

Homework 10 (Posted 16th April, Due during or before 1 p.m. 23rd April (**Please submit to Ms. Drucilla Spanner. Programming assignment due by 25th April, 11.59 p.m.**))

**Policy for Programming Assignment:** Problem 1 has a programming assignment. The design for this one will be posted 23rd April. **The programming assignment is due by 25th April 11.59 p.m.** If your name is John Smith, then name your program as JohnSmith.c and email it to yjkim78@gradient.cis.upenn.edu.

**Problem 1: 10+15** Design an algorithm to find the maximum weighted path from a source to all destinations in a DAG. Analyze its complexity. Prove its correctness. Program your algorithm.

**Problem 2: 10 pts** Give an algorithm to detect the existence of a negative weight cycle in a strongly connected digraph. Analyze its complexity and prove its correctness.

**Problem 3: 5 pts** Give a counterexample to show that Dijkstras algorithm does not generate the shortest paths if edges have negative weights, even if the digraph does not have a nonpositive weight cycle.

**Problem 4: 10 pts** Let  $p(u, v)$  be the shortest path weight from  $u$  to  $v$ . Give an  $O(VE)$  algorithm to find the  $\min_{v \in V} p(v, w)$  for all vertices  $w$  in a digraph. Note that the algorithm should find  $\min_{v \in V} p(v, w)$  for all vertices  $w$  and should run in  $O(VE)$ . Prove its correctness.