The Impact of Imperfect Information in Multi-Channel Wireless Systems

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***Abstract*— We consider a wireless system with multiple channels when each channel has several different transmission states associated with different probabilities of successful transmissions. Hardware limitations and MAC protocols dictate that the channel be switched only after certain intervals, and the channel needs to be selected based on the channel states at the beginning of the interval. We demonstrate that the fundamental relations between QoS metrics like throughput and stability significantly change owing to the lack of complete information. We obtain a randomized channel selection and threshold-type transmission rule that maximizes throughput while using imperfect information. Using this optimal strategy, we numerically quantify the penalty associated with different amounts of imperfections in the available information.**

I. INTRODUCTION

A. Motivation

Wireless networks are being rapidly deployed all over the world. Cellular networks already exist in several parts of the world. In the U.S. several companies like Boingo, Cometa and T-mobile are deploying nationwide IEEE 802.11b based wireless local area networks (Wi-Fi networks) for data and voice communications. Ad hoc networks provide the only means of communication in remote terrains, battle-fields, disaster recovery operations. The success of this proliferation is however contingent upon providing the desired quality of service (QoS) to the users. The following are the main challenges towards this goal.

1) Bandwidth limitations: Wireless channels can support low data rates as compared to wireline networks owing to the limitations in the radio spectrum. For example, due to interference, the same channel can not be used simultaneously in multiple transmissions in a vicinity. Also, the available channels suffer from fading due to obstructions in signal path, mobility of terminals and interference, which in turn leads to variable transmission quality.

2) Imperfect Information: The link schedulers often have access to limited prior information about transmission quality in the available channels. This is sometimes due to fundamental limitations such as random fluctuations in the channels which can not be known apriori. For example, the scheduler can measure the average ratio between the signal and the noise power (SNR) prior to each transmission, and can thereby learn the probability of successful transmission but not the exact outcome. Sometimes, the scheduler deliberately acquires limited information since the cost of acquiring the complete information is prohibitive. For example, a scheduler can measure the SNR in each available channel prior to a transmission, but this introduces significant control overhead.

3) Software limitations: Many of the current day wireless devices have simple and inexpensive hardware. This in turn imposes limitations on the software complexity and thereby the resource allocation strategies that can be executed in these devices. For example, due to the execution complexity of the channel switching protocol, oftentimes a device can switch channels only after a certain minimum delay. This in turn degrades the performance.

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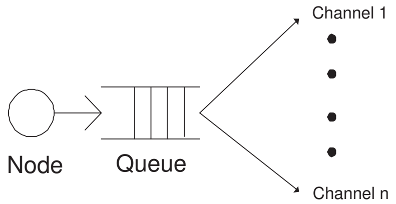

Fig. 1. A node generating traffic at rate λ with access to n channels.

Several existing wireless technologies, e.g., IEEE 802.11a [1], IEEE802.11b [4], IEEE802.11h [2] propose to use multiple channels so as to mitigate the first challenge. A channel can for example be a frequency in a frequency division multiple access network or a code in a code division multiple access network. When multiple channels are available then multiple transmissions can proceed simultaneously in a vicinity if each transmission uses a different channel. Also, at any given time the probability that there exists at least one channel with acceptable transmission quality increases with increase in the number of channels. But, the availability of multiple channels does not mitigate, and sometimes aggravates the other two challenges. This is because the control overhead required for acquiring complete information on all available channels increases linearly with increase in the number of channels. Channel switching also incurs additional software overhead.

The above observations motivate the investigation of the impact of imperfect information and hardware limitations on optimal spectrum utilization in wireless networks with multiple channels. Towards this end, we investigate a partial information based stochastic control problem which is fundamental to a broad range of communication scenarios in multi-channel wireless networks. We demonstrate that imperfect information drastically alters the relations between fundamental QoS metrics like throughput and stability. We thereafter quantify the optimal performance in such a system as a function of the amount of available information and the switching delay.

B. Problem Definition

We consider a node that has access to n different channels (Figure 1). The node generates packets at the rate of λ per unit time and needs to transmit the packets in the available channels. It can transmit in only one channel in a time slot and can transmit at most one packet in each slot. At the beginning of each interval of size L slots, it can obtain information on the transmission conditions which are the probabilities of successful transmissions in the channels. During the interval, the node transmits packets only in the selected channel. In each slot, it knows the probability of successful transmission in only the selected channel, and based on this information decides whether to transmit. The interval size L is a system parameter whose value is determined by the restrictions on control overhead and the minimum switching delay imposed by limitations on software complexity. The goal is to select the channel and the transmission times so as to maximize the throughput (i.e., the rate of successful packet transmissions).

This problem forms the basis of wireless link scheduling in several different wireless networks. We now present some examples. (a) The node can be a terminal communicating with another terminal in an ad hoc network, or (b) a terminal communicating with an access point in a Wi-Fi network or (c) a mobile station communicating with a base station in a cellular network. The channel can be a frequency in each of these cases. In the first two cases the node can use the IEEE 802.11a [1] or IEEE 802.11b [4] protocols for medium access. IEEE 802.11a protocol has 8 channels for indoor use and 4 channels for outdoor use in the 5GHz band, while the IEEE 802.11b protocol has 3 channels in the 2.4GHz band. In a cellular network, a mobile station in a GSM system is assigned a 200 KHz channel which it shares with 7 other MS's in a TDMA frame. It is possible to envision a GSM-like scheme assigning a specific sequence of channels in the 200-KHz frequency channels to some mobile station. In all three cases, a node can transmit in only one channel at a time if it has a single network interface card (NIC).

Now, mobile terminals can also have multiple antennas, e.g., mobile laptops with multiple antennas (antenna arrays) incorporated in the front lid have been developed. Thus, in (a), (b), (c), a channel can also be (i) an antenna or (ii) a polarization state (vertical or horizontal) of an antenna or a (iii) pointing direction of a small transmit array (used to broadly direct power in, say, one of two slightly different directions). If a node has multiple antennas and the antennas represent the channels, the node can simultaneously transmit in all the channels. But, this requires compatible transmission circuits to appropriately distribute the power across the antennas, which may not always be present. However, antenna selection and transmission using one antenna in a slot can be accomplished through software modifications.

The last example where this optimization applies consists of a node in the transmission range of multiple access points. Then the different channels represent links with different access points, and the selection among the channels is equivalent to selection of the appropriate access point. The node can use the IEEE 802.11h [2] MAC protocol that provides 8 channels in the 5GHz band. Note that IEEE 802.11h standard allows the receiver to communicate the received signal strength (RSSRI) which can in turn be used for estimating the probability of successful transmission. A node needs to "associate" with an access point before it can transmit a packet, and typically the association delay is significant leading to a large value of L. But, future association protocols and strategies may allow a node to simultaneously associate with two or more access points. In this case a node may switch access points after moderate values of L.

C. Our Contribution

The throughput needs to be maximized by appropriately selecting the transmission time and the channel based only on partial information. The information is partial in the following sense. (a) The node selects a channel for an entire interval based only on the previous and current transmission conditions in the channels. Thus a node needs to select the channel for future transmissions without knowing the future states. (b) Before deciding whether to transmit at the current time, the node only knows the probability of successful transmission, and not the outcome of the transmission. In Section II we present the channel model and describe some system assumptions. In Section III we demonstrate that this limited information changes fundamental system dynamics and also significantly complicates the computation of the optimal strategy. For example, in many systems a selection strategy maximizes throughput if and only if it stabilizes the system for the maximum possible set of arrival rates[1], e.g., [10], [11]. But, this equivalence does not hold in the system we consider. Thus, the standard technique for proving throughput optimality of a policy via proving that it stabilizes the system for the maximum possible set of arrival rates does not apply.

We now describe some key challenges in designing a policy that maximizes the throughput. First, consider the challenges in determining the optimum transmission times. If a sender transmits only when the probability of successful transmission in the selected channel is at the highest possible value (which may still be less than 1 as the probability of successful transmission is 1 only when the signal to noise ratio is infinite for gaussian noise processes for example), then it may only transmit packets infrequently which leads to low throughput. On the other hand, if a sender transmits irrespective of the probability of successful transmission in the selected channel, then it will transmit packets frequently but many of these packets will be received with error. This also leads to low throughput. Thus, the optimum decision is likely to be somewhere between these extremes.

The next challenge is to select the channel at the beginning of each interval. Consider a system with two channels for this discussion. Intuitively, the channel which is likely to have better transmission condition throughout the interval based on the measurements at the beginning of the interval and the channel statistics must be selected. The challenge here is that the decision is closely coupled with the transmission decision. For example, the transmission strategy may allow the system to transmit only when the selected channel is in certain states ("acceptable states"). Also, the transmission conditions of the channels in the slots in which the sender does not have a packet to transmit does not affect the throughput. Thus, the channel that has better transmission conditions in the slots in which (a) it is in an acceptable state and (b) the sender has a packet to transmit must be selected. These conditions depend upon the transmission policy, the sender's queue length at the beginning of the interval, future arrivals and future transmission states. The bottom-line is that the transmission and channel selection decisions depend on each other and must be jointly optimized using channel and arrival statistics.

In Section IV we have shown that the optimal channel selection and the transmission decisions can be obtained as a solution of a linear program which also quantifies the

[1] A system is stable if the expected queue length at the sender is bounded.

optimum throughput. The optimal transmission decision is to transmit whenever the sender has a packet to transmit and the probability of success in the selected channel exceeds a certain threshold that is selected randomly. The choice of the threshold depends on the channel statistics, L and λ but not on the instantaneous state of any channel. The channel selection is also random and is based on the statistics, L, λ and the channel states in the beginning of each interval.

In Section V, we discuss some salient features of the computation framework we propose and identify some directions for future research.

II. System Model and Definitions

We describe our assumptions about the system. We assume that time is slotted. The sender has infinite buffer. The packet arrival process at a sender evolves as per an irreducible, aperiodic markov chain with K states and stationary distribution $\{a_i\}$. Now, λ_i packets are generated in a slot in which the process is in state i. Also, $\sum_i a_i \lambda_i = \lambda$.

In any slot, the transmission condition in the n channels can be described by a $n-$dimensional joint channel state vector $\vec{u} = (u_1, \ldots, u_n)$ where u_i represents the state of channel i, $u_i \in \{1, \ldots, G\}$, $\forall i, 1 \leq i \leq n$. When channel i is in state u, a packet is successfully transmitted in it with probability $\alpha_i(u)$, $0 \leq \alpha_i(1) < \alpha_i(2) \ldots < \alpha_i(G) \leq 1$, $\forall i$. Thus, the transmission condition in a channel improves as its state u increases. Let S be the set of all possible joint channel state vectors. Now, $\vec{u}$ evolves according to a finite dimensional irreducible aperiodic markov chain with a transition matrix denoted by A and unique stationary distribution $\vec{\pi} = \{\pi(\vec{u}) : \vec{u} \in S\}$. The states of different channels may be correlated. This model has been motivated by multi-state markov models used for modeling fading channels [6], [12], [13]. The arrival process and the channel states are independent.

The slots are divided in intervals of size L. At the beginning of each interval the sender knows the current channel state vector $\vec{u}$ and hence the probabilities of successful transmission in each channel. In any slot, the sender knows the state and hence the probability of successful transmission in the selected channel.

Definition 1: Let $b_{\vec{u},i,j}$ be the probability that channel i is in state j in an arbitrary slot in an interval given that the channel state vector at the beginning of the interval is $\vec{u}$. Let $I(u_i = j)$ be an indicator which is 1 when $u_i = j$. Then,

$$b_{\vec{u},i,j} = \frac{I(u_i = j) + \sum_{k=1}^{L-1} \sum_{\vec{v} \in S: I(v_i = j)} A^k_{\vec{u},\vec{v}}}{L}. \quad (1)$$

We now present some definitions that will be used throughout the paper.

Definition 2: A *transmission policy* is an algorithm that decides in each slot t whether to transmit a packet in the slot.

Definition 3: A *channel selection policy* is an algorithm that selects a channel in the beginning of each interval.

The transmission and channel selection policies determine whether to transmit and which channel to select based on the sender's queue lengths and the observed channel states in the current and all previous slots.

We assume that a transmission policy does not transmit a packet in a slot in which the selected channel i is in a state u such that $\alpha_i(u) = 0$.

Definition 4: *Throughput* is the expected number of packets received successfully per unit time.

Definition 5: A system is *stable* if the mean queue length at the sender is bounded. A *stable transmission policy* is one that stabilizes the system.

Definition 6: The *stability region of a policy* is the set of arrival rates λ for which the sender has finite expected queue lengths when it executes the policy.

Definition 7: The *stability region of the system* is the union of the stability regions of all policies.

Clearly, the stability region of the system is a subset of $[0, 1]$.

Definition 8: A policy *attains the stability region of the system* if its stability region equals that of the system.

Definition 9: A transmission policy is *ϵ-throughput optimal* if its throughput differs from the maximum possible throughput by at most ϵ.

III. Relation between performance metrics in presence of imperfect information

In many stochastic control systems a strategy maximizes throughput if and only if it attains the stability region of the system [8], [11]. The latter happens if there exists a lyapunov function that has a negative drift for the policy for all arrival rates for which there exists at least one policy that stabilizes the system. Many policies have been proved to be throughput optimal by showing that such a lyapunov function exists. We next demonstrate through an example that in presence of imperfect information about transmission conditions, a policy that attains the stability region of the system need not maximize the throughput. Thus, the existing framework for proving throughput optimality does not apply.

Example 1: Consider a system with a single channel which can be in one of two possible transmission states with probabilities of successful transmission 0.2 (state 1) and 0.8 (state 2) respectively. Assume that the states of the channel in different slots are independent. Here the question of channel selection does not arise. The sender generates a packet with probability λ in each slot. The number of packets generated in different slots are independent. Consider two transmission policies Δ_1 and Δ_2. Under Δ_1, the sender transmits whenever it has a packet. Under Δ_2, the sender transmits whenever it has a packet and the channel is in state 2. Let the probability that the channel is in state 1 be β and in state 2 be $1 - \beta$. Thus, the system is stable for all $\lambda < 1$ under Δ_1 and only when $\lambda < 1 - \beta$ under Δ_2. Thus, Δ_1 stabilizes the system whenever some policy stabilizes the system, and if $\beta > 0$, Δ_2 does not satisfy the above property. We now show that depending on λ and β, Δ_1 can have lower throughput than Δ_2. Let $\beta > 0$. For $\lambda \neq 1$, Δ_1's throughput is $\min(\lambda, 1)\,(0.2\beta + 0.8(1 - \beta)) = \min(\lambda, 1)(0.8 - 0.6\beta)$. For $\lambda \neq 1 - \beta$, Δ_2's throughput is $0.8 \min(\lambda, 1 - \beta)$. Thus, Δ_1 can have lower throughput than Δ_2 when (i) $\lambda < 1 - \beta$ (i.e., when both policies stabilize the system) and (ii) when $1 - \beta < \lambda < \min(1, \frac{1-\beta}{1-2/3\beta})$ e.g., $\lambda = 0.202, \beta = 0.8$. In (ii), Δ_2 does not stabilize the system, but still its throughput is higher than that of a stable policy, Δ_1.

We now demonstrate that for some arrival rates the throughputs of all the stable policies may be significantly lower than the maximum throughput.

Example 2: Consider a system with two channels. Channel 1 has a probability of successful transmission ν in each slot. Channel 2 has a probability of successful transmission of 0 (state 1) with probability β and probability of successful transmission $\gamma > 0$ (state 2) with probability $1 - \beta$. Thus, $\alpha_1(1) = \nu$, $\alpha_2(1) = 0, \alpha_2(2) = \gamma$. Assume that the states of the two channels are mutually independent in each slot.

Also, the states of the same channel in different slots are independent. The arrival process is the same as that in the previous example. Let $1-\beta < \lambda < 1$ and $L > 1$. Consider a policy Δ_1 in which the sender always selects channel 2 and transmits whenever it has a packet and channel 2 is in state 2. Thus the sender transmits in at most $1-\beta$ fraction of slots. Since $1-\beta < \lambda$ the system is unstable and the sender always has packets to transmit. The sender's throughput is $\gamma(1-\beta)$. Thus, the maximum throughput is greater than or equal to $\gamma(1-\beta)$. If however the sender always selects channel 1 it can transmit in all slots in which it has a packet to transmit, and therefore the system is stable for all $\lambda < 1$. Now, if λ is close to (but less than) 1, then under a stable policy the sender must select channel 1 most of the times. Thus, the throughput of a stable policy is close to $\nu\lambda$ which is significantly less than $\gamma(1-\beta)$ when $\nu/\gamma << (1-\beta)/\lambda$. For example, let $\lambda = .9, \beta = .4, \gamma = .9, \nu = .1$. Now, $\gamma(1-\beta) = 0.54$. For $L \geq 2$, the maximum throughput of any stable policy is 0.09 which is $1/6$th of the lower bound $\gamma(1-\beta)$ for the maximum possible throughput.

Now, consider a system in which the sender can select a channel in each slot ($L = 1$), knows the state of each channel in each slot, and a channel can be in one of two possible transmission states with probabilities of successful transmission 0 and 1 respectively. Thus, the sender knows which channels will have successful transmission in each slot. Hence, the sender has perfect information in such a system. Now, a policy maximizes the throughput if and only if it attains the stability region of the system. This is because the system stability region will be attained only when the sender selects in each slot a channel that has probability 1 of successful transmission in the slot and transmits a packet in the selected channel if it has a packet to transmit[2]. If no channel has probability 1 of successful transmission in a slot, the sender does not transmit in the slot. Any such policy maximizes the throughput for all arrival rates. *It therefore appears that the equivalence between maximizing throughput and attaining the stability region is lost in the system we consider due to availability of imperfect information about transmission conditions.*

IV. Throughput optimal Channel Selection and Transmission Strategy

We present a channel selection and transmission strategy that maximizes the throughput. We initially consider systems where every channel has a strictly positive probability of successful transmission in each state, i.e., $\alpha_i(u) > 0$ for each i, u. We demonstrated in Example 1 that any policy that attains the stability region of the system need not maximize the throughput in these systems. We now demonstrate that there however exists at least one policy that both maximizes[3] the throughput for all arrival rates and also attains the stability region of the system. We present such a policy. We next present a policy that maximizes the throughput when $\alpha_i(u) = 0$ for some i, u.

We first examine why when $\alpha_i(u) > 0$ for each i, u we expect to obtain a policy that maximizes both the throughput and attains the stability region of the system. The claim is somewhat counter-intuitive as we just showed in Example 2 that for several values of λ, throughputs of all stable policies can be significantly less than the maximum possible throughput. But, $\alpha_2(1) = 0$ in Example 2. Now, when $\alpha_i(u) > 0$ for each i, u, the stability region of the system is $\{\lambda : \lambda < 1\}$[4]. This follows as when the sender always selects channel 1 and transmits packets whenever it has a packet to transmit, the system is stable for all $\lambda \in [0, 1)$. The claim can therefore be restated as there exists a policy that maximizes the throughput and stabilizes the system for all $\lambda \in [0, 1)$. In other words, for all $\lambda \in [0, 1)$ a stable policy maximizes the throughput. We now explain why this is the case. If not, then for some $\lambda \in [0, 1)$ an unstable policy maximizes the throughput. Since the policy is unstable, the sender always has packets to transmit. The sender transmits at a rate μ, where $\mu < \lambda < 1$. Thus the sender does not transmit in all slots. If the sender increases its transmission rate to $\mu + (\lambda-\mu)/2$, its transmission rate is still less than λ. Hence, it continues to be infinitely backlogged. Since every channel has a positive probability of successful transmission in each slot, the sender's throughput increases by at least $\frac{\lambda-\mu}{2}\min_{i,u}\alpha_i(u) > 0$. Thus, the throughput can always be increased by increasing the transmission rate if the policy is unstable. Thus, unstable policies can not maximize the throughput.

We now provide the intuition we use to design a throughput optimal policy when $\alpha_i(u) > 0$ for each i, u. It is worthwhile to note that in practice $\alpha_i(u) > 0$ for each i, u, as probability of successful transmission is 0 only when noise power is infinite. It follows from the previous discussion that a stable policy maximizes the throughput in this case. We therefore need to design a policy that maximizes throughput among all stable policies.

For simplicity, we assume that $\alpha_i(u) = \alpha_j(u)$ for all channels i, j without loss of generality[5]. Note that the individual channels may still be statistically different. For example, if individual channel states are markovian they could have different stationary probabilities because of different transition probabilities.

First consider how to maximize throughput by appropriately selecting a transmission strategy given a channel selection strategy. Since we consider only stable policies, the sender transmits at the rate λ. Now, the sender's throughput is the product of λ and the probability of successful transmission of a packet. The latter is low if the sender transmits irrespective of the probability of successful transmission in the selected channel. On the other hand, the system becomes unstable if the sender transmits only when the selected channel has a very high probability of successful transmission. Thus, intuitively the optimal transmission strategy is threshold-type, i.e., the sender should transmit only when the selected channel has a certain minimum probability of successful transmission. We now define a class of transmission policies known as two-threshold policies.

[2]Note that in any slot the sender can not transmit a packet in a channel whose probability of successful transmission is 0 in the slot

[3]For simplicity of exposition, we consider maximization of throughput equivalent to ϵ-throughput optimality

[4]We could prove that the stability region is a superset of $\{\lambda : \lambda < 1\}$ and subset of $\{\lambda : \lambda \leq 1\}$. For simplicity we consider the stability region as $\{\lambda : \lambda < 1\}$.

[5]If $\alpha_i(u) \neq \alpha_j(u)$ for two channels i, j augment the state space of each channel by introducing states v, w such that $\alpha_i(v) = \alpha_j(u)$ and $\alpha_j(w) = \alpha_i(u)$, and renumber the states so as to make the probabilities of success increase with increase in state. The state space of the joint chain is the cross product of the individual state spaces. The transition probability into the additional states in the joint chain is 0. The transition probabilities among the other states remain the same as before. For the states in the original markov chain, the new markov chain has the same stationary distribution. Thus, the two chains are equivalent for our purpose.

Definition 10: A *two-threshold* (T, q) transmission policy selects threshold T at the beginning of an interval with probability q and Threshold $T+1$ with probability $1-q$. If the selected threshold is u, then during the interval the sender transmits a packet in a slot if and only if it has a packet to transmit and the selected channel is in a state u or higher.

We prove that given any channel selection strategy the transmission strategy that maximizes the throughput among all transmission strategies that stabilize the system is a two-threshold (T, q) transmission policy. We therefore need to consider only two-threshold transmission strategies for maximizing the throughput. We now describe how to compute T, q given the channel selection strategy.

Definition 11: The *average threshold* of a two-threshold(T, q) transmission policy is $T + (1-q)$.

Note that the probability of successful transmission of each packet increases as the average threshold increases. The throughput increases if the average threshold is increased while maintaining system stability.

Definition 12: Let z_u be the probability that the selected channel is in state u.
$\{z_u\}$ is determined by the channel selection strategy.

Under a two-threshold (T, q) transmission policy and a channel selection strategy $\{z_u\}$, the selected channel is in a state which is either greater than or equal to the threshold in $f(q, T, \{z_u\}) = qz_T + \sum_{u=T+1}^{G} z_u$ fraction of slots. Since the system is stable and the sender can transmit only in $f(q, T, \{z_u\})$ fraction of slots, $f(q, T, \{z_u\}) > \lambda$. Now, if the average threshold increases then either q decreases or T increases and $f(q, T, \{z_u\})$ decreases. Since the throughput of a two-threshold (T, q) transmission policy and a channel selection strategy $\{z_u\}$ increases if the average threshold is increased while maintaining stability, and $f(q, T, \{z_u\})$ decreases with increase in the average threshold, the optimal parameters T, q are such that $f(q, T, \{z_u\}) \in (\lambda, \lambda + \epsilon]$ for some suitably small $\epsilon > 0$.

Summarizing, the optimal channel selection and the transmission strategy must then be such that (i) the transmission strategy is a two-threshold(T, q) policy and (ii) $f(q, T, \{z_u\}) \in (\lambda, \lambda + \epsilon]$ for some suitably small $\epsilon > 0$. We can compute the channel selection strategy and T, q in the following two steps.

1) For each T, we compute the q and the channel selection strategy that maximizes the throughput subject to satisfying $f(q, T, \{z_u\}) \in (\lambda, \lambda + \epsilon]$ for some suitably small $\epsilon > 0$.
2) We then select the T that attains the maximum throughput in the previous step.

The resulting T, q and the channel selection strategy clearly maximizes throughput and also attains the system stability region as it stabilizes the system for each $\lambda < 1$ since $f(q, T, \{z_u\}) > \lambda$.

We now describe the computation in step (1). Since the arrival and the channel states are markovian, the channel and the threshold-parameter q must be selected as a function of the channel states at the beginning of an interval [3]. In other words, these selections do not depend on the previous states and decisions. The channel selection is randomized, i.e., given that $\vec{u}$ describes the channel states at the beginning of an interval, channel i is selected with probability $c_i(\vec{u})$, where $\sum_i c_i(\vec{u}) = 1$, for each $\vec{u}$. Furthermore, the selected q is used to determine whether the threshold for the interval will be T or $T+1$. Equivalently, channel i and threshold T $(T+1)$ are selected at the beginning of each interval with probability $x_{i,T}(\vec{u})$ $(x_{i,T+1}(\vec{u}))$, where

$$\sum_i x_{i,T}(\vec{u}) + \sum_i x_{i,T+1}(\vec{u}) = 1, \ \forall \ \vec{u}. \qquad (2)$$

Definition 13: Let $y_{j,k}$ be the probability that the selected channel is in state j and the threshold is k.
Now, $y_{j,k}$ is related to z_j as follows: $y_{j,T} = qz_j$, $y_{j,T+1} = (1-q)z_j$ and $y_{j,k} = 0$ if $k \notin \{T, T+1\}$. Here,

$$y_{j,k} = \sum_{\vec{u} \in S} \pi(\vec{u}) \sum_i x_{i,k}(\vec{u}) b_{\vec{u},i,j}, \ \forall \ j \in \{1, \dots G\}, \quad k = T, T+1. \qquad (3)$$

(Note that $b_{\vec{u},i,j}$ has been defined in (1).) Also, $y_{T,T} + \sum_{j=T+1}^{G} (y_{j,T} + y_{j,T+1})$ is the fraction of slots in which the selected channel is in a state that is greater than or equal to the selected threshold. The sender transmits in these slots if it has packets. This fraction is the same as $f(q, T, \{z_u\})$. Thus,

$$y_{T,T} + \sum_{j=T+1}^{G} (y_{j,T} + y_{j,T+1}) \in (\lambda, \lambda + \epsilon]. \qquad (4)$$

If a packet is transmitted in a slot in which the selected channel is in state u, the packet is successfully transmitted with probability $\alpha(u)$. Thus, if the sender always has packets to transmit, it has a throughput of $\alpha(T)y_{T,T} + \sum_{j \geq T+1} \alpha(j)(y_{j,T} + y_{j,T+1})$. From (4), with a high probability the sender has a packet to send in at least $1-\epsilon$ fraction of slots, where ϵ can be selected as an arbitrarily small positive number. Thus, the throughput is approximately $\alpha(T)y_{T,T} + \sum_{j=T+1}^{G} \alpha(j)(y_{j,T} + y_{j,T+1})$. It follows that the maximum throughput in step (1) can be obtained by maximizing $\alpha(T)y_{T,T} + \sum_{j=T+1}^{G} \alpha(j)(y_{j,T} + y_{j,T+1})$ subject to satisfying constraints (2), (3), (4), which can be attained by solving the following linear program (**LP**).

Maximize: $\alpha(T)y_{T,T} + \sum_{j=T+1}^{G} \alpha(j)(y_{j,T} + y_{j,T+1})$
Subject To:

$$y_{j,k} = \sum_{\vec{u} \in S} \pi(\vec{u}) \sum_i x_{i,k}(\vec{u}) b_{\vec{u},i,j}, \ \forall \ j \in \{1, \dots G\}, \quad k = T, T+1,$$

$$\sum_i x_{i,T}(\vec{u}) + \sum_i x_{i,T+1}(\vec{u}) = 1, \ \forall \ \vec{u} \in S,$$

$$y_{T,T} + \sum_{j=T+1}^{G} (y_{j,T} + y_{j,T+1}) = \lambda + \epsilon. \qquad (5)$$

Definition 14: Let $\Psi(T)$ denote the optimum value of the objective function of **LP**, if **LP** has a feasible solution. Let $\Psi(T) = 0$, if **LP** does not have a feasible solution. Let $x^*(i, T), x^*_{i,T+1}(\vec{u})$ denote the optimum solution of **LP**, if **LP** has a feasible solution.
Let $T^* = \max_{1 \leq T \leq G} \Psi(T)$. Note that the LP is feasible at T^* and $x^*(i, T^*+1)(\vec{u})$ exists.

We describe the Optimum Channel selection and Transmission strategy ("OCT") in Figure 2. The intuition used in designing "OCT" motivates the following optimality result.

Theorem 1: For any $\lambda < 1$ and $0 < \epsilon < 1-\lambda$, the channel selection and transmission strategy ("OCT") described in Figure 2 is $\epsilon-$throughput optimal. Also OCT attains the stability region of the system.

The Throughput Maximizing Strategy OCT
begin

1)Let $\epsilon > 0$ be a parameter.

2) Let the channel state vector at the beginning of an interval be $\vec{u}$. The sender selects channel i and threshold T^* with probability $x^*(i, T^*)(\vec{u})$, and channel i and threshold $T^* + 1$ with probability $x^*(i, T^* + 1)(\vec{u})$.

3) The sender transmits packets in any slot in which it has a packet to transmit and the selected channel is in a state that is greater than or equal to the selected threshold.

end

Fig. 2. Algorithm for the optimum channel selection and transmission strategy

Refer to [5] for a formal proof.

We now relax the assumption that $\alpha(u) > 0$ for all u. Consider a linear program **LP1** which can be obtained by a modification of **LP**. The modification is that the equality in constraint (5) in **LP** is replaced by a " $\leq$ " in **LP1**. Now, consider a Generalized channel selection and transmission strategy ("GOCT") which can be obtained by substituting **LP** by **LP1** in Figure 2.

Theorem 2: For any $\lambda < 1$ and $0 < \epsilon < 1 - \lambda$, GOCT is $\epsilon-$throughput optimal.

As Example 2 demonstrated, when $\alpha(u) = 0$ for some u, all stable policies may attain significantly lower throughput than the maximum possible throughput. Thus, for all sufficiently small ϵ, no $\epsilon-$throughput optimal strategy (e.g., GOCT) may attain the stability region of the system.

V. Discussion and Conclusion

We develop a framework for optimally selecting channels and deciding transmission times in multi-channel wireless systems with imperfect information. Several interesting medium access control protocols [7], [9] have been proposed for selecting channels in context of specific wireless technologies, e.g., IEEE 802.11, which do not however guarantee any performance bound. Our approach is complementary as we identify strategies for optimal spectrum utilization and characterize the optimal performance as a function of system parameters. We now discuss some salient features of the computational framework.

First, the framework computes the optimal strategy for any number of jointly markovian channels with arbitrary number of states and arbitrary correlations, and any value of L. This is clearly advantageous as different channels may have different correlations (e.g., transmission conditions using antennas in the same mobile unit may be strongly correlated but transmission conditions in orthogonal frequencies may be independent), and different number of states depending on the fading and interference conditions. Also, different switching delays will require different values of L in different systems. Furthermore, the number of variables and constraints in the linear programs are polynomial ($O(G^n)$) in the number of states G. Thus, the complexity of computing the optimal strategy is also polynomial in G. Thus, the model can easily accommodate large G which is necessary for slow fading channels [13]. The number of variables and constraints in the linear program is however exponential in the number of channels n. This does not however significantly increase the computation complexity as the number of channels is usually few, e.g., IEEE 802.11b has 3 frequencies, a sender usually has no more than 2 antennas, etc. Furthermore, our linear programs can be solved in a short time using computionally efficient softwares even when the number of variables and constraints is large (e.g., using CPLEX software on a 2-GHz processor 1GB RAM, we computed the optimal strategy for $G = 8$, $n = 10$ in 3 seconds). Nevertheless, determining polynomial complexity computable suboptimal strategies with guaranteeable approximation bounds constitutes an intellectually challenging area of future research.

The policy needs to be computed offline and subsequently executed using a lookup table. Since the number of states of the channels is $O(G^n)$, the storage is again polynomial in G and exponential in n. Again, the storage is not significant as n is usually small. The execution complexity is however $O(1)$ (if arrays are used to store the decisions). The computation of the policy requires knowledge of channel statistics since prediction of the future states is required. The statistics can be obtained by estimation techniques.

When the throughput is Z, the system incurs a loss rate of $\lambda - Z$. Thus, a policy that maximizes throughput also minimizes the loss. Thus, the $\epsilon-$throughput optimal policies presented in this paper are also $\epsilon-$loss optimal. Now, the lost packets can be recovered through retransmissions at different layers or by forward error coding (FEC) provided the loss rate is below a certain acceptable value that depends on the retransmission protocol and the code rate. Thus, minimizing the loss rate certainly augments the efficacy of these recovery schemes. Detailed investigation of loss recovery schemes is beyond the scope of this paper. An interesting topic for future research will be to jointly maximize the throughput and specific loss recovery schemes.

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