Optimizing transmission rate in wireless channels using adaptive probes

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Abstract— **We consider a wireless system with multiple channels where each channel is either on or off, and probing the state of any channel incurs a cost. We present a polynomial time algorithm that determines which channels to probe and also which channel to transmit so as to maximize the difference between the rate of successful transmissions and the cost incurred in probing.**

I. INTRODUCTION

In future wireless networks each node is expected to have access to a large number of channels. The challenge in exploiting multiple channels is that a node will likely have only limited information about the instantaneous transmission quality of the individual channels. A node can learn the instantaneous transmission quality of a channel by transmitting a probe packet in the channel and subsequently measuring the signal to noise ratio. The probing process is however associated with a cost as it consumes additional energy and prevents neighboring users from simultaneously utilizing the channel. In this paper we initiate a study where a node seeks to maximize a utility function that depends on both the rate of successful transmission and the cost accrued in probing the channels, using the available channel statistics. Towards this end, a node needs to determine a jointly optimal (a) *probing policy* which specifies which channels to probe as well as the probing sequence, and (b) *channel selection policy*, which decides which channel to transmit based on the outcomes of the probes.

Problem Definition: We consider a sender U which has access to n channels which are denoted as channels $1, 2, \ldots, n$. Each channel has 2 possible states, $0, 1$. We assume that time is slotted. We assume that a node transmits one packet in each time slot and transmits the packet in only one channel which it need not have probed in the slot. In any slot channel j is in state i with probability p_{ij} independent of its state in other slots and the states of other channels in any slot. The transmission in a slot is successful if and only if the selected channel is in state 1. Whenever U probes a channel i, it learns its state and pays a cost of $c_i \geq 0$. Given $\{c_i\}$ and $\{p_{ij}\}$ we seek a probing and selection policy that maximizes the difference between the expected number of successful transmissions and the expected probing cost before each transmission. Loosely, this utility function represents the "gain" or the "profit" of the sender if the sender receives credit from the receiver for each packet it delivers successfully and needs to additionally compensate the wireless provider for each probe packet it transmits.

Related Work: Recently, Sabharwal *et al* [1], Kanodia *et al* [2] and Ji *et al* [3] have considered variants of the above problem in systems where channels are statistically identical but may have arbitrary number of states. In a companion paper [4], we extend our results to multiple channels with potentially different statistical characteristics and arbitrary number of states, and also review the related literature in more detail.

II. OPTIMAL PROBING AND CHANNEL SELECTION POLICY

We first consider a specific class of probing and selection policies and prove that the optimal policy belongs in this class. Subsequently we show how to find the optimal policy in this class in polynomial time.

Definition 2.1: Given $S \subset \{1, \ldots, n\}, i \notin S$, let $\text{EXHAUST}(S, i)$ denote the class of policies which probe all channels in S in a deterministic order until a probed channel is in state 1 or all channels in S have been probed. It selects the last probed channel if it is in state 1, and selects i otherwise. Channel i is denoted as the *backup* channel.

Lemma 2.1: There exists an $\text{EXHAUST}(S, i)$ policy which is optimal.

Proof: We prove the lemma by induction on the number of channels n. For the base case, $n = 1$, the expected gain is $p_{11} - c_1$ if the optimal policy probes the channel, and p_{11} otherwise. Since $c_1 \geq 0$, the policy that selects the channel without probing is optimal over all possible convex combinations, i.e., randomizations, of the above two policies. Thus, $\text{EXHAUST}(\Phi, 1)$ is an optimal policy in this case.

Assuming the lemma holds for $n = s$, consider a set J of $n = s + 1$ channels. The optimal policy can randomize over the two possibilities: (a) selects a channel without probing or (b) probes a channel. Conditioned on (a), the policy can randomize over the channels which is a convex combination of $\text{EXHAUST}(\Phi, j)$ policies. Now, conditioned on (b), the optimum policy chooses to probe a channel i with some probability. Subsequent to the probe, if i is in state 1, the optimal transmission policy selects i and no further probing will occur. If i is in state 0 then the optimal policy takes the same decisions as that in a system with the s remaining channels and by the induction hypothesis, uses $\text{EXHAUST}(Q, j)$

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policy for some $Q \subset J - \{i\}, j \in (J - Q) - \{i\}$. Thus, the optimal policy in this case is an EXHAUST$(\{i\} \circ Q, j)$ where the $\circ$ denotes the ordering. Therefore conditioned on (b), the optimum policy is a convex combination of EXHAUST policies as well. Therefore the overall policy is a convex combination of EXHAUST policies. Thus, there exists an optimum policy which is EXHAUST(S, i). ■

We next prove that the optimal policy satisfies additional properties, which allows a fast computation of the policy.

Lemma 2.2: Let $S_i = \{j : (1 - p_{1i})p_{1j} > c_j\}$. If EXHAUST$(S, i)$ is an optimum policy, then

1) channels j in S are probed in decreasing order of p_{1j}/c_j, and
2) EXHAUST(S_i, i) policy is an optimum policy.

Proof: Let EXHAUST(S, i) policy be the optimum policy. Wlog. $S = \{k_1, \dots k_{|S|}\}$, where channel k_l is probed before k_{l+1}. Then the expected gain of EXHAUST(S, i) policy is

$$A = \sum_{l=1}^{|S|} (p_{1k_l} - c_{k_l}) \prod_{m=1}^{l-1} (1 - p_{1k_m}) + p_{1i} \prod_{m=1}^{|S|} (1 - p_{1k_m}).$$

We first prove (1). If $p_{1j} = 1$ for some channel j, then the policy that does not probe any channel and always selects channel j maximizes the expected gain. Thus, henceforth, we assume that $p_{1j} < 1$ for all channels j. Let $p_{1k_s}/c_{k_s} < p_{1k_{s+1}}/c_{k_{s+1}}$. Consider a new policy which probes k_{s+1} before k_s but is other-wise similar to the EXHAUST(S, i) policy. Let the expected gain of this new policy be B. Then, $A - B = \prod_{m=1}^{s-1}(1 - p_{1,k_m})(p_{1,s}c_{s+1} - p_{1,s+1}c_s)$. Since $p_{1j} < 1$ for all j, and $p_{1s}/c_s < p_{1,s+1}/c_{s+1}$, $B > A$. Thus, EXHAUST(S, i) is not the optimum policy. The result follows.

We now prove (2). If $S = S_i$ the result follows. Let $S \neq S_i$. Thus, either $S_i \setminus S \neq \phi$ or $S \setminus S_i \neq \phi$.

Let $S \setminus S_i \neq \phi$. Consider some $j \in S \setminus S_i$. From (1), $p_{1k_s}/c_{k_s} \geq p_{1k_{s+1}}/c_{k_{s+1}}$. Thus, $(1 - p_{1i})p_{1k_{|S|}}/c_{k_{|S|}} = \min_{1 \leq l \leq |S|}(1 - p_{1i})p_l/c_l \leq (1 - p_{1i})p_{1j}/c_j \leq 1$. Thus, $k_{|S|} \in S \setminus S_i$. Let $Q = S \setminus \{k_{|S|}\}$. The expected gain of EXHAUST(Q, i) policy with probing sequence $k_1, \dots, k_{|S|-1}$ is $D = \sum_{l=1}^{|S|-1}(p_{1k_l} - c_{k_l})\Pi_{m=1}^{l-1}(1 - p_{1k_m}) + p_{1i}\Pi_{m=1}^{|S|-1}(1 - p_{1k_m})$. Now, $D - A = (c_{k_{|S|}} - (1 - p_{1i})p_{1k_{|S|}})\Pi_{m=1}^{|S|-1}(1 - p_{1k_m})$. Since $(1 - p_{1i})p_{1k_{|S|}} \leq c_{k_{|S|}}$, $D \geq A$. Thus, EXHAUST(Q, i) is an optimum policy, where $Q \subseteq S$ and $|Q \setminus S_i| < |S \setminus S_i|$. Continuing this argument, clearly there exists a T such that $T \subseteq S$ and $T \setminus S_i = \phi$ and EXHAUST(T, i) policy is optimal.

Now let $S_i \setminus S \neq \phi$. If $S \setminus S_i \neq \phi$, let T be as constructed in the above paragraph; otherwise let $T = S$. In both cases, EXHAUST(T, i) policy is optimal. We now show that $S_i \setminus T = \phi$. If not, consider a $j \in S_i \setminus T$. Let $Q = T \cup \{j\}$. The expected gain of EXHAUST(T, i) policy with probing sequence $k_1, \dots k_{|T|}, k_j$ is $C = \sum_{l=1}^{|T|}(p_{1k_l} - c_{k_l})\Pi_{m=1}^{l-1}(1 - p_{1k_m}) + (p_{1j} - c_j)\Pi_{l=1}^{|T|}(1 - p_{1k_l}) + p_{1i}(1 - p_{1j})\Pi_{l=1}^{|T|}(1 - p_{1k_l})$. Now, $C - A = ((1 - p_{1i})p_{1j} - c_j)\,\Pi_{l=1}^{|T|}(1 - p_{1k_l})$. Since $p_{1s} < 1$ for all s and $(1 - p_{1i})p_{1j} > c_j$, $C > A$. This contradicts the optimality of the EXHAUST(T, i). Thus, $S_i \setminus T = \phi$. Thus, $S_i = T$. Hence, EXHAUST(S_i, i) policy is optimal.

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Procedure Determining Best Backup Channel
begin
  Wlog. let channels be numbered in decreasing order of p_{1i}/c_i, i.e., if
  i < j, p_{1i}/c_i > p_{1j}/c_j.
  S_i = {j : (1 - p_{1i}) p_{1j} > c_j, j ≠ i}.
  Compute F_j = \sum_{l=1}^{j-1} (p_{1l} - c_l) \prod_{m=1}^{l-1} (1 - p_{1m}) for each j
    /* Gain of the first j - 1 channels if probed. */
  Compute D_j = \Pi_{l=1}^{j-1} (1 - p_{1l}) for each j
    /* Probability that first j - 1 channels return state "0" */
  For each channel i,
  if i ≤ |S_i| then
    Gain(i) = F_i + (F_{|S_i|+2} - F_i - (p_{1i} - c_i) D_i)/(1 - p_{1i}) + p_{1i}/(1 - p_{1i}) D_{|S_i|+2};
        /* Omit channel i from ordering */
  else
    Gain(i) = F_{|S_i|+1} + p_{1i} D_{|S_i|+1}.
  Let Best-Bkup = arg max_{i=1,...,n} Gain(i).
end

Procedure Optimum Probing and Transmission
begin
  Probe channels j ∈ S_{Best-Bkup} until a probed channel is in state 1 or
  all channels in S_{Best-Bkup} have been probed.
  if the last probed channel j is in state 1, then
    transmit the packet in j;
  else
    transmit the packet in Best-Bkup.
end
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Fig. 1. Pseudo Code for the Optimum Probing and Selection Strategy

■

Lemma 2.2 together with Lemma 2.1 proves that there exists an EXHAUST(S_i, i) policy that is optimal, and this policy probes the channels in S_i in decreasing order of p_{1j}/c_j. The procedure "Determining Best Backup Channel" in Figure 1 determines the policy that has the maximum expected gain among all such policies, and the procedure "Optimum Probing and Selection" executes this policy.

The computation is optimized for efficiency as follows. The set of channels in S_i is a prefix of the channels sorted in decreasing order of $\frac{p_{1j}}{c_j}$, omitting the channel i. We compute the gain of probing channels in each prefix in the sorted order including channel i; channel i's contribution to this gain can be omitted in constant time. The overall running time is dominated by the sorting of the channels, which takes $O(n \log n)$ time. Therefore, we have the following Theorem.

Theorem 2.3: The policy proposed in Figure 1 maximizes the expected gain, and can be executed in $O(n \log n)$ time.

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