Sensor Network Devolution and Breakdown in Survivor Connectivity

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Abstract — **As batteries fail in wireless sensor networks there is an inevitable devolution of the network characterised by a breakdown in connectivity between the surviving nodes of the network. A sharp limit theorem characterising the time at which this phenomena makes an appearance is derived.**

I. System Model

Previous work on dense sensor networks have concentrated on establishing initial connectivity or coverage. There is much less known, however, about how connectivity between the surviving nodes devolves as nodes degrade and fail over time, primarily due to limited battery power at the nodes.

We consider a sensor field comprised of a circle of unit radius in which n sensors are to be dispersed. We will suppose that each sensor can communicate with any other sensor located within a distance r from it. As design parameters we will suppose that the transmission radius $r = r_n$ is a suitably decaying function of the number of sensors n. A known result asserts indeed that $\frac{\log n}{n}$ is a threshold function for the radius at which network connectivity appears abruptly (cf. Gupta and Kumar [1] for the result in the current framework). We will be concerned mainly with the situation when the graph is initially connected.

Each sensor is equipped with a battery which has a finite lifetime determined by the usage patterns of the sensor and the selected transmission radius. We will suppose that the battery lifetimes are independent random variables with a common distribution $G_r(t)$ for the probability that the battery lifetime exceeds t.

II. Breakdown in Survivor Connectivity

The degradation of the network due to sensor losses in time also manifests itself ultimately in a breakdown in connectivity. At the simplest level, such a breakdown occurs when a live node is isolated though connectivity may have broken down before such an occurrence. More formally, what can be said about the connectivity of the *network of survivors* at a given time t? In particular, how long will the network of survivors remain connected in the face of continuing losses?

It is fruitful to think of the setting as follows. Initially, one starts with a connected spatial random graph on n vertices. (Of course, we are assuming tacitly that the communication radius r_n exceeds the critical threshold $\log(n)/n$ so that the network is connected.) At time t a random fraction of the nodes has expired leaving a collection of $S(t)$ survivors with the induced subgraph on those vertices. The situation may be arrived at by an equivalent probabilistic game in which random deletions of vertices (and associated edges) are performed on the original graph with each vertex removed independently from the graph with probability $1 - G(t)$. The number of survivors $S(t)$ is hence binomially distributed with parameters n and $G(t)$. The de Moivre-Laplace theorem tells us that $S(t)$ is concentrated around its mean value $nG(t)$. It can be shown that $S(t) = nG(t) + \mathcal{O}(n^{1/2+\epsilon})$ with asymptotic probability close to 1.

Condition on $S(t) = s$ survivors where $s = nG(t) + \zeta$ and $\zeta = \mathcal{O}(n^{1/2+\epsilon})$. As deletions are performed independently, the locations of the s survivors are independent of each other and uniformly distributed in the unit circle. It follows that $\log(s)/s$ is a threshold function for the transmission radius to ensure survivor connectivity. More precisely, let $\omega(s)$ be any slowly growing function of s. Bear in mind that the transmission radius is still the originally set radius r_n and that $s \sim nG(t)$. We hence obtain that the survivors are disconnected with asymptotic probability approaching 1 if $r_n \leq \bigl(\log(s) - \omega(s)\bigr)/s$ while the survivors are connected with asymptotic probability approaching 1 if $r_n \geq \bigl(\log(s) + \omega(s)\bigr)/s$. Write $\nu = nG(t)$ and take expectation with respect to s to get rid of the conditioning. The concentration of the binomial allows us to focus on $s \sim nG(t)$. It follows that a threshold function for the radius is $\log(\nu)/\nu$ to ensure survivor connectivity. Inverting the system we obtain that *the critical region of time $t = t_n$ where the survivor network passes from connected to disconnected satisfies* $G_{r_n}(t_n) \sim \frac{1}{nr_n} \log \frac{1}{r_n}$.

For concreteness, if the failure distribution is the memoryless distribution seen earlier with $G_{r_n}(t) = e^{-\alpha r_n^4 t}$ and r_n is initially set at just above the critical connectivity threshold, then survivor connectivity breaks sharply around $t_n = n^2 \log\log(n)/\alpha \log^2 n$. More precisely, for any $\epsilon > 0$, the probability that the survivors are connected tends to 1 if $t_n \leq (1-\epsilon)n^2 \log\log(n)/\alpha \log^2 n$ while the probability that the survivors are disconnected tends to 1 if $t_n \geq (1+\epsilon)n^2 \log\log(n)/\alpha \log^2 n$.

References

[1] P. Gupta and P. R. Kumar. Critical power for asymptotic connectivity in wireless networks. *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming*, pages 547–566, 1998.