On the Formalization of Logical Relation Arguments

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Tait's method (a.k.a. proof by logical relations) is a powerful proof technique frequently used for showing foundational properties of languages based on typed lambda-calculi. Historically, these proofs have been difficult to formalize in Twelf, in part because logical relations are difficult to define judgementally.

In [SS06], we present a solution to this problem. Rather than defining the logical relation directly in LF, we define the logical relation as a predicate in an auxiliary assertion logic. This distinction allows us to decompose the argument into two parts: we show the consistency of the assertion logic entails the validity of the argument, and that the assertion logic is consistent. The meat of the argument is in the entailment proof. However, by Gödel's second incompleteness theorem, there will always be assertions logics whose consistency cannot be formalized. In these cases, the entailment proof can be seen as reducing the validity of the theorem at hand to the consistency of the assertion logic. We believe that the proof technique presented here is of broad interest to the community attempting to mechanize the meta-theory programming languages.

References

[SS06] Jeffrey Sarnat and Carsten Schürmann. On the representation of logical relations in Twelf. Technical Report YaleU/DCS/TR-1362, Yale University, 2006.

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