a slot and transmit at most one packet in a slot, the queue length during the cycle can differ from the queue length at the start of the cycle by at most a constant. Thus it is sufficient to prove that the queue lengths at the start of a cycle have a finite average.

Let $B_i(t)$ denote the queue length of terminal i at start of the tth cycle, and $\vec{B}(t) = (B_1(t), \ldots, B_N(t))$. Note that $\vec{B}(t)$ is a markov chain. We need to show that this markov chain has a steady state distribution with a finite mean. Let $V(t)=\sum_{i=1}^{N}B_i^2(t)$. We will show that the quadratic lyapunov function $V(t)$ has negative drift. More precisely, we need to show that $E\left(V(t+1)-V(t)|\vec{B}(t)=\vec{b}\right)<-M$ except for $\vec{b}\in W$, where W is a finite set and M is some positive constant. The result will then follow from standard results in theory of markov chains[1].

Let $W=\{\vec{b}: b_i\geq T$ for some $i, 2T\max_i b_i\left(\sum_{i=1}^{N}a_i-(1-N/T)\right)+\sum_{i=1}^{N}p_i^2+2N(T-N)^2\leq -M\}$. Observe that since $\left(\sum_{i=1}^{N}a_i-(1-N/T)\right)<0$, W includes all possible states $\vec{B}(t)$ except a finite number of states. Let $\vec{b}\in W$. Thus, $\max_i b_i\geq T$. Let $A_i(t)$ be the total number of packet arrivals in the tth cycle for terminal i. Note that $A_i(t)$ is independent of $\vec{B}(t)$ and $EA_i(t)=a_iT$ for all i,t. Let $D_i(t)$ be the total number of packets transmitted in the tth cycle by terminal i. If $\vec{B}(t)\in W$, then $D_j(t)=T-N$ for a terminal j which has th maximum queue length at the start, ($j=\arg\max_i B_i(t)$), and $D_j(t)=0$ otherwise.

$$\begin{aligned}
& E\left(V(t+1)-V(t)|\vec{B}(t)=\vec{b}\right)\\
= \; & E\left(\left(\sum_{i=1}^{N}\left(B_i(t+1)-B_i(t)\right)\left(B_i(t+1)+B_i(t)\right)\right)|\vec{B}(t)=\vec{b}\right)\\
= \; & E\left(\left(\sum_{i=1}^{N}\left(A_i(t)-D_i(t)\right)\left(A_i(t)-D_i(t)+2B_i(t)\right)\right)|\vec{B}(t)=\vec{b}\right)\\
= \; & E\left(\sum_{i=1}^{N}\left(A_i(t)-D_i(t)\right)^2|\vec{B}(t)=\vec{b}\right)+2E\left(\sum_{i=1}^{N}B_i(t)\left(A_i(t)-D_i(t)\right)|\vec{B}(t)=\vec{b}\right) \qquad (1)
\end{aligned}$$

$$E\left(\sum_{i=1}^{N}\left(A_i(t)-D_i(t)\right)^2|\vec{B}(t)=\vec{b}\right)\leq\sum_{i=1}^{N}p_i^2+2N(T-N)^2 \text{ if } \vec{b}\in W \qquad (2)$$

$$\begin{aligned}
E\left(\sum_{i=1}^{N}B_i(t)\left(A_i(t)-D_i(t)\right)|\vec{B}(t)=\vec{b}\right) &= T\sum_{i=1}^{N}a_ib_i-(T-N)\max_i b_i\\
&\leq T\max_i b_i\left(\sum_{i=1}^{N}a_i-(1-N/T)\right) \qquad (3)
\end{aligned}$$

From definition of W, (1), (2) and (3)

$$E\left(V(t+1)-V(t)|\vec{B}(t)=\vec{b}\right)\leq -M, \text{ if } \vec{b}\in W$$

The result follows.

The scheduling policy proposed here shows that in a broadcast network a distributed scheduling can attain the same throughput as the optimum centralized scheme, using queue length information. The distributed scheme need only know bounds on the sum of the arrival rates. However, the distributed scheme is not the best in terms of average packet delay, etc. Ongoing effort is directed towards improving the delay properties while retaining the throughput optimality.

References

[1] S. Meyn and R. Tweedie. *Markov Chains and Stochastic Stability.* Springer Verlag