Higher-order polymorphism

January 26, 2009

1 Definitions

$termvar,\ x,\ y,\ z$
$tyvar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ n,\ m$

$t,\ u$	::=		term:
	$\|$	x	variable
	$\|$	$\lambda x : S . t$	abstraction
	$\|$	$t\ t'$	application
	$\|$	$\lambda X :: K . t$	type abstraction
	$\|$	$t\ [\ T\ ]$	type application
v	::=		value:
	$\|$	$\lambda x : T . t$	abstraction value
	$\|$	$\lambda X :: K . t$	type abstraction value
$T,\ S$	::=		types:
	$\|$	X	type variable
	$\|$	$T\ T'$	operator application
	$\|$	$S \rightarrow S'$	type of function
	$\|$	$\forall X :: K . S$	universal type
Γ	::=		contexts:
	$\|$	$\emptyset$	empty context
	$\|$	$\Gamma, x : S$	term variable binding
	$\|$	$\Gamma, X :: K$	type variable binding
K	::=		kinds:
	$\|$	$*$	kind of proper types
	$\|$	$K \Rightarrow K'$	kind of operators

$\boxed{t \rightarrow t'}$ Evaluation

$$\frac{t_1 \rightarrow t_1'}{t_1\ t_2 \rightarrow t_1'\ t_2} \quad \text{E\_APP1}$$

$$\frac{t_2 \rightarrow t_2'}{v_1\ t_2 \rightarrow v_1\ t_2'} \quad \text{E\_APP2}$$

$$\frac{}{(\lambda x : T_{11} . t_{12})\ v_2 \rightarrow t_{12}\{ v_2 / x \}} \quad \text{E\_APPABS}$$

$$\frac{t_1 \;\to\; t_1'}{t_1\,[\,T_2\,] \;\to\; t_1'\,[\,T_2\,]} \quad \text{E\_TAPP}$$

$$\frac{}{(\,\lambda X :: K_{11} . t_{12}\,)\,[\,T_2\,] \;\to\; t_{12}\,\{\,T_2\,/\,X\,\}} \quad \text{E\_TAPPTABS}$$

$\boxed{\vdash\ \Gamma}$ Context well-formed

$$\frac{}{\vdash\ \emptyset} \quad \text{C\_EMPTY}$$

$$\frac{\vdash\ \Gamma \quad \Gamma \vdash T :: * \quad x \notin dom(\Gamma)}{\vdash\ \Gamma, x : T} \quad \text{C\_VAR}$$

$$\frac{\vdash\ \Gamma \quad X \notin dom(\Gamma)}{\vdash\ \Gamma, X :: K} \quad \text{C\_TVAR}$$

$\boxed{\Gamma \vdash T :: K}$ Kinding

$$\frac{\vdash\ \Gamma \quad X :: K \in \Gamma}{\Gamma \vdash X :: K} \quad \text{K\_TVAR}$$

$$\frac{\Gamma \vdash T_1 :: K_{11} \Rightarrow K_2 \quad \Gamma \vdash T_2 :: K_{11}}{\Gamma \vdash T_1\ T_2 :: K_2} \quad \text{K\_APP}$$

$$\frac{\Gamma \vdash T_1 :: * \quad \Gamma \vdash T_2 :: *}{\Gamma \vdash T_1 \;\to\; T_2 :: *} \quad \text{K\_ARROW}$$

$$\frac{\Gamma, X :: K_1 \vdash T_2 :: *}{\Gamma \vdash \forall X :: K_1 . T_2 :: *} \quad \text{K\_ALL}$$

$\boxed{\Gamma \vdash t : S}$ Typing

$$\frac{\vdash\ \Gamma \quad x : S \in \Gamma}{\Gamma \vdash x : S} \quad \text{T\_VAR}$$

$$\frac{\Gamma \vdash S_1 :: * \quad \Gamma, x : S_1 \vdash t_2 : T_2}{\Gamma \vdash \lambda x : S_1 . t_2 : S_1 \;\to\; S_2} \quad \text{T\_ABS}$$

$$\frac{\Gamma \vdash t_1 : S_{11} \;\to\; S_{12} \quad \Gamma \vdash t_2 : S_{11}}{\Gamma \vdash t_1\ t_2 : S_{12}} \quad \text{T\_APP}$$

$$\frac{\Gamma, X :: K \vdash t_1 : S}{\Gamma \vdash \lambda X :: K . t_1 : \forall X :: K . S} \quad \text{T\_TABS}$$

$$\frac{\Gamma \vdash t_1 : \forall X :: K . S \quad \Gamma \vdash T_1 :: K}{\Gamma \vdash t_1\,[\,T_1\,] : S\,\{\,T_1\,/\,X\,\}} \quad \text{T\_TAPP}$$

Theorem 1 (Preservation) *If $\emptyset \vdash t : T$ and $t \;\to\; t'$ then $\emptyset \vdash t' : T$*

Theorem 2 (Progress) *If $\emptyset \vdash t : T$ then either t is a value or there exists a t' such that $t \;\to\; t'$.*