

$termvar, x, y, z$		
$tyvar, X, Y, Z$		
$index, i, j, n, m$		
$t, u$	$::=$	term:
	$x$	variable
	$\lambda x : S. t$	abstraction
	$t t'$	application
	$\lambda X : K. t$	type abstraction
	$t [ T ]$	type application
$v$	$::=$	value:
	$\lambda x : T. t$	abstraction value
	$\lambda X : K. t$	type abstraction value
$T, S, U$	$::=$	types:
	$X$	type variable
	$T T'$	operator application
	$S \rightarrow S'$	type of function
	$\forall X : K. S$	universal type
	$\lambda X : K. S$	type abstraction
$NF$	$::=$	(weak-head) normal form types
	$X$	
	$NE S$	
	$S \rightarrow T$	
	$\forall X : K. S$	
	$\lambda X : K. S$	
$NE$	$::=$	neutral types, paths
	$X$	
	$NE S$	
	$S \rightarrow T$	
	$\forall X : K. S$	
$\Gamma$	$::=$	contexts:
	$\emptyset$	empty context
	$\Gamma, x : S$	term variable binding
	$\Gamma, X : K$	type variable binding
$K$	$::=$	kinds:
	$*$	kind of proper types
	$K \Rightarrow K'$	kind of operators

$t \rightarrow t'$  Evaluation

$$\frac{t_1 \rightarrow t'_1}{t_1 t_2 \rightarrow t'_1 t_2} \quad \text{E\_APP1}$$

$$\frac{t_2 \rightarrow t'_2}{t_1 t_2 \rightarrow t_1 t'_2} \quad \text{E\_APP2}$$

$$\frac{}{(\lambda x : T_{11}. t_{12}) v_2 \rightarrow t_{12} \{ v_2 / x \}} \quad \text{E\_APPABS}$$

$$\frac{t_1 \rightarrow t'_1}{t_1 [T_2] \rightarrow t'_1 [T_2]} \quad \text{E\_TAPP}$$

$$\frac{}{(\lambda X : K_{11}. t_{12}) [T_2] \rightarrow t_{12} \{ T_2 / X \}} \quad \text{E\_TAPPTABS}$$

$\boxed{\Gamma \vdash T : K}$  Kinding

$$\frac{X :: K \in \Gamma}{\Gamma \vdash X : K} \quad \text{K\_TVAR}$$

$$\frac{\Gamma \vdash T_1 : K_{11} \Rightarrow K_2 \quad \Gamma \vdash T_2 : K_{11}}{\Gamma \vdash T_1 T_2 : K_2} \quad \text{K\_APP}$$

$$\frac{\Gamma \vdash T_1 : * \quad \Gamma \vdash T_2 : *}{\Gamma \vdash T_1 \rightarrow T_2 : *} \quad \text{K\_ARROW}$$

$$\frac{\Gamma, X : K_1 \vdash T_2 : *}{\Gamma \vdash \forall X : K_1. T_2 : *} \quad \text{K\_ALL}$$

$$\frac{\Gamma, X : K_1 \vdash T_2 : K_2}{\Gamma \vdash \lambda X : K_1. T_2 : K_1 \Rightarrow K_2} \quad \text{K\_ABS}$$

$\boxed{\Gamma \vdash S \equiv T : K}$  type equality

$$\frac{\Gamma \vdash T : *}{\Gamma \vdash T \equiv T : *} \quad \text{Q\_REFL}$$

$$\frac{\Gamma \vdash T \equiv S : K}{\Gamma \vdash S \equiv T : K} \quad \text{Q\_SYMM}$$

$$\frac{\Gamma \vdash S \equiv U : K \quad \Gamma \vdash U \equiv T : K}{\Gamma \vdash S \equiv T : K} \quad \text{Q\_TRANS}$$

$$\frac{\Gamma \vdash S_1 \equiv T_1 : * \quad \Gamma \vdash S_2 \equiv T_2 : *}{\Gamma \vdash S_1 \rightarrow S_2 \equiv T_1 \rightarrow T_2 : *} \quad \text{Q\_ARROW}$$

$$\frac{\Gamma, X : K_1 \vdash S_2 \equiv T_2 : *}{\Gamma \vdash \forall X : K_1. S_2 \equiv \forall X : K_1. T_2 : *} \quad \text{Q\_ALL}$$

$$\frac{\Gamma, X : K_1 \vdash S_2 \equiv T_2 : K_2}{\Gamma \vdash \lambda X : K_1. S_2 \equiv \lambda X : K_1. T_2 : K_1 \Rightarrow K_2} \quad \text{Q\_ABS}$$

$$\frac{\Gamma \vdash S_1 \equiv T_1 : K_1 \Rightarrow K_2 \quad \Gamma \vdash S_2 \equiv T_2 : K_1}{\Gamma \vdash S_1 S_2 \equiv T_1 T_2 : K_2} \quad \text{Q\_APP}$$

$$\frac{\Gamma, X : K_{11} \vdash T_{12} : K_2 \quad \Gamma \vdash T_2 : K_{11}}{\Gamma \vdash (\lambda X : K_{11}. T_{12}) T_2 \equiv T_{12} \{ T_2 / X \} : K_2} \quad \text{Q\_APPABS}$$

$$\frac{\Gamma, X : K_1 \vdash T_1 X \equiv T_2 X : K_2}{\Gamma \vdash T_1 \equiv T_2 : K_1 \Rightarrow K_2} \quad \text{Q\_EXT}$$

$\boxed{\Gamma \vdash t : S}$  Typing

$$\frac{x : S \in \Gamma}{\Gamma \vdash x : S} \quad \text{T\_VAR}$$

$$\frac{\Gamma \vdash S_1 : * \quad \Gamma, x : S_1 \vdash t_2 : T_2}{\Gamma \vdash \lambda x : S_1. t_2 : S_1 \rightarrow S_2} \quad \text{T\_ABS}$$

$$\frac{\Gamma \vdash t_1 : S_{11} \rightarrow S_{12} \quad \Gamma \vdash t_2 : S_{11}}{\Gamma \vdash t_1 t_2 : S_{12}} \quad \text{T\_APP}$$

$$\frac{\Gamma, X:K \vdash t : S}{\Gamma \vdash \lambda X:K.t : \forall X:K.S} \quad \text{T\_TABS}$$

$$\frac{\Gamma \vdash t : \forall X:K.S \quad \Gamma \vdash T : K}{\Gamma \vdash t[T] : S\{T/X\}} \quad \text{T\_TAPP}$$

$$\frac{\Gamma \vdash t : S \quad \Gamma \vdash S \equiv T : *}{\Gamma \vdash t : T} \quad \text{T\_EQ}$$

$\boxed{\Gamma \vdash_i t : S}$  Inference algorithm

$$\frac{x:S \in \Gamma}{\Gamma \vdash_i x : S} \quad \text{A\_VAR}$$

$$\frac{\Gamma \vdash S_1 : * \quad \Gamma, x:S_1 \vdash_i t_2 : T_2}{\Gamma \vdash_i \lambda x:S_1.t_2 : S_1 \rightarrow S_2} \quad \text{A\_ABS}$$

$$\frac{\Gamma \vdash_i t_1 : S_1 \quad \Gamma \vdash_i t_2 : S_2 \quad S_1 \rightarrow_{whnf} (S_{11} \rightarrow S_{12}) \quad \Gamma \vdash S_2 \Leftrightarrow S_{11} : *}{\Gamma \vdash_i t_1 t_2 : S_{12}} \quad \text{A\_APP}$$

$$\frac{\Gamma, X:K \vdash_i t : S}{\Gamma \vdash_i \lambda X:K.t : \forall X:K.S} \quad \text{A\_TABS}$$

$$\frac{\Gamma \vdash_i t : S_1 \quad S_1 \rightarrow_{whnf} \forall X:K.S \quad \Gamma \vdash T : K}{\Gamma \vdash_i t[T] : S\{T/X\}} \quad \text{A\_TAPP}$$

$\boxed{S \rightarrow_{wh} T}$  Weak-head reduction

$$\frac{}{(\lambda X:K.S) T \rightarrow_{wh} S\{T/X\}} \quad \text{QAR\_BETA}$$

$$\frac{S_1 \rightarrow_{wh} S'_1}{S_1 T_1 \rightarrow_{wh} S'_1 T_1} \quad \text{QAR\_APP\_}$$

$\boxed{S \rightarrow_{whnf} NF}$  Weak-head normalization

$$\frac{}{NF \rightarrow_{whnf} NF} \quad \text{QAN\_REDUCE}$$

$$\frac{S \rightarrow_{wh} T \quad T \rightarrow_{whnf} NF}{S \rightarrow_{whnf} NF} \quad \text{QAN\_NORMAL}$$

$\boxed{\Gamma \vdash S \Leftrightarrow T : K}$  Algorithmic equivalence

$$\frac{S \rightarrow_{whnf} NE_1 \quad T \rightarrow_{whnf} NE_2 \quad \Gamma \vdash NE_1 \Leftrightarrow NE_2 : *}{\Gamma \vdash S \Leftrightarrow T : *} \quad \text{QAT\_BASE}$$

$$\frac{\Gamma, X:K_1 \vdash S X \Leftrightarrow T X : K_2}{\Gamma \vdash S \Leftrightarrow T : K_1 \Rightarrow K_2} \quad \text{QAT\_ARROW}$$

$\boxed{\Gamma \vdash NE_1 \Leftrightarrow NE_2 : K}$  Algorithmic path equivalence (neutral terms)

$$\frac{X::K \in \Gamma}{\Gamma \vdash X \Leftrightarrow X : K} \quad \text{QAP\_VAR}$$

$$\frac{\Gamma \vdash NE_1 \Leftrightarrow NE_2 : K_1 \Rightarrow K_2 \quad \Gamma \vdash T_1 \Leftrightarrow T_2 : K_1}{\Gamma \vdash NE_1 T_1 \Leftrightarrow NE_2 T_2 : K_2} \quad \text{QAP\_APP}$$

$$\frac{\Gamma \vdash S_1 \Leftrightarrow S_2 : * \quad \Gamma \vdash T_1 \Leftrightarrow T_2 : *}{\Gamma \vdash S_1 \rightarrow T_1 \Leftrightarrow S_2 \rightarrow T_2 : *} \quad \text{QAP\_ARRT}$$

$$\frac{\Gamma, X:K \vdash S \Leftrightarrow T : *}{\Gamma \vdash \forall X:K. S \Leftrightarrow \forall X:K. T : *} \quad \text{QAP\_ALLT}$$

Definition rules:            40 good      0 bad  
Definition rule clauses: 78 good      0 bad