Stochastic Control Techniques for Throughput Optimal Wireless Multicast

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Abstract

We design transmission strategies for MAC layer multicast which maximize the utilization of available bandwidth. Bandwidth efficiency of wireless multicast can be improved substantially by exploiting the fact that several receivers can be reached at the MAC layer by a single transmission. However, this feature can be effectively exploited only by resolving several decision problems. The fundamental relations between QoS parameters such as throughput, stability and loss change on account of the multicast nature of transmissions, e.g., a strategy that maximizes the system throughput does not necessarily maximize the stability region or minimize packet loss. We explore the trade-off between the QoS parameters, and provide optimal transmission strategies which maximize the system throughput subject to stability and loss constraints.

1 Introduction

Many of the current day wireless applications need one to many (*multicast*) communication, e.g., conference meetings, sensor networks, rescue and disaster recovery and military operation. Most of the research in wireless multicast has been directed towards the development of end-to-end error recovery and routing protocols [1, 2, 4, 5, 11, 12]. Though the overall network performance depends on the efficiency of the underlying scheduling strategy used at the medium access control (MAC) layer, MAC layer multicast has not been adequately explored. Our research is directed towards filling this void.

Wireless communication is inherently broadcast in nature, i.e., all the nodes in the transmission range of a sender can receive a transmission from the sender (e.g., Figure 1). Hence, it suffices to transmit each packet once in order to reach all the intended receivers, which may provide significant bandwidth and power advantage. Multicast in the wireless case can be more effi-
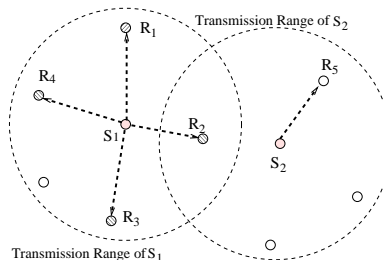
Figure 1: An example to demonstrate the advantages and the challenges associated with wireless multicast. The figure shows two senders S_1, S_2 and 5 receivers R_1 to R_5. Dashed circle indicates the communication range of a sender. A single transmission from S_1 can reach all its receivers, $R_1, \ldots, R_4$.

cient than in the wireline counterpart as in the latter a packet has to be transmitted from a sender to each of the receivers separately.

Though the broadcast nature of wireless transmissions provides a possible approach to improve the efficiency of the multicast communication, it also imposes various difficulties. A multicast specific challenge is that some but *not all* the receivers may be ready to receive. For example, in Figure 1, when sender S_2 is transmitting to receiver R_5, receiver R_2 can not receive the transmission from sender S_1 as both the transmissions will collide at R_2. However, receivers R_1, R_3 and R_4 can still receive the transmission. The readiness state of a receiver depends on the network load and the transmission quality of the channel. The policy decision is whether the sender S_1 should transmit or it should wait till all the receivers are ready. A policy of not to transmit until a sufficient number of receivers are ready may render the system unstable. On the other hand, if the sender transmits irrespective of the readyness states of the receivers, then the transmitted packet may be lost at several receivers that were not ready. The resulting packet loss at the receivers may be unacceptably high. The throughput may be low at both extremes and maximum somewhere in between. This is because the transmission rate is low at the first and packets do not reach most receivers in the last case. Thus, there is a multicast specific trade-off between throughput, stability and packet loss.

In section 2, we define our system model. In section 3 we show that the fundamental relations between QoS parameters such as throughput, loss and stability change on account of the multicast nature of transmissions, e.g., a strategy that maximizes the system throughput does not necessarily maximize the stability region or minimize packet loss. In section 4, we propose a policy that decides when a sender should transmit a packet so as to maximize the throughput subject to (a) system stability and (b) packet loss constraints at the receivers. The optimal strategy is to transmit only when the number of ready receivers is above a certain threshold. This threshold based policy is simple to implement once the optimal threshold is known, as the sender need not know the individual readiness states of the receivers. In section 4, we show how to compute the optimal threshold, which depends on the statistics of the arrival and the receiver readiness process. Next, we present an adaptive approach that computes the threshold based on the estimates of the statistics obtained from system observations. Refer to technical report [3] for proofs.

The threshold based scheme is a generalization of a protocol proposed by Tang *et al.*, where a sender transmits whenever at least one receiver is ready [7, 8]. We refer to this approach as *Threshold*-1 strategy. Other existing multiple access strategies for wireless multicast are *Threshold*-0, which is used in IEEE 802.11, and *unicast based multicast* [9]. The former transmits a packet irrespective of the existing transmissions and the readiness states of the receivers. This causes packet loss at the receivers because of collision due to second hop interference. The latter attains multicast by transmitting a packet to each receiver separately in round robin fashion [9], and thus does not exploit the broadcast nature of wireless medium. In technical report [3], we analyze the existing approaches and show using numerical performance evaluation that the proposed optimal policy provides significantly more efficient usage of bandwidth.

2 System Model

The objective is to study a wireless network with several MAC layer multicast sessions. Each multicast session comprises of a sender and a set of receivers (multicast group). At the MAC layer, all the receivers are within the transmission range of the sender. Refer to Figure 2 for an example scenario.

We consider a single multicast session in isolation with G receivers (refer to Figure 3). The impact of the network and the channel errors on the multicast session is that the receivers are not always ready to receive. This may happen because of a transmission in the neighbor-
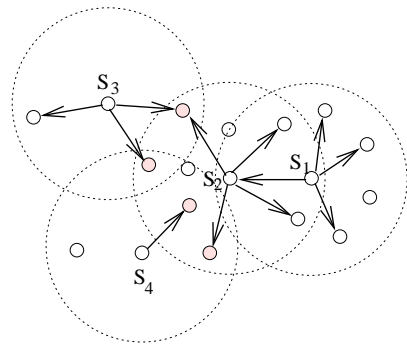

Figure 2: The figure shows four MAC layer multicast sessions. Nodes S_1 to S_4 are the senders for multicast groups 1 to 4 respectively. Arrows from a sender point to its designated receivers. Note that the node S_2 is a sender for multicast group 2, while it is a receiver for multicast group 1.
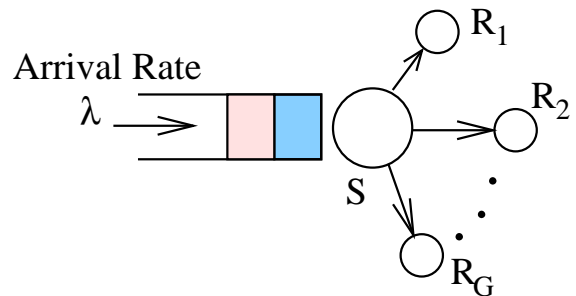

Figure 3: Figure shows an isolated MAC layer multicast session. The packets arrive at the rate λ at sender S. The receivers are R_1 to R_G.

hood of a receiver, bursty channel error, or on account of the power saving mode. Thus, the receiver readiness states are correlated in the same time slot, and across the time slots. We model the readiness process of all the receivers as a markov chain (MC) with an arbitrary Transition Probability Matrix (TPM) $\widehat{P}$. A state of the MC is a G dimensional vector $\vec{j} = [j_1 \ j_2 \ \dots \ j_G]$, where the component j_l is 1 if the lth receiver is ready and it is 0 otherwise. Let $\mathcal{S}$ be the state space of the receiver readiness process. We assume that the $2^G \times 2^G$ TPM $\widehat{P}$ is irreducible, aperiodic and time-homogeneous. Thus, $\widehat{P}$ has a unique stationary distribution $\vec{B} = \{B_{\vec{j}} : \vec{j} \in \mathcal{S}\}$, which depends on the network load, channel characteristics and the power saving scheme. Let b_u be the steady state probability that u receivers are ready to receive, $b_u = \sum_{\vec{j}:\sum_{l=1}^{G} j_l = u} B_{\vec{j}}$, $b_u > 0$, for each u. We refer to the probability distribution $\vec{b} = [b_0 \ b_1 \ \dots \ b_G]$ as the aggregate stationary distribution of the receiver readiness process.

A sender queries the readiness state of receivers by control packet exchange. A sender decides whether or not to transmit a packet depending on the transmission strategy, availability of packet and the result of

the query. Every receiver maintains its readiness state throughout the transmission. This assumption is justified because the time scale of a change of transmission quality is large as compared to packet sizes. Also, the level of interference does not change during a packet transmission. This is because in several MAC protocols (e.g., IEEE 802.11), the exchange of control messages prevents a new transmission during an ongoing transmission, in the reception range of the receiver. The sender backs off for a random duration before querying the system again, irrespective of the transmission decision, so as to allow other senders to use the shared medium. The structure of the multiple access protocol described above is similar to IEEE 802.11.

We assume a slotted time axis. The number of packets arriving in a slot constitute an irreducible, aperiodic markov chain with a finite number of states. The expected number of arrivals in a slot under the stationary distribution is denoted as λ. The packet lengths and back-off durations are independent and identically distributed (iid) random variables with arbitrary probability distributions and means $\mathbf{E}[V]$ and $\mathbf{E}[X]$ respectively. Let $\mathbf{E}[A]$ and $\mathbf{E}[\bar{A}]$ be the expected number of arrivals in the duration of back-off and a packet transmission respectively, under the stationary distribution for the arrival process. Then, $\mathbf{E}[A] = \lambda\mathbf{E}[X]$ and $\mathbf{E}[\bar{A}] = \lambda\mathbf{E}[V]$.

We consider data traffic and assume FIFO selection of packets for transmission. We consider three Quality of Service (QoS) measures: (a) *throughput* (b) *packet loss* (c) *system stability*.

Definition 1 *A reward attained by the system for a transmission is the number of receivers that receive the packet successfully.*

Definition 2 *Throughput is the expected reward attained by the system per unit time.*

Definition 3 *The packet loss at a receiver is the fraction of transmitted packets that are either not received or received in error at the receiver. A packet loss constraint specifies an upper bound (L) on the sum of the packet losses at all the receivers in the multicast group.*

Definition 4 *The sample points are the epochs at which the sender samples (queries) the receiver readiness state.*

Definition 5 *A transmission policy is an algorithm that decides whether or not to transmit a packet at a sample.*

Definition 6 *A system is said to be stable if the mean queue length at the sender is bounded. A stable transmission policy is one which stabilizes the system.*

Definition 7 *The stability region of a transmission policy is the maximum value of λ for which it stabilizes the system. The stability region of the system is the maximum value of λ for which some transmission policy stabilizes the system.*

The class of policies include offline strategies that assume prior knowledge of packet arrivals in all (including future) slots and the readiness states at all (including future) samples. The offline strategies know the readiness states in all slots apriori in the special case that the sender samples the system every slot, i.e., when every packet has length 1 slot and there is no backoff. We present online strategies which maximize the throughput subject to stability and packet loss in this large class, despite taking the transmission decision based on the current packet availability and the number of ready receivers at the current sample.

A small loss tolerance increases the throughput and the stability region of the system significantly in wireless multicast [3]. The lost information can be recovered by using coding redundancy, or a higher layer reliable protocol. We impose a constraint on the sum of the packet loss at the receivers as individual receivers can often retrieve lost packets from other receivers who have received the packet. A sender may achieve the required loss characteristics by transmitting a packet several times till a sufficient number of receivers receive the packet, e.g., in Figure 1 sender S_1 may transmit a packet to the receivers R_1, R_3 and R_4 even when the receiver R_2 is not ready and then retransmit the packet when receiver R_2 becomes ready. But each additional transmission causes additional power consumption. Therefore, we assume that a packet can be transmitted only once at the MAC layer.

3 Relation between throughput, stability and packet loss

We first investigate the relation between throughput and stability for multicast transmission. In the unicast case, a throughput optimal strategy is one that attains the stability region of the system (Definition 7) [6, 10]. Exclude policies that transmit even when no receiver is ready. Then, in Fig 3 if $G = 1$, a policy that transmits whenever the sender has a packet and the receiver is ready, maximizes the throughput and attains the stability region of the system. This relation between throughput optimality and system stability does not hold in the multicast case. Let $G = 2$

in Fig 3. A policy that transmits when at least one receiver is ready attains the stability region of the system. However, the policy that transmits only when both receivers are ready has a smaller stability region but can provide a higher throughput for appropriate choice of the system parameters. Assume that each receiver is ready with a probability of $p = 0.1$ in each slot independent of the other receiver and other slots. Let $EX = 1, EV = 1000, \lambda = 1/1050$. Then the throughputs of the two policies are 1.058×10^{-3} and 1.818×10^{-3} respectively. The first (second) policy renders the system stable (unstable). If the arrival rates are such that both the policies stabilize the system, then the first policy has a throughput of 1.11λ, and the second policy has a throughput of 2λ. Thus, in the multicast case a policy that maximizes the throughput need not attain the stability region of the system, and vice-versa. The equivalence between throughput maximization and attaining stability region in the unicast case is because a transmitted packet always fetches a reward of 1 unit. However, the reward obtained by a transmitted packet depends on the readiness states of the receivers and can be anywhere between 1 and G for multicast transmission. While transmitting packets at a rate equal to the arival rate is enough for guaranteeing stability in both cases, maximization of the multicast throughput depends also on the readiness states of the receivers during the transmissions.

We investigate the relation between throughput and packet loss now. First, consider a stable system. The throughput of a transmission policy is λR, where R is the average reward received by the policy per transmission. Further, the average loss experienced by all the receivers in the system is $G - R$. Thus a throughput optimal policy minimizes the average loss at the receivers for stable systems. However, this relation does not hold for unstable systems. An unstable system is saturated in the sense that the sender always has a packet to transmit. Let $G = 2$ in Fig 3. Now, let one receiver be ready with probability p in one slot, while the other is always ready. Let $EX = EV = 1$. Consider a policy that transmits only when both the receivers are ready, and another that transmits with probability q if only one receiver is ready and with probability 1 if both the receivers are ready. Let $\lambda > p + q(1 - p)$. The transmission rates are p and $p + q(1 - p)$ respectively. Thus, neither policy is stable. The throughputs are $2p$ and $2p + q(1-p)$ respectively. The losses are 0 and $\frac{q(1-p)}{p+q(1-p)}$ respectively. Thus, for $p, q \in (0, 1)$, the second policy has both higher throughput and higher loss, and hence the maximization of throughput is not equivalent to the minimization of packet loss.

We now discuss whether the saturated region is interesting from practical considerations. From the discussion above, if the total loss for the policy that maxi-

mizes the throughput subject to stability is more than the required loss constraint, then the loss constraint can not be guaranteed by any stable policy. It is always possible to satisfy the loss constraint if the stability requirement is relaxed. For example, a policy that sets threshold G w.p. 1 has zero packet loss, but can render the system unstable. Thus there is a tradeoff between packet loss and stability. If meeting the loss constraint is more important, then we need to consider transmission policies that achieve the required loss characteristics even at the cost of stability.

4 Throughput Optimal Transmission Policy

In subsection 4.1, we obtain a transmission policy that maximizes the throughput subject to attaining system stability. Next, in subsection 4.2, we obtain a transmission policy that maximizes the throughput subject to satisfying the packet loss constraint. In each subsection, we provide algorithms that decide the parameters of the optimum strategies without using any information about the system statistics. We first present some definitions.

Definition 8 *The busy samples are the sample points at which the sender's queue is non-empty.*

Definition 9 *A single-threshold transmission policy(T) is a policy that transmits a packet at every busy sample with T or more ready receivers. The parameter T is the threshold.*

Definition 10 *A two-threshold transmission policy(T,q) is a policy that sets threshold T for a given sample w.p. q or a threshold $T + 1$ w.p. $1 - q$ and transmits in accordance with the threshold.*

Definition 11 *A stable transmission policy Δ is called ϵ-throughput optimal if no other stable transmission policy can achieve throughput more than ϵ plus that achieved by Δ.*

4.1 Throughput Optimality subject to Stability
We first describe the stability region of the system. The service time of a packet is the difference between the times at which the packet finishes transmission and reaches the head of line position in the queue. The expected service time must be less than the expected packet inter-arrival time for the system to be stable. The sum of the transmission time plus one back-off duration is the lower bound on the service time of a packet for any transmission policy. Hence, for stability we need $\lambda(\mathbf{E}[X] + \mathbf{E}[V]) < 1$, i.e.,

$$\mathbf{E}[A] + \mathbf{E}[\bar{A}] < 1. \qquad (1)$$

We show that if (1) is satisfied, we can choose a threshold T and a probability q such that the corresponding two-threshold policy(T, q) is ϵ-throughput optimal.

Theorem 1 *Let the stability condition (1) hold. For every $\epsilon > 0$, there exists a choice of parameters T and q such that the corresponding two-threshold policy(T, q) is ϵ-throughput optimal with probability 1. The optimal values of the parameters T and q are*

$$T^* = \arg\max_{0 \leq i \leq G} \left\{ \frac{\mathbf{E}[A] + \widehat{\epsilon}\mathbf{E}[X]}{1 - \mathbf{E}[\bar{A}]} \leq \sum_{u=i}^{G} b_u \right\}, \quad (2)$$

$$q^* = \frac{1}{b_{T^*}} \left[\frac{\mathbf{E}[A] + \widehat{\epsilon}\mathbf{E}[X]}{1 - \mathbf{E}[\bar{A}]} - \sum_{u=T^*+1}^{G} b_u \right], \quad (3)$$

$$\text{where} \quad \widehat{\epsilon} = \min \left\{ \frac{\epsilon}{G}, \frac{1 - (\mathbf{E}[A] + \mathbf{E}[\bar{A}])}{\mathbf{E}[X]} \right\}. \quad (4)$$

The throughput of the two-threshold policy(T^*, q^*) (denoted as Δ^*) can be lower bounded as

$$\Omega^{\Delta^*} \geq \frac{(T^* q^* b_{T^*} + \sum_{u=T^*+1}^{G} u b_u)(1 - \mathbf{E}[\bar{A}])}{\mathbf{E}[X]} - \epsilon \ w.p. \ 1. \quad (5)$$

The computation of the optimal parameters provided in equations (2) and (3) of Theorem 1 depend on $\vec{b}, E[A], E[\bar{A}], E[X]$. Next, we design an adaptive approach, $\hat{\Delta}(t)$ which computes T^* and q^* accurately without prior knowledge of $\vec{b}$.

Let $n_r(t)$ be the number of samples with r ready receivers and $n(t)$ be the number of samples till time t. Let $\vec{b}(t) = [\frac{n_0(t)}{n(t)} \ \frac{n_1(t)}{n(t)} \ \ldots \ \frac{n_G(t)}{n(t)}]$. Now, estimates $\hat{T}(t)$ and $\hat{q}(t)$ for T^*, q^* are computed by substituting $\vec{b}(t)$ with its estimate $\vec{\hat{b}}(t)$ in equations (2) and (3).

Theorem 2 *Let there exist a T such that $\sum_{u=T+1}^{G} b_u < \frac{E[A]}{1-E[A]} < \sum_{u=T}^{G} b_u$.*
Let $0 < \epsilon < \frac{1-E[\bar{A}]}{E[X]} \left(\sum_{u=T}^{G} b_u - \frac{E[A]}{1-E[A]} \right)$.

$$\text{Then,} \ \lim_{t \to \infty} \hat{T}(t) = T^* \quad w.p. \ 1 \quad (6)$$

$$\lim_{t \to \infty} \hat{q}(t) = q^* \quad w.p. \ 1 \quad (7)$$

Since $\lambda > 0, EX > 0, EV > 0, 0 < \frac{E[A]}{1-E[A]} < 1$ from (1). Also, $b_u > 0$, for each u, $\sum_{u=T+1}^{G} b_u = 0$ for $T = G$ and $\sum_{u=T}^{G} b_u = 1$ for $T = 0$. Thus, there always exists a T such that $\sum_{u=T+1}^{G} b_u < \frac{E[A]}{1-E[A]} \leq \sum_{u=T}^{G} b_u$. We assume a strict inequality in the theorem.

The outputs $\hat{T}(t)$ and $\hat{q}(t)$ converge to T^*, q^* even when $E[X], E[A], E[\bar{A}]$ are substituted with their estimates in equations (2) and (3).

4.2 Throughput Optimality subject to Packet Loss Constraint

For stable systems throughput maximization is equivalent to loss minimization. Thus, we will assume a saturated system throughout this subsection. We show that for appropriate choice of parameters T and q, a two-threshold policy(T, q) Δ_S^* maximizes the throughput subject to any given loss constraint. First, we quantify the throughput for a two-threshold policy(T, q), $\Delta_{(T,q)}$.

Lemma 1 *For a saturated system, the throughput $(\Omega^{\Delta_{(T,q)}})$ and the mean reward achieved per transmission $(R^{\Delta_{(T,q)}})$ by a two threshold policy(T, q), $\Delta_{(T,q)}$ is as follows.*

$$\Omega^{\Delta_{(T,q)}} = \frac{qTb_T + \sum_{r=T+1}^{G} rb_r}{\mathbf{E}[X] + \mathbf{E}[V](qb_T + \sum_{u=T+1}^{G} b_u)} \quad w.p. \ 1$$

$$R^{\Delta_{(T,q)}} = \frac{qTb_T + \sum_{r=T+1}^{G} rb_r}{qb_T + \sum_{u=T+1}^{G} b_u} \quad w.p. \ 1.$$

We next show that a single-threshold policy maximizes the throughput in a saturated system.

Theorem 3 *A single-threshold policy(T_S) attains the maximum possible throughput in the saturated case, if $T_S = \arg\max_{0 \leq T \leq G} \left\{ \Omega^{\Delta_{(T,1)}} \right\}$.*

The optimum threshold T_S can now be computed from Lemma 1.

Under any policy Δ, the expected total loss at the receivers is $G - R^\Delta$ like in stable systems. The difference with stable systems is that the throughput is not a monotonic increasing function of the expected reward. This explains the observation that a throughput optimal transmission policy need not minimize the loss in the saturated case unlike that for stable systems. So the policy proposed in Theorem 3 may not satisfy the required loss constraint. We present a two-threshold transmission policy that maximizes the throughput subject to satisfying the loss constraint $L \geq 0$.

Theorem 4 *For a saturated system, the two-threshold policy(T_S^*, q_S^*) Δ_S^* (Fig. 4) maximizes the throughput subject to satisfying the loss constraint L. The throughput attained by Δ_S^* is*

$$\Omega^{\Delta_S^*} = \frac{q_S^* T_S^* b_{T_S^*} + \sum_{r=T_S^*+1}^{G} rb_r}{\mathbf{E}[X] + \mathbf{E}[V](q_S^* b_{T_S^*} + \sum_{u=T_S^*+1}^{G} b_u)} \quad w.p. \ 1.$$

The expressions for the throughput and reward per packet of a two-threshold policy obtained in Lemma 1 can be used in the computations in Figure 4.

A two-threshold(T_S^*, q_S^*) policy Δ_S^* for saturated systems
begin

  if ($R^{\Delta^{(0,1)}} \geq G - L$) **then**
    $T_S^* = \arg\max_{0 \leq T \leq G} \left\{ \Omega^{\Delta^{(T,1)}} \right\}$ and $q_S^* = 1$,
  else
    Find $T_M = \arg\max_{0 \leq u \leq G} \{ R^{\Delta^{(u,1)}} < G - L \}$
    Find $T_1 = \arg\max_{T_M+1 \leq u \leq G} \{ \Omega^{\Delta^{(u,1)}} \}$
    Let Δ_1 be a single-threshold policy with parameter T_1
    Let Δ_2 be a two-threshold policy with parameters (T_M, q_2),
    where $q_2 = \dfrac{\sum_{r=T_M+1}^{G} (r+L-G)b_r}{b_{T_M}(G-L-T_M)}$
    if ($\Omega^{\Delta_1} \geq \Omega^{\Delta_2}$) **then**
      $T_S^* = T_1$ and $q_S^* = 1$
    else
      $T_S^* = T_M$ and $q_S^* = q_2$

end

Figure 4: Pseudo code of an algorithm which maximizes the throughput subject to loss constraints for a saturated system

Adaptive policies can be designed for saturated systems like in section 4.1. Let $\hat{T}_S(t)$, $\hat{T}_S^*(t)$ and $\hat{q}_S^*(t)$ be the values of the parameters obtained in Theorem 3 and Figure 4, if $\vec{b}$ is replaced by its estimate $\hat{\vec{b}}(t)$. If $R^{\Delta^{(0,1)}} > G-L$, or there exists a T such that $R^{\Delta^{(T,1)}} < G - L < R^{\Delta^{(T+1,1)}}$, then w.p. 1, $\lim_{t\to\infty} \hat{T}_S(t) = T_S$, $\lim_{t\to\infty} \hat{T}_S^*(t) = T_S^*$, and $\lim_{t\to\infty} \hat{q}_S^*(t) = q_S^*$.

5 Conclusion

We investigate bandwidth efficient transmission strategies for wireless multicast networks. We establish that the relation between QoS parameters like throughput, loss and stability change on account of the multicast nature of transmissions. The maximization of the throughput is no longer equivalent to attaining the stability region of the system or minimization of packet loss. We show that threshold based transmission policies maximize the throughput subject to stability and packet loss constraints, and present an adaptive approach to compute the parameters of the optimum policies without any knowledge of system statistics. The sender only needs to know the number of ready reeceivers in each slot, and not the individual readiness states of the receivers, to implement the threshold based policies.

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