

# Spatial Mismatch, Search Effort and Urban Spatial Structure

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**Abstract:** The aim of this paper is to provide a new mechanism for the spatial mismatch hypothesis. Spatial mismatch can here be the result of optimizing behavior on the part of the labor market participants. In particular, the unemployed can choose low amounts of search and long-term unemployment if they reside far away from jobs. They choose voluntarily not to relocate close to jobs because the short-run gains (low land rent and large housing consumption) are big enough compared to the long-run gains of residing near jobs (higher probability of finding a job).

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## 1. Introduction

The spatial mismatch hypothesis, first formulated by Kain [18], states that, residing in urban segregated areas distant from and poorly connected to major centers of employment growth, black workers face strong geographic barriers to finding and keeping well-paid jobs. In the U.S. context, where jobs have been decentralized and blacks have stayed in the central part of cities, the main conclusion of the spatial mismatch hypothesis is to put forward the distance to jobs as the main culprit for the high unemployment rates among blacks.

Since the study of Kain, dozens of empirical studies have been carried out trying to test this hypothesis (see the surveys by Holzer [15], Kain [19] and Ihlanfeldt and Sjoquist [16]). The usual approach is to relate a measure of labor-market outcomes, based on either individual or aggregate data, to another measure of job access, typically some index that captures the distance from residences to centers of employment. The weight of the evidence suggests that bad job access indeed worsens labor-market outcomes, confirming the spatial mismatch hypothesis.

The theoretical foundations behind these empirical results remain however unclear. If researchers do agree on the causes (housing discrimination, social interactions) and on the consequences of the spatial mismatch hypothesis (higher unemployment rates and lower wages for black workers), the economic mechanisms and thus the policy implications are difficult to identify.

A first theoretical view developed by Brueckner and Martin [7] and Brueckner and Zenou [8] is to argue that suburban housing discrimination skews black workers towards the Central Business District (CBD) and thus keeps black residences remote from the suburbs. Since black workers who work in the Suburban Business District (SBD) have more costly commutes, few of them will accept SBD jobs, which makes the black CBD labor pool large relative to the SBD pool. Under either a minimum-wage or efficiency wage model, this enlargement of the CBD pool leads to a high unemployment rate among CBD workers. The policy recommendation emerging from this model is to subsidize the commuting costs of black workers, so as to improve job access.

Wasmer and Zenou [37] have proposed a different theory for the spatial mismatch hypothesis. Using a search-matching model, they state that distance to jobs prevents black workers from obtaining job information, thus isolating them from employment centers. Indeed little information reaches the area where blacks live, which lowers their search efficiency and thus their probability of finding a job. The policy implications of this model are thus quite different

than those of the previous one. The model suggests that the government should provide information about jobs, thus lowering the search cost of inner-city black residents.

Another theoretical view has been proposed by Coulson, Laing and Wang [10]. Using also a search-matching model, they assume that the fixed entry cost of firms is greater in the CBD than in the SBD and that workers are heterogeneous in their disutility of transportation (or equivalently in their search costs). These two fundamental assumptions are sufficient to generate an equilibrium in which central city residents experience a higher rate of unemployment than suburban residents and suburban firms create more jobs than central firms (higher job vacancy rate). Their model yields the same policy implications that the two models above since improvements in the efficiency of the matching function and/or in the transportation infrastructure yield a lower level of unemployment. They propose however another policy that is more specific to their model. The government should reduce the differential in the fixed entry cost in order to partially alleviate the spatial mismatch; for example, by subsidizing the entry of firms in the CBD. Such policies have been implemented in the U.S. through the enterprise zone programs (Papke [25], Boarnet and Bogart [5] and Mauer and Ott [23]). The basic idea is to designate a specific urban (or rural) area, which is depressed, and target it for economic development through government-provided subsidies to labor and capital.<sup>1</sup>

In the present paper, we propose an alternative theoretical approach to explain the spatial mismatch hypothesis. Using a search-matching model with endogenous housing consumption and location, we show that distance to jobs is harmful because it implies low search intensities. There is in fact a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed. Indeed, locations near jobs are costly in the short run (both in terms of high rents and low housing consumption), but allow higher search intensities which in turn increase the long-run prospects of reemployment. Conversely, locations far from jobs are more desirable in the short run (low rents and high housing consumption) but allow only infrequent trips to jobs and hence reduce the long-run prospects of reemployment. Therefore, for workers residing further away from the CBD, it is optimal to spend the minimal search effort whereas workers residing close to jobs provide high search effort.

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<sup>1</sup>For a general survey on the theoretical foundations of the spatial mismatch, see Gobillon, Selod and Zenou [12].

In this context, spatial mismatch can be the result of optimizing behavior on the part of the labor market participants since the unemployed can *choose* low amounts of search and long-term unemployment. This implies that the standard US-style mismatch arises because inner-city blacks choose to remain in the inner-city and search only little. They do not relocate to the suburbs because the short run-long run gap is big enough to make locations near the jobs too expensive. The policy implications are therefore quite different. In particular, “Moving to Opportunity” programs (such as the so-called Gautreaux program) are just the correct policy device to reduce mismatch, rather than lower search costs in some other way.

More precisely, a spatial labor market model is developed in which both job-matching behavior and residential-location behavior are treated simultaneously. Since time is discrete, search intensity is the fraction of the period during which the unemployed are actively searching. Equilibrium for this system involves the interaction of two markets: a spatially concentrated (CBD) labor market in which unemployed workers compete for jobs, and a spatially dispersed land market in which all workers compete for residential land. The most important linkage between these markets is in terms of the differing job-search intensities chosen by unemployed workers at various distances from the CBD.

We first show that there is a non-linear decreasing relationship between the residential distance to jobs of the unemployed and their search intensity  $s$ . In fact, individuals living sufficiently close to jobs search every day,  $s = 1$ , whereas those residing far away provide a minimum search intensity,  $s = s_0$ . Workers living in between these two areas see a decrease in their search intensity from  $s = 1$  to  $s = s_0$ . We then embed this result (the fact that the unemployed’s search intensities are location dependent) into an urban equilibrium in which all individuals (including the unemployed) endogenously choose their residential location. This is one of the main difficulties that we had to overcome. In a classification theorem (see Theorem 2), we show that only three urban configurations are compatible with the decreasing relationship between search intensity and location. These possible equilibrium location patterns are shown to differ only in terms of whether the unemployed workers occupy the central core around the CBD, the periphery of the city, or possibly both.

Finally, since our purpose is to shed some light on the spatial mismatch hypothesis, we focus on two urban equilibria: the *core-periphery urban equilibrium*, in which the unemployed reside either close to jobs (and provide full

search intensity  $s = 1$ ) or far away from jobs (and provide a positive minimal level of search  $s = s_0$ ) and the *segregated* equilibrium where the unemployed are always far away from jobs. We show that each equilibrium is unique, and we give a set of sufficient conditions for its existence.

The remainder of the paper is organized as follows. Section 2 sets up the model and describes the land and labor markets. In section 3, we demonstrate our first result, namely the non-linear and decreasing relationship between the residential distance to jobs of the unemployed and their search intensity. Section 4 is devoted to our classification theorem that shows that only three urban configurations are compatible with the negative relationship between search intensity and location. In section 5, we show the existence and the uniqueness of the core-periphery equilibrium and of the segregated equilibrium. Finally, we analyze some of the policy implications of our model in section 6.

## 2. The model

Consider a population of  $N$  *workers* who live in a monocentric city where all jobs are concentrated in the central business district (CBD). All employed workers earn the same prevailing *daily wage*,  $w$ , and all unemployed workers receive a *daily unemployment benefit*,  $b$  (where it is assumed that  $b < w$ ). Employed workers commute to the CBD each day, and unemployed workers also travel to the CBD to search for jobs. Hence all workers desire to be near the CBD, and compete for residential land on this basis. This urban system is thus characterized by two interdependent markets: a *labor market* in which unemployed workers compete for jobs at the CBD, and a *land market* in which all worker compete for land near to the CBD. We now model each of these markets in turn, and then consider the relevant interactions between them.

### 2.1. The labor market

Since our focus is on the spatial behavior of workers and their match with firms, we cannot use directly the standard *macroeconomic* matching function (Mortensen and Pissarides [24] and Pissarides [27]). Instead, we need to spell out the micro scenario that leads to a well behaved matching function. For that, the present labor market is based on the model of job-matching behavior developed in Smith and Zenou [35], hereafter referred to as [SZ]. It is in fact a variation of the standard urn-ball model where the system steady state is approximated by an exponential-type matching function as the population

becomes large (see among others Hall [14], Pissarides [26], Blanchard and Diamond [4]). Let us describe it more precisely.

In our model it is assumed that (i) jobs are completely specialized in terms of skill requirements, and that (ii) workers are heterogeneous in terms of their skill endowments. Thus *job matching* here constitutes a process whereby heterogeneous workers allocate themselves to jobs with different skill requirements. Heterogeneity of workers does not here imply any superiority or inferiority among their abilities. Rather, all are assumed to possess the same level of general human capital, which is manifested in a variety of different skills (as for example college graduates with degrees in different fields). Hence all workers are assumed to have the same chance of being qualified for any given job, as modeled by a common *qualification probability*,  $\gamma$ .

In this context, the actual *job matching process* can be described as follows. At any point in time (time is discrete), each worker is either employed or unemployed, and only unemployed workers are assumed to search for jobs. Since individual jobs are completely specialized, their creation and closing can be regarded as independent events. In particular, job creations and job closings are here modeled as a simple ‘birth and death’ process in which ‘births’ are governed by a *job-creation rate*,  $\lambda$  (denoting the mean number of jobs per worker created each day) and ‘deaths’ are governed by a *job-closure rate*,  $\rho$  (denoting the probability that any currently existing job will be closed on a given day). This process is taken to depend on the general state of economy, and hence is treated as exogenous to the labor market. As mentioned above, the *daily wage*,  $w$ , is assumed to be the same for all jobs and (for sake of simplicity) is here assumed to be given exogenously. As in [SZ], the behavioral day-to-day scenario for the job market model on a given day,  $t$ , can be summarized as follows:

- At the beginning of day  $t$  those unemployed workers currently seeking work travel to the job market (CBD). All current job vacancies are posted, and are offered at the going wage  $w$ . Each searcher applies for a single job. No additional prior information about jobs is available, and there is no communication between searchers. Hence searchers choose jobs at random, and more than one searcher may apply for the same job.
- As mentioned above, each job applicant has the same probability,  $\gamma$ , of satisfying all qualifications for the given job. If more than one applicant is qualified for a job, the employer chooses a qualified applicant at random. Otherwise the job is not filled on day  $t$ .

- At the end of day  $t$  each successful applicant is notified, and is requested to start work on the following day. In addition, decisions are made by employers as to which jobs are no longer profitable and should be closed. For currently active jobs which are closed, layoff notices are distributed to workers. Moreover, for jobs which are filled that day and then closed, the successful (but unlucky) applicants are also given notices. Finally, those currently vacant jobs which are closed are simply removed from the postings at the beginning of the next day. As mentioned above, all jobs (active or vacant) have the same chance,  $\rho$ , of being closed on day  $t$ .
- In addition, those new job opportunities which have arisen during the day (at rate,  $\lambda$ , per worker) are added to the vacant job postings for the next day.

For the present it is assumed that the residential locations of all workers (both employed and unemployed) are given. In this context, the key decision problem for each unemployed worker is to determine his *search intensity*,  $s$ , which we here take to be the fraction of days he travels to the CBD in search of work. If the average value of this fraction over all unemployed workers is designated as the *mean search intensity*,  $\bar{s}$ , then on any given day, the probability that a randomly sampled worker will appear at the job market is by definition  $\bar{s}$ . Hence if the unemployment pool is large, then it follows (from the Weak Law of Large Numbers) that the fraction of unemployed workers appearing at the market each day is well approximated by  $\bar{s}$ . This system parameter,  $\bar{s}$ , is also assumed to be given for the present.

In this context, it is shown in [SZ] that if jobs creations are characterized by the birth-and-death process described above, then there is a unique steady-state distribution of unemployment and job vacancy levels for each set of parameters  $(\rho, \lambda, \bar{s}, \gamma)$ . Moreover as population size,  $N$ , becomes large, this distribution converges in probability to its mean value, characterized by a steady-state *unemployment rate*,  $u$ , (representing the fraction of workers unemployed on each day), and steady-state *vacancy rate*,  $v$ , (representing the number of vacant jobs per worker on each day). These steady-state values are given by the unique solution of the following steady-state equations:<sup>2</sup>

$$v + (1 - u) = \lambda / \rho \tag{2.1}$$

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<sup>2</sup>This steady state equilibrium can be compared to that of the standard matching model (Mortensen-Pissarides [24], Pissarides [27]), by noting that the Beveridge curve in their model is very similar to our steady-state condition (2.2). However, we do not use the

$$\rho (1 - u) = (1 - \rho) u \bar{s} p_h \quad (2.2)$$

where the *hiring probability*,  $p_h$  (i.e., the probability that a randomly sampled job searcher will be hired on a given day) is given by:<sup>3</sup>

$$p_h = \frac{v}{u \bar{s}} \left[ 1 - e^{-(\gamma \bar{s} u / v)} \right] \quad (2.3)$$

In particular, it is shown that the limiting number of jobs per worker in the system at steady state is given by  $\lambda/\rho$ . Hence, noting that the number of active jobs per worker is precisely the fraction of employed workers,  $1 - u$ , it follows that equation (2.1) is simply an accounting identity relating the number of vacant jobs and active jobs to total jobs per worker. Similarly, noting that  $\rho (1 - u)$  is the number of active jobs per worker closed on a given day, and that  $(1 - \rho) u \bar{s} p_h$  is the number of active jobs created on a given day (i.e., the fraction of vacant jobs which are filled and not closed), it follows that equation (2.2) amounts simply to the requirement that the number of active jobs per worker remain constant in the steady state (this equation corresponds to the standard Beveridge curve in the matching literature). If we now let  $d = \frac{\lambda}{\rho} - 1$ , and solve for  $v$  in (2.1) as

$$v = u + d \quad (2.4)$$

then (2.1) through (2.4) are seen to imply that the steady-state unemployment rate,  $u$ , must satisfy the single equation

$$\rho (1 - u) = (1 - \rho) (u + d) \left( 1 - e^{-\gamma \frac{\bar{s} u}{u + d}} \right) \quad (2.5)$$

In terms of our present notation, it is shown in [SZ] (Lemma A.2 and Theorem 1.2) that

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standard free entry condition to close the labor market equilibrium but our steady-state condition (2.1) results from the underlying birth-death process on vacancies.

<sup>3</sup>This hiring probability corresponds to the following aggregate matching function:

$$m(u, v) = v \left[ 1 - e^{-(\gamma s u / v)} \right]$$

which has the standard properties (increasing in both its arguments and concave, and homogeneous of degree 1). Observe that the *individual* probability to find a job for a job seeker with search intensity  $s$  is given by:

$$s p_h = \frac{s}{\bar{s}} \frac{m(u, v)}{u}$$

**Theorem 1 (Labor Market Steady State).** *For each mean search intensity,  $\bar{s} \in [0, 1]$ , there exists a unique solution,  $u(\bar{s})$ , to (2.5). In addition,  $u(\bar{s})$  a positive decreasing differentiable function of  $\bar{s}$  with  $u(0) = 1$ .*

Notice in particular that there is always a *positive* unemployment rate in the steady state, regardless of how many jobs are being created. This is a consequence of the *frictional unemployment* inherent in the job-matching process itself. There is always some chance that an unemployed worker will not be hired on a given day, regardless of how many jobs are available.

For our later purposes, it is also important to notice that one can solve for  $\bar{s}$  in terms of  $u$  in (2.5), and obtain the following explicit form for the inverse function:

$$\bar{s} = \psi(u) = - \left( \frac{u+d}{\gamma u} \right) \ln \left[ 1 - \left( \frac{\rho}{1-\rho} \right) \left( \frac{1-u}{u+d} \right) \right] \quad (2.6)$$

This relation allows one to determine for each unemployment rate,  $u$ , the unique mean search intensity level,  $\bar{s}$ , which will support  $u$  as a steady state.

## 2.2. The land market

As stated in the introduction, all jobs are assumed to be located at the center (CBD) of a large metropolitan area. In a manner similar to Smith and Zenou [33], this metropolitan area is taken to be representable by a *circular monocentric* city, in which the CBD is the unique center of all business activity and in which all commuting distances are measured as straight-line distances to the CBD. Hence individual *locations*,  $x$ , are identified with distances for the CBD. In addition the city is assumed to be *closed* with fixed total population,  $N$ .<sup>4</sup> As in the labor market model above (which appealed to large-number approximations), the population,  $N$ , is here treated as a continuum in which the influence of individual workers is vanishingly small. Residential land (here synonymous with housing) is rented by workers from absentee landlords. In the terminology of Fujita [11], the present model is thus a *closed city model under absentee land ownership* with *land intensity*,

$$L(x) = 2\pi x \quad (2.7)$$

at each distance  $x$  from the CBD. A key point in this model is that individuals are now free to consume any amount of land consistent with their budgets. This

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<sup>4</sup>This implies in particular that there is no in-migration or out-migration from the city. In addition, there are no births or deaths of workers, so that individuals are assumed to be ‘infinitely lived’.

relaxation is of particular importance in that it allows unemployed workers to compete for locations near the CBD, by consuming small amounts of land (and living in crowded conditions) if necessary.

As in the labor market model above, workers can in principle change employment states from day to day. Loss of employment involves a change in income from the daily wage,  $w$ , to the daily unemployment benefit,  $b$ , and visa versa. Hence, given the prevailing *rent gradient*,  $R(x)$ , at each location  $x$ , this change of income and employment status may motivate individuals to change their location (or at least in the amount of housing consumed at their current location). All such changes are assumed to be instantaneous, and are governed only by individual utility-maximizing behavior.<sup>5</sup> This decision problem for newly unemployed workers is complicated by the fact that finding a new job will involve some level of search intensity,  $s$ . In the labor market setting above, unemployed workers must travel to the CBD to find jobs, so that high levels of search intensity require frequent trips to the CBD. This leads to a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed. On the one hand, locations near the CBD are costly in the short run (both in terms of high rents and crowded living conditions), but allow higher search intensities which in turn increase the long-run prospects of reemployment. Conversely, locations far from the CBD are more desirable in the short run (low rents and uncrowded conditions) but allow only infrequent trips to the CBD and hence reduce the long-run prospects of reemployment.

To model this basic trade-off, we begin by assuming that all workers have identical preferences among consumptions bundles  $(q, z)$  of *land (housing)*,  $q$ , and *composite good*,  $z$ , representable by a log-linear utility

$$U(q, z) = q^\alpha z^\beta \tag{2.8}$$

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<sup>5</sup>In particular, there are assumed to be no relocation costs, either in terms of time or money. This is a simplifying assumption, which is quite standard in urban economics. It implies that workers change location as soon as they change employment status. In the context of labor markets in which workers tend to experience long unemployment spells (for example black workers), it is a rather good approximation since, when workers become unemployed, they will be less able to pay land rents and, after some time, they will have to relocate in cheaper places. This assumption could be relaxed by assuming for example that workers only care about their expected utility, i.e. the fraction of their lifetime spent employed and unemployed (this is the case if the discount rate is equal to zero) so that, whatever their employment status, they always stay in the same location. This will however complicate the analysis without changing our main result on the relationship between search intensity and distance to jobs.

with  $\alpha, \beta > 0$ , where it is also assumed that  $\alpha + \beta < 1$ .<sup>6</sup> However the budget constraints for employed and unemployed workers are different. Each *employed* worker living at location,  $x$ , has the standard budget constraint

$$qR(x) + cx + z = w \quad (2.9)$$

where  $z$  is taken as the numeraire good with unit price,  $R(x)$ , is the prevailing (daily) rent per unit of land at  $x$ , and where  $c$  is the daily round-trip cost of commuting to the CBD. However, an *unemployed* worker at  $x$  not only has a different daily income,  $b$ , but also has different travel costs depending on his chosen level of search intensity,  $s$ . Hence the relevant budget constraint for each such unemployed worker is of the form

$$qR(x) + scx + z = b \quad (2.10)$$

where, for example, searching every other day ( $s = 1/2$ ) would yield an average daily travel cost of  $cx/2$ . If one denotes the *unemployed state* for workers by ‘0’, and the *employed state* by ‘1’, then maximizing utility (2.8) subject to (2.9) yields the following *land demand for employed workers* at  $x$ :

$$q_1(x) = \frac{\alpha}{\alpha + \beta} \cdot \frac{w - cx}{R(x)} \quad (2.11)$$

Similarly, maximizing (2.8) subject to (2.10) yields the following *land demand for unemployed workers* at  $x$ :

$$q_0(x) = \frac{\alpha}{\alpha + \beta} \cdot \frac{b - s(x)cx}{R(x)} \quad (2.12)$$

We can now derive the following indirect utility

$$U_1(x) = a(w - cx)^{\alpha + \beta} R(x)^{-\alpha} \quad (2.13)$$

for each *employed* worker at  $x$ , where  $a = [\alpha/(\alpha + \beta)]^\alpha [\beta/(\alpha + \beta)]^\beta$  and the following indirect utility

$$U_0(s, x) = a(b - scx)^{\alpha + \beta} R(x)^{-\alpha} \quad (2.14)$$

for each *unemployed* worker at  $x$ , where in this case  $s$  is now included as a relevant choice variable.<sup>7</sup>

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<sup>6</sup>This property, which implies ‘diminishing marginal utility on rays’ [i.e.,  $U(\lambda q, \lambda z) < \lambda U(q, z)$  for all  $\lambda > 0$ ], insures that the optimal-search-intensity problem discussed below has a differentiable maximum. It is important to emphasize here that (unlike the standard urban economic model) the utility function in (2.8) is necessarily *cardinal* in nature, so that properties such as diminishing marginal utility on rays are behaviorally meaningful. See footnote 9 below for further discussion of this point.

<sup>7</sup>At this point it should be noted that there is a basic difference between the present

### 3. Optimal search intensities in the city

To model the trade-off outlined above, we focus on the decision problem for an unemployed worker at location  $x$  who is currently considering his choice of search intensity,  $s$  (which for simplicity can be regarded as the choice of a roulette wheel to use each morning in deciding whether to search that day). To weigh alternative choices, he must evaluate the expected future consumption streams resulting from each choice of  $s$ . At each point of time in the future the worker will be in one of two states: unemployed (0) or employed (1). Hence, if we now assume that the present value of future consumption bundles for all workers is representable by a common *utility discount rate*,  $\sigma \in (0, 1)$ , and if we designate the expected discounted utility streams starting in each state as the *lifetime values*,  $V_0$  and  $V_1$ , of these states,<sup>8</sup> then (by employing the same arguments as in [SZ]) it can be shown that  $V_0$  and  $V_1$  satisfy the following identities:<sup>9</sup>

$$V_0 = \left( \frac{1 - e_0}{1 - \sigma} \right) U_0 + e_0 V_1 \quad (3.1)$$

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utility formulation and that in [SZ]. In that paper the basic utility tradeoff for all workers was postulated to be in terms of income versus leisure time. In a spaceless world with no travel costs, it can be argued that *time costs* represent the key variable cost in job search. Such costs of course continue to be important when space is introduced. But in the present model, we have endeavored to keep the framework as simple as possible by focusing only on the *travel costs* associated with spatial job search. A more satisfactory approach would of course encompass both types of costs (including the time spent in travel itself).

<sup>8</sup>To be more precise, preferences over *consumption streams*, i.e., sequences of daily consumption bundles,  $\omega = [(q_t, z_t) : t = 1, 2, \dots]$ , are taken to be representable by a *discounted utility function* of the form  $V(\omega) = \sum_t \sigma^t U(q_t, z_t)$ , where  $U$  is the utility in (2.8). Behavioral conditions for the existence of such representations (including ‘impatience’ for consumption and ‘time stationarity’ of preferences) are given in Koopmans [21]. Of particular importance for our present purposes is uniqueness of these representations: the behavioral discount rate,  $\sigma$ , is *unique*, and the consumption utility,  $U$ , is unique up to a linear transformation. Hence utility is necessarily *cardinal* in nature, and in fact, has the same measurement status as money [if it is assumed that  $U(0, 0) = 0$ , as in (2.8)]. It is thus perfectly meaningful to treat  $V(\omega)$  as the realization of a well defined random variable,  $V$ , with different conditional distributions depending on the initial employment status of the worker. Hence the lifetime values,  $V_0$  and  $V_1$ , are the corresponding conditional means of  $V$  given initial states ‘0’ and ‘1’, respectively.

<sup>9</sup>It is easy to see that (3.1) and (3.2) correspond to the two following more intuitive Bellman equations:

$$\begin{aligned} V_0 &= U_0 + \sigma [s p_h V_1 + (1 - s p_h) V_0] \\ V_1 &= U_1 + \sigma [\rho V_0 + (1 - \rho) V_1] \end{aligned}$$

$$V_1 = \left( \frac{1 - e_1}{1 - \sigma} \right) U_1 + e_1 V_0 \quad (3.2)$$

where

$$e_0 = \frac{s\sigma p_h}{1 - \sigma + s\sigma p_h} \quad (3.3)$$

$$e_1 = \frac{\sigma\rho}{1 - \sigma + \sigma\rho} \quad (3.4)$$

(and where dependence of the  $V$ 's and  $U$ 's on  $x$  and  $s$  is suppressed).

By substituting [(3.3),(3.4)] into [(3.1),(3.2)] and solving these equations simultaneously, one may express  $V_0$  and  $V_1$  in terms of  $U_0$  and  $U_1$  as follows:

$$V_0 = \frac{(1 - \sigma + \sigma\rho)U_0 + (s\sigma p_h)U_1}{(1 - \sigma)(1 - \sigma + \sigma\rho + s\sigma p_h)} \quad (3.5)$$

$$V_1 = \frac{(1 - \sigma + s\sigma p_h)U_1 + (\sigma\rho)U_0}{(1 - \sigma)(1 - \sigma + \sigma\rho + s\sigma p_h)} \quad (3.6)$$

Returning to our basic decision problem, suppose that an unemployed worker at  $x$  is currently reconsidering his search intensity level,  $s$ . To characterize his optimal choice of  $s$  as an equilibrium condition, it is convenient to assume that the system is in steady state with some *mean search intensity*,  $\bar{s}$ . Associated with this mean intensity level is a steady-state *hiring probability* (2.3) which we again denote by  $p_h = p_h(\bar{s})$ . In addition, we also assume that the current *lifetime values*,  $V_0$  and  $V_1$ , of both employed and unemployed workers are constant at all locations (as they must be in equilibrium to ensure that no workers are motivated to relocate). In addition we note that  $w > b$  implies desirability of employment, and hence that  $V_1 > V_0$  in equilibrium. Under these conditions, we ask whether there is some choice of  $s$  for the unemployed worker at  $x$  which will improve his current lifetime value, i.e. for which  $V_0(s, x) > V_0$ . Assuming that perturbations in the search intensity,  $s$ , of this single individual cannot influence population values, we may treat both  $p_h$  and  $V_1$  as constants in this decision problem. However,  $U_0$  and  $e_0$  are seen from (2.14) and (3.3) to be directly influenced by the choice of  $s$ . Hence it follows from these expressions, together with (3.1), that worker's lifetime value,  $V_0(s, x)$ , can be written as:

$$\begin{aligned} V_0(s, x) &= \left( \frac{1 - e_0(s)}{1 - \sigma} \right) U_0(s, x) + e_0(s) V_1 \\ &= \frac{a(b - scx)^{\alpha+\beta} R(x)^{-\alpha} + \sigma p_h s V_1}{1 - \sigma + \sigma p_h s} \end{aligned} \quad (3.7)$$

Finally, to rule out the possibility of a zero level of optimal search intensity, we assume that some minimal amount of travel to the CBD is required (for purchase of the composite good,  $z$ ), and hence that there is always some incentive for unemployed workers to live in the city.<sup>10</sup> Assuming that  $w > b$  and that all search costs other than travel are zero, it then follows that *unemployed workers are motivated to apply for jobs on every visit to the CBD*. Hence there is a corresponding *minimal search intensity* level, which we denote by  $s_0 > 0$ .<sup>11</sup> The relevant decision problem for this unemployed worker is thus to choose a value of  $s \in [s_0, 1]$  which maximizes (3.7). Observe also from (2.14) that positive utility is only achievable with positive net income,  $b - scx$ , so that location choices,  $x$ , must always be restricted to the interval  $[0, \frac{b}{s_0c}]$ . We have the following result:

**Proposition 1 (Optimal Search Intensities).**

- At each location  $x$ , there is a unique search intensity  $s$  that maximizes (3.7).
- For any prevailing hiring probability,  $p_h$ , and constant lifetime values,  $V_0, V_1$ , the optimal search intensity function,  $s(x)$ , for unemployed workers is given for each location,  $x \in [0, \frac{b}{s_0c}]$ , by

$$s(x) = \begin{cases} 1 & \text{for } x \leq x(1) \\ \frac{\alpha+\beta}{1-(\alpha+\beta)} \left[ \frac{b}{(\alpha+\beta)cx} - \frac{(1-\sigma)V_0}{\sigma p_h (V_1 - V_0)} \right] & \text{for } x(1) < x < x(s_0) \\ s_0 & \text{for } x \geq x(s_0) \end{cases} \quad (3.8)$$

where

$$x(s) = \frac{b}{sc} \cdot \frac{s\sigma p_h (V_1 - V_0)}{(\alpha + \beta)(1 - \sigma)V_0 + [1 - (\alpha + \beta)]s\sigma p_h (V_1 - V_0)} \quad (3.9)$$

**Proof.** See the Appendix.

The following comments are in order. First, using the first order condition (A.3) in the Appendix, we can easily see the trade off faced by the unemployed

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<sup>10</sup>If the optimal search intensity for an unemployed worker were zero, then since unemployment benefits are taken to be exogenous, there would be no incentive to stay in the city.

<sup>11</sup>We note in passing that the existence of a minimal positive search intensity,  $s_0$ , implies that the steady-state mean search intensity,  $\bar{s}$ , can be no less than  $s_0$ , and hence must also be positive.

when they decide their optimal search intensity level. The left hand side is the short-run utility loss from a marginal increase in search intensity, and the right hand side is the corresponding long-run utility gain from future employment. Indeed, on the one hand, there is a direct and short-run cost of searching more today  $-\partial U_0(s, x)/\partial s$  since it implies higher commuting costs<sup>12</sup> and a lower housing consumption, and thus lower instantaneous utility. On the other, there is a long-run gain of searching more today  $\sigma p_h [V_1 - V_0(s, x)]$  since it increases the marginal chance to obtain a job (remember that the individual probability to obtain a job is  $s p_h$ ) and the corresponding life-time surplus of being employed. This leads to a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed. Indeed, locations near the CBD are costly in the short run (both in terms of high rents and low housing consumption), but allow higher search intensities which in turn increase the long-run prospects of reemployment. Conversely, locations far from the CBD are more desirable in the short run (low rents and high housing consumption) but allow only infrequent trips to the CBD and hence reduce the long-run prospects of reemployment. Therefore, for workers residing further away from the CBD ( $x \geq x(s_0)$ ), it is optimal to spend the minimal search effort  $s_0$  whereas it is the contrary ( $s = 1$ ) for workers residing close to jobs ( $x \leq x(1)$ ). Second, this result sheds some light on the spatial mismatch hypothesis. Indeed, as stated in the introduction, distance to jobs is here harmful because it decreases search intensity. Workers who live further away from jobs spend minimal search effort because the short-run gains (low rent and large housing consumption) outweigh the long-run gains (higher probability to find a job). Third, from (3.8), it is clear that  $s(x)$  is *continuous, nonincreasing, and strictly decreasing on*  $[x(1), x(s_0)]$  (as shown in the top half of Figure 1). Over the decreasing range in particular, this function embodies the continuous trade-off described above. The optimal search intensity  $s(x)$  decreases at locations further from the CBD, as unemployed workers compensate for losses in long-run job prospects by short-run gains in net income (maintaining a constant lifetime value level,  $V_0$ ). Finally, if we take the value of  $s(x)$  for interior locations, i.e. for  $x(1) < x < x(s_0)$ , it is easy to verify that it varies negatively with commuting costs  $c$  and the lifetime value of the unemployed  $V_0$ , and positively with the hiring probability  $p_h$  and the lifetime value of the employed  $V_1$ . The intuition is straightforward since when  $c$  or  $V_0$  is high and

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<sup>12</sup>Commuting costs have to be taken here in a broader sense as long as it measures access to employment activities. For example, including time commuting costs in our framework will imply that the marginal cost of an increased search leads to a reduced leisure time.

when  $p_h$  or  $V_1$  is low, then workers reduce their search effort since either costs of searching are too high or the rewards of searching are too low. Concerning the discount rate  $\sigma$ , one can verify that it is positively correlated with  $s(x)$  so that putting more weight on today's gain increases search intensity.

Is this result consistent with empirical studies? In fact, most studies have shown that workers' search intensity is negatively related to their residential distance to jobs. For example, Seater [31] has found that workers searching further away from the residence are less productive than those who search closer to where they live. Barron and Gilley [6] and Chirinko [9] have also found that there are diminishing returns to search when people live far away from jobs. Rogers [28] has also demonstrated that access to employment is a significant variable in explaining the probability of leaving unemployment.

*[Insert Figure 1 here]*

## 4. The different urban land use equilibria

So far, we have determined the optimal search intensity of the unemployed at each location in the city. The key question now is how the urban land use equilibrium looks. In other words, knowing this function  $s(x)$ , where do the unemployed and the employed locate in the city? The basic trade-off for the employed is between commuting costs and housing consumption whereas for the unemployed, it is between commuting/search costs, housing consumption and search intensity (and thus the duration of unemployment). In order to determine the urban land use equilibrium, we have to define the bid rent function of each group of workers.<sup>13</sup>

### 4.1. Bid rents and locational equilibrium patterns

Given the utilities and lifetime values above, we now define the equilibrium bid-rents which are possible for any set of equilibrium values  $(p_h, V_0, V_1)$  with  $V_1 > V_0$ . Turning first to employed workers, we may observe from (3.2) and (3.4) that their equilibrium utility level,  $U_1$ , is constant over locations, and is given by

$$U_1 = \left( \frac{1 - \sigma}{1 - e_1} \right) (V_1 - e_1 V_0)$$

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<sup>13</sup>The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance  $x$  from the CBD is ready to pay in order to achieve the equilibrium utility level of his/her group.

$$= (1 - \sigma)V_1 + \sigma\rho(V_1 - V_0) \quad (4.1)$$

Hence it follows from the form of the indirect utility in (2.13) that the relevant *bid rent function*,  $R_1(x)$ , for employed workers at each location,  $x \in [0, \frac{w}{c}]$ , is given by the relation:

$$\begin{aligned} a(w - cx)^{\alpha+\beta} R_1(x)^{-\alpha} &= U_1 = (1 - \sigma)V_1 + \sigma\rho(V_1 - V_0) \\ \Rightarrow R_1(x) &= \left[ \frac{a(w - cx)^{\alpha+\beta}}{(1 - \sigma)V_1 + \sigma\rho(V_1 - V_0)} \right]^{\frac{1}{\alpha}} \end{aligned} \quad (4.2)$$

The bid rent function for unemployed workers is considerably more complex, in that it depends on the optimal search intensity level at each location. To specify this function observe first from (3.1) and (3.3) that the equilibrium utility,  $U_0(x)$ , at each location,  $x \in [0, \frac{b}{s_0c}]$  is given [in a manner paralleling (4.1)] by

$$\begin{aligned} U_0(x) &= \left( \frac{1 - \sigma}{1 - e_0(x)} \right) (V_0 - e_0(x)V_1) \\ &= (1 - \sigma)V_0 - s(x)\sigma p_h(V_1 - V_0) \end{aligned} \quad (4.3)$$

Hence the indirect utility in (2.14) yields the following *bid rent function*,  $R_0(x)$ , for unemployed workers at each location,  $x \in [0, \frac{b}{s_0c}]$ :

$$\begin{aligned} a[b - s(x)cx]^{\alpha+\beta} R_0(x)^{-\alpha} &= U_0(x) = (1 - \sigma)V_0 - s(x)\sigma p_h(V_1 - V_0) \\ \Rightarrow R_0(x) &= \left[ \frac{a[b - s(x)cx]^{\alpha+\beta}}{(1 - \sigma)V_0 - s(x)\sigma p_h(V_1 - V_0)} \right]^{\frac{1}{\alpha}} \end{aligned} \quad (4.4)$$

where  $s(x)$  is given by (3.8) above. [An instance of this (piecewise continuously differentiable) bid rent function is shown in the bottom half of Figure 1, where the curve represents a typical ‘slice’ through the two-dimensional rent surface].

It should be clear that the bid rents are calculated such that the lifetime utilities of both the employed and the unemployed workers, respectively,  $V_1$  and  $V_0$ , are spatially invariant. Compare for example an unemployed worker residing close to jobs and another unemployed worker living far away from jobs. The former has a lower search (commuting) cost and a higher chance to find a job but consume less land whereas the latter has a higher search (commuting) cost and a lower chance to find a job but consume more land. The bid rent defined by (4.4) exactly compensates these differences by ensuring that these two workers obtain the same lifetime utility  $V_0$ . This is not true for the current utility of the unemployed  $U_0(x)$  because, as can be seen in (4.3), the land rent

does not compensate for  $s(x)$ . In fact, the unemployed residing close to jobs have a lower current utility than the ones living far away from jobs because they provide more search intensity (indeed, using (4.3), it is easy to see that  $U'_0(x) > 0$ ). However, because they provide more search intensity, they have a higher chance to find a job, and thus in the long-run they compensate the short-run disadvantage so that all unemployed workers obtain  $V_0$ .

If there is also postulated to be an exogenous level of *agricultural rent* (or *opportunity rent*),  $R_A$ , which is uniform in space, then it follows by standard competitive arguments land at each location is assigned to the highest bidder. This implies in particular that the *equilibrium land rent function*,  $R(x)$ , must be given at all locations,  $x$ , by

$$R(x) = \max\{R_0(x), R_1(x), R_A\} \quad (4.5)$$

In addition, land at  $x$  can only be occupied by workers (employed or unemployed) if their bid rents are maximal. More precisely, if the *population densities* of employed workers and unemployed workers at  $x$  are denoted respectively by  $\eta_1(x)$  and  $\eta_0(x)$ , then at equilibrium we must have

$$\eta_i(x) > 0 \Rightarrow R_i(x) = R(x) , \quad i = 0, 1 \quad (4.6)$$

Finally, we have the usual ‘land capacity’ condition that no more land be consumed than is available, and ‘land filling’ condition that all land with rents higher than agricultural rent must be occupied by workers. To state these conditions precisely, observe that, from above, the optimal land demand for employed workers at  $x$  is given by (2.11) whereas the optimal land demand for unemployed workers at  $x$  is given by (2.12), [with  $s = s(x)$ ]. In terms of these land demands, the *land capacity condition* and *land filling condition* take the respective forms [see for example in Fujita (1989, p.102)]

$$q_0(x)\eta_0(x) + q_1(x)\eta_1(x) \leq L(x) \quad (4.7)$$

$$R(x) > R_A \Rightarrow q_0(x)\eta_0(x) + q_1(x)\eta_1(x) = L(x) \quad (4.8)$$

where  $L(x) = 2\pi x$ . Conditions [(4.6),(4.7)(4.8)] can be given a sharper form in the present model as we now show.

## 4.2. Classification of equilibrium land use patterns

With the non-linear bid rents defined by (4.2) and (4.4), different urban configurations can emerge. Indeed, the land market being perfectly competitive, all

workers propose different bid rents at different locations and (absentee) landlords allocate land to the highest bids. So depending on the different steepness of the bid rents (as captured by their slopes), at each location, the employed can outbid the unemployed or can be outbid by the unemployed. An example of the equilibrium rent function defined by (4.5) is shown in Figure 2. In particular, this figure illustrates a case where unemployed workers occupy both a central core of locations and a peripheral ring of locations about the CBD, separated by an intermediate ring of employed workers. Other urban configurations may also emerge. For example, the unemployed can occupy the core of the city and the employed the suburbs. The reverse pattern may also prevail. Since we want to focus on interesting urban configurations in which the unemployed workers can outbid the employed workers for peripheral land in equilibrium, we assume

$$w < \frac{b}{s_0} \quad (4.9)$$

Because this possibility is of considerable interest for our present purposes, we shall assume (4.9) throughout the analysis to follow.<sup>14</sup>

But while (4.9) does allow for this possibility, it is by no means sufficient. Hence the main result of this section is to show that the conditions above imply that in equilibrium there are exactly *three* possible locational configurations of workers:<sup>15</sup>

**Theorem 2 (Equilibrium Location Patterns).** *In equilibrium there are exactly three possible locational patterns:*

- (i) *a central core of unemployed surrounded by a peripheral ring of employed,*
- (ii) *a central core of employed surrounded by a peripheral ring of unemployed,*
- (iii) *both a central core and peripheral ring of unemployed separated by an intermediate ring of employed.*

This theorem shows that, in a framework where workers' search intensity is location dependent (see Proposition 1), different urban equilibrium configurations can emerge. In the first one (i), referred to as the *Integrated Equilibrium*, the unemployed reside close to the CBD, have high search intensities and experience short unemployment spells. In the second one (ii), referred to as the

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<sup>14</sup>Observe that if one relaxes condition (4.9) and instead assumes  $w > b/s_0$ , then it strongly restricts the set of urban equilibria since the equilibrium that is more likely to prevail is the one where the unemployed reside close to jobs and the employed at the periphery of the city. If condition (4.9) holds, then the analysis is much more richer since three types of urban equilibria can emerge, including the one described above.

<sup>15</sup>The proof of Theorem 2 is available in Smith and Zenou [34].

*Segregated Equilibrium*, the employed occupy the core of the city and bid away the unemployed in the suburbs. In this case, the latter tend to stay unemployed for a longer time since their search intensity is quite low. Finally, the third case (*iii*), referred to as the *Core-Periphery Equilibrium*, is when there are two categories of unemployed: the short-run ones who reside close to jobs and the long-term ones who live at the periphery of the city (see Figure 2).

In Wasmer and Zenou [37] where the relationship between search effort and distance to jobs is assumed instead of being derived (like here), only two equilibria can emerge: (*i*) and (*ii*). We would thus like now to study the third type of equilibrium, the core-periphery equilibrium, since it has not yet been investigated, even though it is quite relevant. Furthermore, this equilibrium encompasses the two other ones since the first equilibrium (*i*) is a limiting case of the core-periphery equilibrium when  $x_p = x_f$  ( $x_p$  is the border between the employed and the long term-unemployed workers, and  $x_f$  is the city-fringe; see Figure 2) while the second equilibrium (*ii*) is a limiting case of the core-periphery equilibrium when  $x_c = 0$  ( $x_c$  is the border between the short-run unemployed and the employed workers; see Figure 2).

The key question is to see under which conditions what equilibrium prevails. Since we know from (4.2) and (4.4) that both bid rents  $R_1(x)$  and  $R_0(0)$  are continuous, twice differentiable, decreasing and convex, we have:

- (1) If  $R_1(0) > R_0(0)$  and  $R_1(x_f) < R_0(x_f)$ , then there is a unique Segregated Equilibrium (*ii*);
- (2) If  $R_1(0) < R_0(0)$  and  $R_1(x_f) > R_0(x_f)$ , then there is a unique Integrated Equilibrium (*i*);
- (3) If  $R_1(0) < R_0(0)$ ,  $R'_1(0) < R'_0(0)$  and  $R_1(x_f) < R_0(x_f)$ , then there is a unique Core-Periphery Equilibrium (*iii*).

Of course, because it is so cumbersome (since  $x_f$ ,  $V_0$ ,  $V_1$ ,  $p_h$  and  $\rho$  are all endogenous variables), the exact conditions on the exogenous parameters are impossible to determine analytically. We have here implicit conditions that link endogenous and exogenous variables.

Let us now focus on the more general equilibrium (*iii*) (since the others are just a particular case of (*iii*)). We will characterize it, shows its existence and uniqueness.

[Insert Figure 2]

## 5. The Core-Periphery equilibrium

To define the core-periphery equilibrium, we first collect all the relevant parameters for the problem. For any probabilities,  $\rho, \sigma, \gamma, s_0 \in (0, 1)$ , and scalars,  $\lambda, \alpha, \beta, b, w, c, N, R_A > 0$  with  $\alpha + \beta < 1$  and  $s_0 w < b < w$ , we may define an *admissible parameter vector*,  $\theta = (\rho, \sigma, \gamma, s_0, \lambda, N, \alpha, \beta, b, w, c, R_A)$ . Next, for any given *lifetime values*,  $V_0, V_1$ , with  $V_1 > V_0$ , and *hiring probability*,  $p_h \in (0, 1)$ , we define the following set of functions. First, let the function  $s$  be defined by (3.8) with ranges,  $x(1)$  and  $x(s_0)$ , given by (3.9). In terms of  $s$  and  $(V_0, V_1, p_h)$ , we may then define the additional functions,  $U_0, R_0, R_1$ , and  $R$ , respectively by (4.3), (4.4), (4.2), (4.5). Using  $R_0, R_1$ , and  $R$ , we next define the *indicator functions*,  $\delta_i, i = 0, 1$ , specifying the relevant regions occupied by unemployed and employed workers, respectively:

$$\delta_i(x) = \begin{cases} 1 & , R_i(x) = R(x) \\ 0 & , otherwise \end{cases} \quad , \quad i = 0, 1 \quad (5.1)$$

It should be noted that the validity of this characterization of the location pattern is made possible by the more technical version of Theorem 2 (Theorem A.1 plus Lemma 5 in section A.2 of the Appendix of Smith and Zenou [34]), which shows that these indicator functions are ambiguous only on a set of measure zero [i.e., that the equality  $R_0(x) = R_1(x)$  holds only on a set of measure zero in the interval of relevant distances,  $x$ ]. Hence one can now sharpen the general set of locational equilibrium conditions [(4.6),(4.7),(4.8)] above by noting in the present case that at almost every distance,  $x$ , at most one of the population densities,  $\eta_0(x)$  and  $\eta_1(x)$ , can be positive. Hence, by substituting (2.11) and (2.12) into (4.8), and observing that by definition,  $R_i(x) = R(x)$  iff  $\delta_i(x) = 1$ , it follows that the appropriate *population densities*, must have the form

$$\eta_0(x) = \frac{L(x)}{q_0(x)} = 2\pi x \left( \frac{\alpha + \beta}{\alpha} \right) \frac{R_0(x)}{b - s(x)cx} \quad (5.2)$$

$$\eta_1(x) = \frac{L(x)}{q_1(x)} = 2\pi x \left( \frac{\alpha + \beta}{\alpha} \right) \frac{R_1(x)}{w - cx} \quad (5.3)$$

At this point, it is important to reiterate that all of the above functions are completely defined by the *lifetime values*,  $V_0, V_1$ , and *hiring probability*,  $p_h$ . With these functions, we can now give a formal general definition of equilibrium as follows:

**Definition 1 (General).** For any admissible parameter values,

$\theta = (\rho, \gamma, s_0, \lambda, N, \sigma, \alpha, \beta, b, w, c, R_A)$ , a nonnegative vector  $\xi = (V_0, V_1, p_h, u, \bar{s}, N_0, N_1)$  is said to be an equilibrium for  $\theta$  iff  $\xi$  satisfies the following five conditions [where  $d = 1 - \frac{\lambda}{\rho}$ , and where the functions  $(s, U_0, R_0, R_1, R, \delta_0, \delta_1, \eta_0, \eta_1)$  are given by the constructions above]:

$$p_h = \frac{u + d}{u\bar{s}} \left(1 - e^{-\gamma \frac{u\bar{s}}{u+d}}\right) \quad (5.4)$$

$$\rho(1 - u) = (1 - \rho)u\bar{s}p_h \quad (5.5)$$

$$\bar{s} = \frac{1}{N_0} \int s(x)\delta_0(x)\eta_0(x)dx \quad (5.6)$$

$$N_i = \int \delta_i(x)\eta_i(x)dx, \quad i = 0, 1 \quad (5.7)$$

$$N = N_0 + N_1 \quad (5.8)$$

The first two conditions follow from [(2.3),(2.4),(2.5)] and define the labor market steady state, given the mean search intensity,  $\bar{s}$ . Condition (5.6) defines  $\bar{s}$  in terms of the search intensities,  $s(x)$ , and population densities,  $\eta_0(x)$ , at each location  $x$  occupied by unemployed workers [i.e., with  $\delta_0(x) = 1$ ]. Finally, condition (5.7) defines the population totals for employed and unemployed workers, together with the accounting condition (5.8) that all workers are either employed or unemployed.<sup>16</sup>

While this definition is conceptually quite simple in that it gives a *finite-dimensional* characterization of equilibrium [in terms of the scalar variables  $(V_0, V_1, p_h, u, \bar{s}, N_0, N_1)$ ], it is not very tractable analytically. In particular, indicator functions such as  $\delta_0$  and  $\delta_1$  are difficult to analyze in practice. However, by employing Theorem 2 (and its more technical counterpart, Theorem A.1 in section A.2 of the Appendix of Smith and Zenou [34]), one can give a more explicit characterization of these indicator functions. In particular, it follows from Theorem 2 that employed workers will always live in a single connected ring, and hence that the positive support of the indicator function,  $\delta_1$ , must be closed interval,  $[x_c, x_p]$ , with end points given by<sup>17</sup>

$$x_c = \min\{x \geq 0 : \delta_1(x) > 0\} \quad (5.9)$$

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<sup>16</sup>Note that the bid-rent and population density conditions [(4.5),(4.6),(4.7),(4.8)] stated above are not made explicit in this formulation, but rather are implicit in the definitions of the indicator functions,  $\delta_0$  and  $\delta_1$ .

<sup>17</sup>Given the possibility of ‘trivial intersection points’ (as in Lemma 5 in section A.2 of the Appendix of Smith and Zenou [34]), a more technically correct version of these conditions would be to replace ‘min’ in (5.9) by ‘essential infimum’ and ‘max’ in (5.10) by ‘essential supremum’.

$$x_p = \max\{x \geq 0 : \delta_1(x) > 0\} \quad (5.10)$$

In addition, it follows that unemployed workers will live in at most two distinct rings, the first given by  $[0, x_c]$  and the second by  $[x_p, x_f]$ , where  $x_f$  is the *frontier location* (or *city edge*) as characterized by

$$x_f = \min\{x \geq 0 : R(x) = R_A\} \quad (5.11)$$

Hence, in the present case, it is possible to remove the indicator functions above, and replace [(5.6),(5.7)] by a more explicit set of conditions involving only the density functions  $(\eta_0, \eta_1)$  and the boundary variables  $(x_c, x_p, x_f)$ .

This plan is now carried out for an important subclass of equilibria (the core-periphery ones), which illustrate all the main features of the above model, and which are sufficiently tractable to allow a detailed analysis of equilibria. The equilibrium bid-rent configuration shown in Figure 2 yields a simple type of core-periphery location pattern. Notice in particular that there are only *two* search intensity levels for unemployed workers: all unemployed workers in the central core search with *full intensity*,  $s = 1$ , and all in the peripheral ring search with *minimum intensity*,  $s = s_0$ . These constant-search-intensity patterns are particularly easy to analyze, as should be evident from (3.8). Moreover, Theorem 2 shows that essentially all equilibrium properties of the system can be studied in terms of these simple cases. For in the other two possible locational patterns, it is clear that so long as the equilibrium bid-rent curves,  $R_0$  and  $R_1$ , do not cross in the region  $[x(1), x(s_0)]$ , only maximal and minimal search intensities will be involved. Moreover, the case illustrated in Figure 2, where the region  $[x(1), x(s_0)]$  is shown to be relatively small, is in fact quite typical. This assertion is supported by the following result, which shows that if utility is ‘almost linearly homogeneous’ in the sense that  $\alpha + \beta$  is close to one, then the interval  $[x(1), x(s_0)]$  is necessarily very small:

**Proposition 2.** *If  $\alpha + \beta \approx 1$ , then in equilibrium  $|x(1) - x(s_0)| \approx 0$ .*

**Proof:** It is enough to observe from (3.9) that for any given lifetime values and hiring probability  $(V_0, V_1, p_h)$ , the locations  $x(1)$  and  $x(s_0)$  have a common limiting value,  $\frac{b \sigma p_h (V_i - V_0)}{c (1 - \sigma) V_0}$ , as  $\alpha + \beta \rightarrow 1$ . ■

Hence if diminishing marginal utility (along rays) is sufficiently small, then equilibrium can be safely assumed to involve only maximal and/or minimal search intensities for unemployed workers.

With these observations, we now restrict attention to the constant-search-intensity case. In particular, we focus on the class of *core-periphery equilibria*, which involve both constant maximal search intensity in a central unemployment core  $[0, x_c]$ , and minimal search intensity in a peripheral unemployment ring  $[x_p, x_f]$ . The other two equilibrium possibilities (equilibria (i) and (ii)) with constant search intensities can then be regarded as limiting cases in which either  $x_c = 0$  or  $x_p = x_f$ .

Our first objective is to give a formal definition of core-periphery equilibria which specializes the general definition above, and which allows a more detailed analysis of both existence and uniqueness properties. To do so, we first observe from (4.3) that in equilibrium,  $U_0(x) \equiv U_0[s(x)]$ , so that each region with constant search intensity must necessarily involve constant utility. For unemployed workers in the core region (with  $s = 1$ ), this equilibrium *core utility level*,  $U_0$ , must satisfy

$$U_0^c = (1 - \sigma)V_0 - \sigma p_h(V_1 - V_0) \quad (5.12)$$

and for those in the peripheral region (with  $s = s_0$ ), the corresponding *peripheral utility level*, which we denote by  $U_0^p$ , must satisfy

$$U_0^p = (1 - \sigma)V_0 - s_0 \sigma p_h(V_1 - V_0) \quad (5.13)$$

Moreover, by evaluating (3.5) at both  $s = 1$  and  $s = s_0$ , we obtain the identity

$$\frac{(1 - \sigma + \sigma \rho)U_0^c + (\sigma p_h)U_1}{1 - \sigma + \sigma \rho + \sigma p_h} = \frac{(1 - \sigma + \sigma \rho)U_0^p + (s_0 \sigma p_h)U_1}{1 - \sigma + \sigma \rho + s_0 \sigma p_h} \quad (5.14)$$

which can be solved for  $U_0^p$  to yield

$$U_0^p = \tau(p_h)U_0^c + [1 - \tau(p_h)]U_1 \quad (5.15)$$

where

$$\tau(p_h) = \frac{(1 - \sigma + \sigma \rho) + s_0 \sigma p_h}{(1 - \sigma + \sigma \rho) + \sigma p_h} \in (0, 1) \quad (5.16)$$

It is worth noting at this point that since  $w > b$  of course implies that  $U_1 > U_0^c$  in equilibrium, and since the positivity of steady-state unemployment levels,  $u$  (Theorem 1) implies from (5.4) that steady-state hiring probabilities,  $p_h$ , are always positive, it follows from the convex combination in (5.15) that in every core-periphery equilibrium one must have

$$U_0^c < U_0^p < U_1 \quad (5.17)$$

This again underscores the essential difference between unemployed workers in the central core and those in the periphery. Those in the central core are giving up short-run utility for long-run utility gains. Hence, if the lifetime value,  $V_0$ , of all unemployed workers is the same, then the short-run utility of those in the periphery must be greater than for those in the central core. These constant utility levels  $(U_0^c, U_0^p, U_1)$  will also turn out to be more useful for analysis than the more general lifetime values  $(V_0, V_1)$ . Hence the present equilibrium conditions will be developed in terms of  $(U_0^c, U_0^p, U_1)$ .

Next we observe that the (outer) *core boundary point*,  $x_c$ , and the (inner) *peripheral boundary point*,  $x_p$ , can now be characterized as intersections between these constant-utility curves as follows. First observe that since the bid rent for core unemployed workers and employed workers must be the same at  $x_c$ , it follows from (2.13) and (2.14) that in equilibrium,

$$\frac{U_0^c}{U_1} = \left( \frac{b - cx_c}{w - cx_c} \right)^{\alpha + \beta} \quad (5.18)$$

Similarly, since the bid rent for peripheral unemployed workers and employed workers must be the same at  $x_p$ , it also follows from (2.13) and (2.14) that in equilibrium,

$$\frac{U_0^p}{U_1} = \left( \frac{b - s_0 cx_p}{w - cx_p} \right)^{\alpha + \beta} \quad (5.19)$$

A final consequence of these constant utility levels is to yield more explicit expressions for the population densities in (5.2) and (5.3). First, by solving for rent  $R(x)$  in (2.13) and substituting this into (2.11) it follows from (5.3) that the equilibrium *employment density*,  $\eta_1(x)$ , is now given for all  $x \in [x_c, x_p]$  by

$$\eta_1(x) = 2\pi x \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_1} \right)^{\frac{1}{\alpha}} (w - cx)^{\frac{\beta}{\alpha}} \quad (5.20)$$

Similarly, by setting  $s(x) = 1$ , solving for  $R(x)$  in (2.14), and substituting this into (2.12), it follows from (5.2) that the equilibrium *core unemployment density*,  $\eta_0^c(x)$ , is given for all  $x \in [0, x_c]$  by

$$\eta_0^c(x) = 2\pi x \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_0^c} \right)^{\frac{1}{\alpha}} (b - cx)^{\frac{\beta}{\alpha}} \quad (5.21)$$

The same procedure with  $s(x) = s_0$  also yields the equilibrium *peripheral unemployment density*,  $\eta_0^p(x)$ , defined for all  $x \in [x_p, x_f]$  by

$$\eta_0^p(x) = 2\pi x \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_0^p} \right)^{\frac{1}{\alpha}} (b - s_0 cx)^{\frac{\beta}{\alpha}} \quad (5.22)$$

Given these population densities and corresponding boundary points, it follows that the integrals in (5.7) can now be calculated explicitly. In particular, if  $N_1$  again denotes the equilibrium *employment level*, and if  $N_0^c$  and  $N_0^p$  now denote the equilibrium *core unemployment level* and *peripheral unemployment level*, respectively, then  $N_0^c$  can be calculated explicitly as

$$\begin{aligned}
N_0^c &= \int_0^{x_c} \eta_0^c(x) dx \\
&= 2\pi \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_0^c} \right)^{\frac{1}{\alpha}} \left\{ - \left( \frac{\alpha x_c}{c(\alpha + \beta)} \right) (b - cx_c)^{\frac{\alpha + \beta}{\alpha}} + \right. \\
&\quad \left. \left( \frac{\alpha}{c(\alpha + \beta)} \right) \left( \frac{\alpha}{c(2\alpha + \beta)} \right) \left[ b^{\frac{2\alpha + \beta}{\alpha}} - (b - cx_c)^{\frac{2\alpha + \beta}{\alpha}} \right] \right\} \quad (5.23)
\end{aligned}$$

Similarly,  $N_1$  is now given by:

$$\begin{aligned}
N_1 &= \int_{x_c}^{x_p} \eta_1(x) dx \\
&= 2\pi \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_1} \right)^{\frac{1}{\alpha}} \left\{ \left( \frac{\alpha x_c}{c(\alpha + \beta)} \right) (w - cx_c)^{\frac{\alpha + \beta}{\alpha}} - \right. \\
&\quad \left( \frac{\alpha x_p}{c(\alpha + \beta)} \right) (w - cx_p)^{\frac{\alpha + \beta}{\alpha}} + \left( \frac{\alpha}{c(\alpha + \beta)} \right) \left( \frac{\alpha}{c(2\alpha + \beta)} \right) \cdot \\
&\quad \left. \left[ (w - cx_c)^{\frac{2\alpha + \beta}{\alpha}} - (w - cx_p)^{\frac{2\alpha + \beta}{\alpha}} \right] \right\} \quad (5.24)
\end{aligned}$$

and  $N_0^p$  is given by:

$$\begin{aligned}
N_0^p &= \int_{x_p}^{x_f} \eta_0^p(x) dx \\
&= 2\pi \left( \frac{\alpha + \beta}{\alpha} \right) \left( \frac{a}{U_0^p} \right)^{\frac{1}{\alpha}} \left\{ \left( \frac{\alpha x_p}{s_0 c (\alpha + \beta)} \right) (b - s_0 cx_p)^{\frac{\alpha + \beta}{\alpha}} - \right. \\
&\quad \left( \frac{\alpha x_f}{s_0 c (\alpha + \beta)} \right) (b - s_0 cx_f)^{\frac{\alpha + \beta}{\alpha}} + \left( \frac{\alpha}{s_0 c (\alpha + \beta)} \right) \left( \frac{\alpha}{s_0 c (2\alpha + \beta)} \right) \cdot \\
&\quad \left. \left[ (b - s_0 cx_p)^{\frac{2\alpha + \beta}{\alpha}} - (b - s_0 cx_f)^{\frac{2\alpha + \beta}{\alpha}} \right] \right\} \quad (5.25)
\end{aligned}$$

Given these equilibrium population levels, we next observe that the single most important simplification made possible by present constant-search-intensity hypothesis is the determination of the equilibrium *mean search intensity level*,  $\bar{s}$ . In particular, since the relevant search intensity function  $s(x)$  has only two values, it now follows that equilibrium condition (5.6) can be replaced by the much simpler form

$$\bar{s} = \frac{N_0^c + s_0 N_0^p}{N_0^c + N_0^p} \quad (5.26)$$

Hence the steady-state model of the labor market can be completely specified in terms of the three population variables  $(N_0^c, N_0^p, N_1)$ . In particular, since the equilibrium *unemployment rate*,  $u$ , is given by

$$u = \frac{N_0^c + N_0^p}{N} = \frac{N - N_1}{N} \quad (5.27)$$

it follows that the inverse relation in (2.6) now yields a single equilibrium condition relating  $N_0^c$  and  $N_0^p$ :

$$\frac{N_0^c + s_0 N_0^p}{N_0^c + N_0^p} = \psi \left( \frac{N - N_1}{N} \right) \quad (5.28)$$

This, together with the accounting identity

$$N_0^c + N_0^p + N_1 = N \quad (5.29)$$

allows one to determine unique values of  $N_0^c$  and  $N_0^p$  for each employment level,  $N_1$ . In addition, by substituting (5.26) and (5.27) into (5.4) it follows that the hiring probability  $p_h$  can then be determined as

$$p_h = \frac{N_0^c + s_0 N_0^p}{N_0^c + N_0^p + Nd} \left( 1 - e^{-\gamma \frac{N_0^c + s_0 N_0^p}{N_0^c + N_0^p + Nd}} \right) \quad (5.30)$$

To complete the equilibrium conditions for the present core-periphery case, recall that the boundary points,  $(x_c, x_p, x_f)$  must satisfy certain additional consistency conditions. First, it follows by hypothesis that full search intensity,  $s = 1$ , is optimal for core unemployed workers, and hence from (3.8) that the core boundary point,  $x_c$ , must satisfy

$$x_c \leq x(1) \quad (5.31)$$

Similarly, minimal search intensity,  $s = s_0$ , is assumed to be optimal for peripheral unemployed workers, so that the peripheral boundary point,  $x_p$ , must satisfy

$$x_p \geq x(s_0) \quad (5.32)$$

Finally, it also follows by definition that bid rent for peripheral unemployed workers must equal the agricultural rent,  $R_A$ , at the frontier location,  $x_f$ . Hence, by letting  $x = x_f$ ,  $s(x) = s_0$ , and  $U_0(x) = U_0^p$  in (2.14) [or (4.4)] it follows that at the frontier location we must have

$$R_A = \left( \frac{a}{U_0^p} \right)^{\frac{1}{\alpha}} (b - s_0 c x_f)^{\frac{\alpha+\beta}{\alpha}} \quad (5.33)$$

This completes the set of equilibrium conditions for the core-periphery case. Hence we have:

**Definition 2 (CP-Equilibrium).** For any admissible parameter vector,  $\theta = (\rho, \gamma, s_0, \lambda, N, \sigma, \alpha, \beta, b, w, c, R_A)$ , a vector of values,  $\xi = (N_0^c, N_0^p, N_1, p_h, U_0^c, U_0^p, U_1, x_c, x_p, x_f)$ , is said to be a core-periphery (CP) equilibrium for  $\theta$  iff conditions [(5.15), (5.18), (5.19), (5.23), (5.24), (5.25), (5.28), (5.29), (5.30), (5.31), (5.32), (5.33)] are satisfied.

Given this definition, we are able to show the existence and the uniqueness of the core-periphery (CP) equilibrium.<sup>18</sup>

## 6. Discussions and policy implications

In our model, there is room for government intervention because, as in the standard search-matching literature (Mortensen and Pissarides [24], Pissarides [26]), market failures are caused by search externalities. There are in fact two types of search externalities: *negative intra-group externalities* (more searching workers reduces the job-acquisition rate) and *positive inter-group externalities* (more searching firms increases the job-acquisition rate).

We would like now to show how the present paper provides a new economic mechanism for the spatial mismatch hypothesis and thus new policy implications. Since our goal is to give a theoretical explanation of the spatial mismatch hypothesis, we focus now on equilibria (ii) (the *Segregated Equilibrium*) and (iii) (the *Core-Periphery Equilibrium*) because in both cases some (or all) unemployed reside far away from jobs.

In both the segregated equilibrium and the core-periphery equilibrium, the unemployed *decide* to reside far away from jobs and thus *voluntarily choose* low amounts of search and long-term unemployment. In this context, the standard US-style mismatch arises because inner-city blacks choose to remain in the inner-city and search only little. They do not relocate to the suburbs (in our model this is the core, but in the US mismatch it is the suburbs) because the short-run gains (low rent and large housing consumption) outweigh the long-run gains of residing near jobs (higher probability of finding a job). As a result, *in both the segregated equilibrium and the core-periphery equilibrium, the spatial mismatch stems from voluntary choices of workers and not from imposed restrictions such as housing discrimination.*

Observe that in both equilibria, the unemployed workers provide too little search effort and thus tend to have long unemployment spells because they prefer short-run over long-run gains. In other words, *the opportunity costs*

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<sup>18</sup>The proof of existence and uniqueness is available in Smith and Zenou [34].

(captured here by land rents, density and leisure time or commuting costs) of not working (or even not participating to the labor market) are too low to motivate these workers to search more. As a result, moving these workers to other areas where these opportunity costs are higher (higher land rents, lower commuting costs) will induce them to provide higher search levels. “Moving to Opportunity” (MTO) programs are thus the correct policy device to reduce mismatch, rather than lowering search costs in some other way.<sup>19</sup>

There have been several MTO programs implemented in the U.S. The starting point was the Gautreaux program, implemented in 1976 in the Chicago metropolitan area, which gave housing assistance (i.e. vouchers and certificates) to tenants in order to help diminish the financial constraints preventing low-income families from relocating to better neighborhoods (Goering, Stebbins and Siewert [13], Turner [36]). Using quasi-experimental methods, the different evaluations of the Gautreaux program suggest that the displaced workers greatly improve their educational as well as their labor market outcomes (Rosebaum [29]). However, one of the main drawbacks of the Gautreaux program was that blacks were less likely to move because of racial discrimination in the housing market. More recently, the MTO program has been launched by the U.S. Department of Housing and Urban Development (HUD) in Baltimore, Boston, Chicago, Los Angeles and New York since 1994. In these programs, the housing discrimination problem was overcome through the provision of additional services such as housing counseling and landlord outreach. To avoid selection biases, participating families were randomly assigned to one of three groups: (i) the ‘experimental’ or ‘MTO’ group, which received housing assistance and mobility counseling and was required to move to low-poverty neighborhoods (i.e. tracts with a population poverty rate not exceeding 10%); (ii) the ‘comparison’ or ‘Section 8’ group, which received housing assistance

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<sup>19</sup>The policy implications would have been quite different if residential segregation had been the result of voluntary choices of workers wishing to share a common culture with their neighbors or to interact in their own language (see among others Akerlof [1], Akerlof and Kranton [2], Ihlanfeldt and Scafidi [17], Selod and Zenou [32] and Battu, McDonald and Zenou [3], who have all emphasized the importance of voluntary choices in the explanation of urban segregation of black workers). If, for example, black workers voluntarily want to live together, then it is difficult to move them to predominant white areas. In the present model, in particular in the segregated equilibrium, location choices are decided by comparing short-versus long-term gains and there is no desire to live among similar workers (the extension to black and white workers is straightforward). So workers are ready to move and will then benefit from the policy since it changes the trade offs and induces them to provide higher effort levels.

and could move anywhere; and (iii) the ‘control’ group, which received no vouchers or certificates and could move on their own. The results of this MTO program for most of the five cities mentioned above show a clear improvement of the well-being of participants and better labor market outcomes (Ladd and Ludwig [22], Katz, Kling and Liebman [20], Rosenbaum and Harris [30]).

Our paper is obviously very much in favor of the MTO programs. In light of our results, it predicts that, relative to the ‘control’ group, displaced workers (from low- to high-rental-housing areas) should provide higher search effort. If labor market participation is a good ‘proxy’ for search effort, then the findings of Rosenbaum and Harris [30] confirm the predictions of our model. Indeed, using the survey data from the MTO program in Chicago, the findings of these authors, based on interviews an average of 18 months after families moved from public housing to higher rental housing areas, show an increase in labor force participation and employment. More precisely, Rosenbaum and Harris [30] show that: ‘After moving to their new neighborhoods, the Section 8 respondents were far more likely to be actively participating in the labor force (i.e. working or looking for a job), while for MTO respondents, a statistically significant increase is evident only for employment per se.’

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## A. Appendix. Proof of Proposition 1

Let us first show that there is a unique maximum at each location  $x$ . To solve this problem, we begin by partially differentiating (3.7) with respect to  $s$ ,

$$\frac{\partial}{\partial s} V_0(s, x) = \frac{-a(\alpha + \beta)(b - scx)^{\alpha + \beta - 1} R(x)^{-\alpha} cx + \sigma p_h V_1 - V_0(s, x) \sigma p_h}{1 - \sigma + \sigma p_h s} \quad (\text{A.1})$$

Hence the first-order condition,  $(\partial/\partial s)V_0(s, x) = 0$ , is seen to hold iff the numerator is zero, which [by using (2.14)] can be rewritten as

$$\frac{U_0(s, x)(\alpha + \beta)cx}{b - scx} = \sigma p_h [V_1 - V_0(s, x)] \quad (\text{A.2})$$

or equivalently

$$-\frac{\partial U_0(s, x)}{\partial s} = \sigma p_h [V_1 - V_0(s, x)] \quad (\text{A.3})$$

To establish the uniqueness of solutions to (A.2) we partially differentiate (A.1) once more [and substitute (A.1) into the result] to obtain:

$$\frac{\partial^2}{\partial s^2} V_0(s, x) = \frac{-aR(x)^{-\alpha}(\alpha + \beta)[1 - (\alpha + \beta)](b - scx)^{\alpha+\beta-2}cx - 2\sigma p_h \frac{\partial}{\partial s} V_0(s, x)}{1 - \sigma + \sigma p_h s} \quad (\text{A.4})$$

Finally, observing that the sign of (A.4) depends on the numerator, and that the first term in the numerator negative (for positive net incomes) we may conclude that

$$\frac{\partial}{\partial s} V_0(s, x) \geq 0 \Rightarrow \frac{\partial^2}{\partial s^2} V_0(s) < 0 \quad (\text{A.5})$$

In particular this implies that stationary points of (3.7) can only be local maxima, and thus [by continuity of (A.1)] that there is at most one stationary point. Thus, at each location  $x$  there is *at most one solution to (A.2)*.

Let us now prove the second part of the proposition.

First, note that in equilibrium this optimal lifetime value must agree with the prevailing lifetime value,  $V_0$ , for unemployed workers, i.e., that  $V_0(s, x) = V_0$  in (A.2). Note also from (3.1) and (3.3) that in equilibrium we must have

$$U_0(s, x) = (1 - \sigma)V_0 - \sigma p_h (V_1 - V_0) \quad (\text{A.6})$$

Hence, by substituting these results into (A.2) and solving for  $s$ , we obtain

$$s(x) = \frac{\alpha + \beta}{1 - (\alpha + \beta)} \left[ \frac{b}{(\alpha + \beta)cx} - \frac{(1 - \sigma)V_0}{\sigma p_h (V_1 - V_0)} \right] \quad (\text{A.7})$$

with unique inverse function,  $x(s)$ , given by (3.9).

In terms of this inverse function (3.9), it follows at once from (A.1) that

$$\frac{\partial}{\partial s} V_0(s, x) \geq 0 \Leftrightarrow x \leq x(s) \quad (\text{A.8})$$

Let us now prove parts (i), (ii), and (iii) of (3.8). They are established respectively as follows:

- (i) [ $x < x(1)$ ] Observe from (A.8) and (A.5) that  $x < x(1) \Rightarrow \frac{\partial}{\partial s} V_0(1, x) > 0 \Rightarrow \frac{\partial^2}{\partial s^2} V_0(1, x) < 0$ , so that  $V_0(\cdot, x)$  must be increasing near  $s = 1$ . Hence if there is some  $s_1 \in [s_0, 1)$  with  $V_0(s_1, x) > V_0(1, x)$ , then it follows from the continuity of (A.1) that  $V_0(\cdot, x)$  must achieve a differentiable minimum at some point interior to  $[s_1, 1]$ . But since this contradicts (A.5), it follows that no such  $s_1$  can exist, and hence that  $V_0(1, x)$  is maximal.

- (ii) [ $x > x(s_0)$ ] Again by (A.8),  $x > x(s_0) \Rightarrow \frac{\partial}{\partial s} V_0(s_0, x) < 0$ , so that  $V_0(\cdot, x)$  must be decreasing near  $s = s_0$ . Hence if there is some  $s_1 \in (s_0, 1]$  with  $V_0(s_1, x) > V_0(s_0, x)$ , then it again follows from the continuity of (A.1) that  $V_0(\cdot, x)$  must achieve a differentiable minimum interior to  $[s_0, 1]$ , which contradicts (A.5). Thus  $V_0(s_0, x)$  must be maximal.
- (iii) [ $x(1) \leq x \leq x(s_0)$ ] Finally, it also follows from (A.8) that  $x(1) \leq x \Rightarrow \frac{\partial}{\partial s} V_0(1, x) \geq 0$ , and  $x \leq x(s_0) \Rightarrow \frac{\partial}{\partial s} V_0(s_0, x) \leq 0$ , so that by continuity there is some  $s \in [s_0, 1]$  with  $\frac{\partial}{\partial s} V_0(s, x) = 0$ . Hence  $s = s(x)$  in (A.7), and we may conclude from the uniqueness of differentiable maxima observed above that  $V_0[s(x), x]$  must be maximal. ■

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**Figure 1: Bid Rent for the Unemployed**

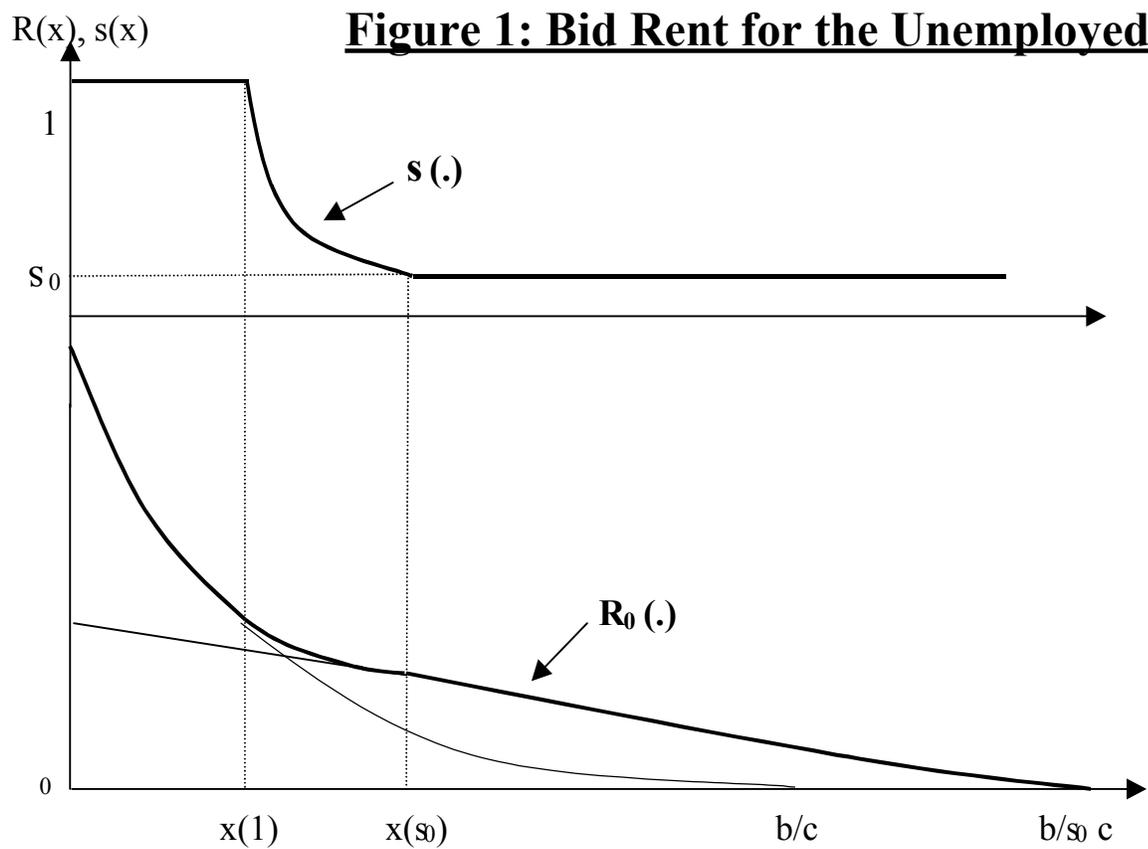


Figure 2: Core-Periphery Equilibrium

