Higher-order polymorphism without kinds

CIS 670 Spring 2009

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1 Definitions

$termvar, x, y, z$
$tyvar, X, Y, Z$
$index, i, j, n, m$

t, u	$::=$		term:
	$\mid$	x	variable
	$\mid$	$\lambda x : S.t$	abstraction
	$\mid$	$t\ t'$	application
	$\mid$	$\lambda X.t$	type abstraction
	$\mid$	$t\,[\,T\,]$	type application
v	$::=$		value:
	$\mid$	$\lambda x : T.t$	abstraction value
	$\mid$	$\lambda X.t$	type abstraction value
T, S	$::=$		types:
	$\mid$	X	type variable
	$\mid$	$T\ T'$	operator application
	$\mid$	$S \rightarrow S'$	type of function
	$\mid$	$\forall X.S$	universal type
Γ	$::=$		contexts:
	$\mid$	$\emptyset$	empty context
	$\mid$	$\Gamma, x : S$	term variable binding

$\boxed{t \rightarrow t'}$ Evaluation

$$\frac{t_1 \rightarrow t_1'}{t_1\, t_2 \rightarrow t_1'\, t_2} \quad \text{E\_APP1}$$

$$\frac{t_2 \rightarrow t_2'}{v_1\, t_2 \rightarrow v_1\, t_2'} \quad \text{E\_APP2}$$

$$\frac{}{(\,\lambda x : T_{11}.t_{12}\,)\, v_2 \rightarrow t_{12}\,\{\, v_2 \,/\, x\,\}} \quad \text{E\_APPABS}$$

$$\frac{t_1 \rightarrow t_1'}{t_1\,[\,T_2\,] \rightarrow t_1'\,[\,T_2\,]} \quad \text{E\_TAPP}$$

$$\frac{}{(\,\lambda X.t_{12}\,)\,[\,T_2\,] \rightarrow t_{12}\,\{\, T_2 \,/\, X\,\}} \quad \text{E\_TAPPTABS}$$

$\boxed{\Gamma \vdash t : S}$ Typing

$$\frac{x : S \in \Gamma}{\Gamma \vdash x : S} \quad \text{T\_VAR}$$

$$\frac{\Gamma, x : S_1 \vdash t_2 : T_2 \quad x \notin dom(\Gamma)}{\Gamma \vdash \lambda x : S_1 . t_2 : S_1 \to S_2} \quad \text{T\_ABS}$$

$$\frac{\Gamma \vdash t_1 : S_{11} \to S_{12} \quad \Gamma \vdash t_2 : S_{11}}{\Gamma \vdash t_1\, t_2 : S_{12}} \quad \text{T\_APP}$$

$$\frac{\Gamma \vdash t_1 : S \quad X \notin rng(\Gamma)}{\Gamma \vdash \lambda X . t_1 : \forall X . S} \quad \text{T\_TABS}$$

$$\frac{\Gamma \vdash t_1 : \forall X . S}{\Gamma \vdash t_1 [\, T_1 \,] : S \{\, T_1 / X \,\}} \quad \text{T\_TAPP}$$

2 Metatheory

Lemma 1 (Context substitution) *If* $X \notin rng(\Gamma)$ *then* $\Gamma \{\, S / X \,\} \equiv \Gamma$

Lemma 2 (Type substitution in terms) *If* $\Gamma \vdash t : T$ *then* $\Gamma \{\, S / X \,\} \vdash t \{\, S / X \,\} : T \{\, S / X \,\}$.

Lemma 3 (Term substitution in terms) *If* $\Gamma_1, x : T_1 , \Gamma_2 \vdash t : T_2$ *and* $\Gamma_1 \vdash u : T_2$ *then* $\Gamma_1 , \Gamma_2 \vdash t \{\, u / x \,\} : T_2$.

Theorem 4 (Preservation) *If* $\emptyset \vdash t : T$ *and* $t \to t'$ *then* $\emptyset \vdash t' : T$

Lemma 5 (Canonical Forms) *If* $\emptyset \vdash v : T_1 \to T_2$ *then* v *is* $\lambda x : S . t$. *If* $\emptyset \vdash v : \forall X . T$ *then* v *is* $\lambda X . t$.

Theorem 6 (Progress) *If* $\emptyset \vdash t : T$ *then either* t *is a value or there exists a* t' *such that* $t \to t'$.