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A Relaxation Approach to Dynamic Sensor Selection in Large-Scale Wireless Networks

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Abstract

Wireless sensor networks (WSNs) require more complex sensor selection strategies than other distributed networks to perform optimal state estimation. In addition to constraints associated with distributed state estimation, wireless sensor networks have limitations on bandwidth, energy consumption, and transmission range. This paper introduces and empirically evaluates a dynamic sensor selection strategy. A discrete-time Kalman filter is used for state estimation. At each time step, a subset of sensors is selected to gather data on the following time step because of power and bandwidth constraints that prohibit using all of the sensors. A standard criterion for selecting this subset of sensors is to maximize the information to be gained by minimizing a function of the next-step error covariance matrix. We propose a relaxation of this non-convex combinatorial optimization problem and demonstrate its applicability to large-scale sensor networks. The proposed dynamic sensor selection strategy is compared empirically to other dynamic and static sensor selection strategies with respect to state estimation performance of a convection-dispersion field arising from the problem of surface-based monitoring of CO_2 sequestration sites.

1. Introduction

The convergence of sensing, computing and communication in low cost, low power devices is enabling a revolution in the way we interact with the physical world. When networked together in Wireless Sensor Networks (WSNs), such technology allows interaction with the physical world at a level of spatial and temporal granularities unthinkable just a few years back. There has been a surge of applications for long-term monitoring of dynamic fields such as environmental conditions or critical chemical concentrations over large physical spaces. In these applications, algorithms like Kalman filters can be used to perform model-based state estimation based on lumped-parameter models of the physical phenomena. WSN operating constraints often make it difficult, however, to collect data from every sensor at the sampling rate required for effective monitoring. Data rates from individual sensors may be limited by low power requirements for long-term operation, which can be on the order of years, and transmission bandwidth may limit the number of sensors from which data can be collected for each sampling period.

These considerations have led to the development of dynamic sensor selection strategies that choose a subset of sensors to report at each sampling period with the objective of rotating the requests for sensor readings while reducing the state estimation error from the Kalman filter. Estimation error reduction is usually achieved by minimizing a function of the one-step estimation error covariance matrix for each sample. This is in general a non-convex, combinatorial optimization problem. However, heuristic or approximate methods can be used to obtain a suboptimal solutions with reasonable computation.

Previous researchers have addressed this problem in various ways. [13] applies V-lambda filtering to choose between *a priori* selection matrices, determined to adequately represent the sensor space. [11] and [6] perform a greedy entropy minimization problem using Bayesian filtering to select an optimal sensor set for object tracking. [7] applies a branch and bound method for determining the best single sensor selection at each time step. [9] presents a stochastic algorithm for selecting sensors, which takes into account the accuracy of a sensor measurement. Other sensor selections schemes are surveyed in [15]. The principle shortcoming of the previous work is scalability to large-scale networks and large-scale state estimation problems. None of the demonstrations thus far have considered networks larger than 70 sensors or state spaces with more than 10 variables.

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This paper introduces a relaxation approach to dynamic sensor selection for large-scale WSNs and large-scale dynamic fields. The combinatorial nature resides in the fact that the choice of sensors is binary. In our approach, the elements of the sensor selection matrix are continuous variables ranging from zero to one. This is sufficient to yield a convex optimization problem. The sensor subset of size M is then selected to correspond to the largest M values in the optimal continuous-parameter sensor selection matrix. We implemented this technique to the problem of surface monitoring of CO_2 sequestration sites and compared our simulation results to existing state-of-the-art methods.

The following section introduces notation and formulates the problem of dynamic sensor selection for state estimation using Kalman filters. Section 3 presents the proposed relaxation method for dynamic sensor selection. Section 4 develops a lumped-parameter state-space model for the convection-dispersion of atmospheric gas concentrations as the basis for WSN monitoring of CO_2 sequestration sites. Section 5 presents simulation results for a field of 81 sensors. The final section summarizes the work presented in this paper and identifies directions for future research.

2. Problem Formulation

State estimation for discrete-time dynamic systems can be accomplished with reduced-order sensing [1]. We begin by modeling discrete-time dynamic systems in state space form as:

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + w(k) \\ y(k) &= C_k x(k) + v(k), \end{aligned} \tag{1}$$

where $w(k)\ N(0, W_k)$ is the input noise, $v(k)\ N(0, V_k)$ is the measurement noise, and A_k, B_k, and C_k are matrices of dimension $n \times n$, $n \times m$, and $l \times n$ respectively at time step k. To introduce reduced-order sensing, the discrete-time dynamic system in (1) becomes:

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + w(k) \\ y(k) &= Z_{k|k-1} C_k x(k) + Z_{k|k-1} v(k), \end{aligned} \tag{2}$$

where $Z_{k|k-1}$ is the sensor selection matrix at time step k based on the information available at time step $k-1$. $Z_{k|k-1}$ has the following properties :

- dimension $p \times l$ where $p \leq l$
- $rank(Z_{k|k-1}) = p$
- contains exactly one entry of 1 per row (and the rest 0)
- $Z_k Z_k^T = I$

We propose a discrete-time Kalman filter to perform state estimation for the state variables in (2). Assuming the system in (2), the governing equations for a discrete-time Kalman filter can be written as:

$$\begin{aligned} \hat{x}_{k+1|k} &= A_k \hat{x}_{k|k} + B_k u_k \\ P_{k+1|k} &= A_k P_{k|k} A_k^T + W_k \\ K_k &= P_{k|k-1} C_k^T Z_{k|k-1}^T (Z_{k|k-1} C_k P_{k|k-1} C_k^T Z_{k|k-1}^T \\ &\quad + Z_{k|k-1} V_k Z_{k|k-1}^T)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (y(k) - Z_{k|k-1} C_k \hat{x}_{k|k-1}) \\ P_{k|k} &= P_{k|k-1} - K_k Z_{k|k-1} C_k P_{k|k-1}, \end{aligned} \tag{3}$$

where $\hat{x}_{k+1|k}$ and $\hat{x}_{k|k}$ are the *a priori* and *a posteriori* state variable estimate for $x(k)$, $P_{k|k}$ is the error covariance, and K_k is the Kalman gain. W_k is the input noise covariance and V_k is the measurement noise covariance (assumed to be diagonal) [10]. A discrete algebraic riccati equation (DARE) results from substituting K_k and $P_{k+1|k}$ into the covariance measurement update $P_{k|k}$. Applying the matrix inversion lemma [17] and simplifying, the DARE becomes:

$$\begin{aligned} P_{k+1|k+1} = ((A_k P_{k|k} A_k^T + W_k)^{-1} \\ + C_{k+1}^T V_{k+1}^{-1} Q_{k+1|k} C_{k+1})^{-1}, \end{aligned} \tag{4}$$

where $Q_{k+1|k} = Z_{k+1|k}^T Z_{k+1|k}$ is a diagonal matrix. $Q_{k+1|k}[i,i] = 1$ if the sensor corresponding to $y_i(k+1)$ is selected and $Q_{k+1|k}[i,i] = 0$ otherwise, where $Q_{k+1|k}[i,i]$ is the ith diagonal element of $Q_{k+1|k}$. Consequently, $Z_{k+1|k}$ is the matrix formed by the p rows of $Q_{k+1|k}$ containing a 1.

3. Dynamic Sensor Selection Strategy

The 1-step optimal sensor selection strategy at time step k is the collection of sensors maximizing the information related to the state variable values at time step $k+1$. One information-maximizing method is to minimize the *trace* of the next step error covariance matrix, $P_{k+1|k+1}$ [3]. But $P_{k+1|k+1}$ is not a convex function with respect to the sensor selection matrix $Z_{k+1|k}$. This makes the problem intractable for large-scale applications. As a computationally feasible solution, we propose a suboptimal dynamic sensor selection strategy via a relaxation of the original optimization problem. To begin, let us introduce the matrix $\hat{Q}_{k+1|k}$ with the following properties:

- $\hat{Q}_{k+1|k}$ is diagonal.
- $trace(\hat{Q}_{k+1|k}) = p$.

- $0 \leq \hat{Q}_{k+1|k}[i,i] \leq 1, \forall i \in [1,l]$.

Substituting $\hat{Q}_{k+1|k}$ in (4) yields:

$$\begin{aligned}\bar{P}_{k+1|k+1} = ((&A_k P_{k|k} A_k^T + W_k)^{-1} \\ &+ C_{k+1}^T V_{k+1}^{-1} \hat{Q}_{k+1|k} C_{k+1})^{-1}\end{aligned} \tag{5}$$

In (5) $\bar{P}_{k+1|k+1}$ is an estimate of $P_{k+1|k+1}$. To improve computation complexity, we introduce $\hat{P}_{k+1|k+1}$ as:

$$\hat{P}_{k+1|k+1} = 3I - 3\bar{P}_{k+1|k+1}^{-1} + (\bar{P}_{k+1|k+1}^{-1})^2, \tag{6}$$

where $\hat{P}_{k+1|k+1}$ approximates $\bar{P}_{k+1|k+1}$ using a 2nd-order matrix inversion approximation [16].

Using this relaxation, the $trace(\hat{P}_{k+1|k+1})$ becomes a convex function of $\hat{Q}_{k+1|k}$, and its minimum can be easily found using convex optimization. According to our relaxation, the selected sensors in $Q_{k+1|k}$ will be the largest p elements of $\hat{Q}_{k+1|k}$. This selection strategy is summarized below in 2 steps:

- Optimization Step: The diagonal elements of $\hat{Q}_{k+1|k}$ are chosen, through convex optimization, to minimize $trace(\hat{P}_{k+1|k+1})$.
- Selection Step: Choose $Q_{k+1|k}$ such that $Q_{k+1|k}[i,i] = 1$ if $\hat{Q}_{k+1|k}[i,i]$ is one of the largest p elements of $\hat{Q}_{k+1|k}$, otherwise, $Q_{k+1|k}[i,i] = 0$.

The proposed dynamic sensor selection strategy is compared to the optimal solution in the following sections for a discrete-time system of dimension n by varying the number of sensors p.

4. Surface Monitoring of CO_2 Concentrations

Most experts claim greenhouse gases produced from burning fossil fuels are adversely affecting the environment. The United States Department of Energy is especially interested in carbon dioxide CO_2 sequestration to reduce emissions of this gas produced from burning fossil fuels in power generation plants [2]. This is usually accomplished by storing CO_2 in underground sites. WSNs can be employed to monitor sequestration sites for leaks [18]. Due to the nature of the problem the monitoring needs to be performed over large areas. Due to power and communication constraints, acquiring data from every sensor each sampling period is impractical. Thus, near-optimal sensor selection strategies become crucial in order to accurately estimate the CO_2 concentration level with limited sensing.

To develop a discrete-time finite-dimensional model for surface CO_2 leakage, we begin with the following continuous-time partial differential equation (PDE) describing a convection-dispersion process [4]:

$$\frac{\delta c(p,t)}{\delta t} + \phi(p,t)\frac{\partial c(p,t)}{\partial p} = \alpha(p,t)\frac{\partial^2 c(p,t)}{\partial p^2}, \tag{7}$$

with the surface (z = 0) boundary condition

$$-\alpha_z(p,t)\frac{\delta c(p,t)}{\delta z}\ |_{p=(x,y,0)} = \lambda(x,y,t)$$

where $\phi(p,t) = [\phi_x(p,t), \phi_y(p,t), \phi_z(p,t)]^T$ and $\alpha(p,t) = [\alpha_x(p,t), \alpha_y(p,t), \alpha_z(p,t)]^T$ are the advection and dispersion coefficients, respectively, and $\lambda(x,y,t)$ is the boundary condition at $z = 0$ (the surface).

We assume the sensors are on the surface. Since there are no observations at $z > 0$, an approximation is needed for $\frac{\delta^2 c(p,t)}{\delta z^2}$ at $z = 0$. This is achieved by assuming:

- $\frac{\delta^2 c(p,t)}{\delta x^2} = \frac{\delta^2 c(p,t)}{\delta y^2} = \frac{\delta^2 c(p,t)}{\delta z^2}$, (symmetric diffusion).

Applying these assumptions to (7), we use the following estimate for $\frac{\delta^2 c(p,t)}{\delta z^2}$ at $z = 0$:

$$\frac{\delta^2 c}{\delta z^2} = \frac{\frac{\delta c}{\delta t} + \phi_x\frac{\delta c}{\delta x} + \phi_y\frac{\delta c}{\delta y} - \frac{\phi_z}{\alpha_z}\lambda(x,y,t)}{\alpha_x + \alpha_y + \alpha_z}, \tag{8}$$

where, for convenience, we write c to mean $c(p,t)$ and similarly for α and ϕ. Substituting (8), into (7) yields:

$$\begin{aligned}\frac{\delta c}{\delta t} = \frac{\alpha_x + \alpha_y + \alpha_z}{\alpha_x + \alpha_y}&\left(\alpha_x\frac{\delta^2 c}{\delta x^2} + \alpha_y\frac{\delta^2 c}{\delta y^2}\right) \\ &-\phi_x\frac{\delta\ c}{\delta x} - \phi_y\frac{\delta\ c}{\delta y} + \frac{\phi_z}{\alpha_z}\lambda(x,y,t),\end{aligned} \tag{9}$$

with boundary and initial conditions :

$$\begin{aligned}c(\infty,\infty,0,t) &= c_0 \\ c(x,y,0,0) &= c_0.\end{aligned}$$

Equation (9) is similar to the PDE described in [5], differing by scaling only. The scaling in (9) accounts for the nonmeasurable effects of vertical advection and diffusion inherent in atmospheric monitoring problems where planar sensing is assumed.

For simplicity, we assume the sensors are placed on the surface in an $N \times M$ grid. These points define the locations of the concentrations used as state variables in a lumped-parameterize model of the concentration dynamics. This discrete-time state space model for (9) is created by applying an Euler approximation to (9) and discretizing in time [8] [14], given linear, time-varying dynamics:

$$\begin{aligned}x(k+1) &= A_k x(k) + B_k u(k) + w(k) \\ y(k) &= x(k) + v(k),\end{aligned} \tag{10}$$

where $x(k) = [c_1(k), c_2(k), \ldots, c_{(n-1)M+m}(k), \ldots, c_{NM}(k)]^T$ is the set of concentration state variables corresponding to $c(n\Delta_x, m\Delta_y, 0, k\Delta_t)$, $u(k) = \{\bar{\lambda}_1(k), \bar{\lambda}_2(k), \ldots, \bar{\lambda}_{(n-1)M+m}(k), \ldots, \bar{\lambda}_{NM}(k)\}$ is the set of source inputs corresponding to the values of $\lambda(p, k\Delta_t)$ on the surface regions $p \in \{[(n-1)\Delta_x/2, (n+1)\Delta_x/2], [(m-1)\Delta_y/2, (m+1)\Delta_y/2], 0\}$, $w(k)$ and $v(k)$ are the process and measurement noise, respectively, $y(k)$ are the noisy measurements, and $n \in \{1, \ldots, N\}$ and $m \in \{1, \ldots, M\}$. A_k and B_k are both square matrices of dimension $NM \times NM$ representing the lumped parameter state dynamics governing $x(k)$ according to (9).

In this application, we do not have $apriori$ knowledge of the sources. We consider the onset of constant leaks, which are the inputs to the system, $u(k)$. To apply Kalman filtering to detect the values of these system inputs, we augment the state vector with the input vectors. Thus, the discrete-time state space system in (10) is written as :

$$\begin{aligned} z(k+1) &= \hat{A}_k z(k) \\ y(k) &= \hat{C}\left(z(k) + v(k)\right), \end{aligned} \tag{11}$$

where

$$z(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}$$
$$\hat{A}_k = \begin{bmatrix} A_k & B_k \\ 0 & I \end{bmatrix}$$
$$\hat{C} = \begin{bmatrix} I & 0 \end{bmatrix}$$

The state space system in (11) has no inputs, only states. The Kalman filter described in (3) estimates $z(k)$ based on the noisy sensor measurements $y(k)$. For this convection-dispersion example, the DARE in (4) becomes :

$$P_{k+1|k+1} = \left[\left(\hat{A}_k P_{k|k} \hat{A}_k^T\right)^{-1} + \hat{C}^T V_{k+1}^{-1} Q_{k+1|k} \hat{C}\right]^{-1}. \tag{12}$$

Using (11) and (12), the following section shows simulation results for the near-optimal dynamic sensor selection strategies outlined in the previous section.

5. Simulation Results

The dynamic sensor selection strategy was implemented in MATLAB [12]. To perform the optimization step in the proposed sensor selection strategy, the MPT toolbox was employed. The following simulation results demonstrate the performance of the proposed dynamic sensor selection strategy.

For the simulations, we define the normalized PDE parameters as:

- $N = M = 9$, (81 sensors)
- $\lambda(\frac{1}{2}, \frac{1}{2}, 0, t) = 1\frac{mol}{(10m)^3 5min}$, else $\lambda(x, y, z, t) = 0$
- $\alpha_x(p,t) = \alpha_y(p,t) = \alpha_z(p,t) = \frac{1}{100}\frac{(10m)^2}{5min}$
- $\phi_x(p,t) = \frac{1}{10} cos(\frac{\pi t}{10}) \frac{10m}{5min}$
- $\phi_y(p,t) = \frac{1}{10}\frac{10m}{5min}$
- $\Delta_x = \Delta_y = \frac{1}{8}(10m)$
- $\Delta_t = 1(5min)$

A simulation of 200 minutes (40 time steps) was performed using the parameters above. Figure 1 shows the CO_2 concentration levels vs. time step and normalized space.

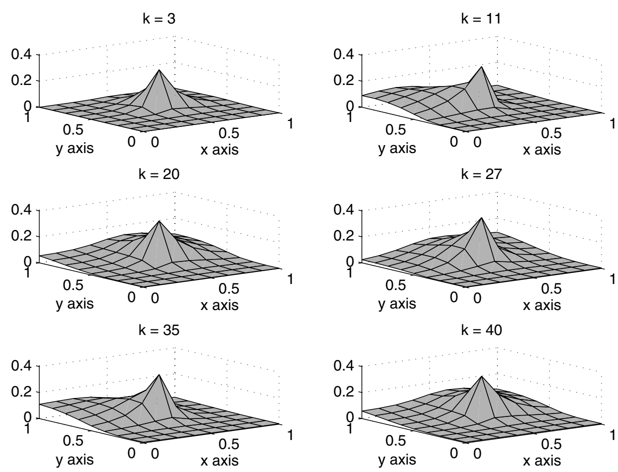

Figure 1. CO_2 concentration vs. space and time step.

The dynamic sensor selection strategy presented in Sect. 3 was simulated for various integer values of p where $1 \leq p \leq 81$. Due to the complexity of finding the optimal sensor selection matrix at each time step, we limit the number of possible sensor selection matrices to a set of $10,000$ distinct matrices and choose them *a priori*. Figure 2 shows the simulated results of the approximated optimal solution and the solution using our proposed strategy for the system in Fig. 1. At each time step, the $trace$ of the error covariance is normalized against the $trace$ of the error covariance using full order sensing ($p = n$). In Figs. 2 and 3, the solid line represents the approximated optimal value, while the circles and dashed line correspond to the dynamic sensor selection strategy.

The average computation time for the proposed sensor selection strategy was 0.6 seconds per time step for the field of 81 sensors. The computation time for testing $10,000$

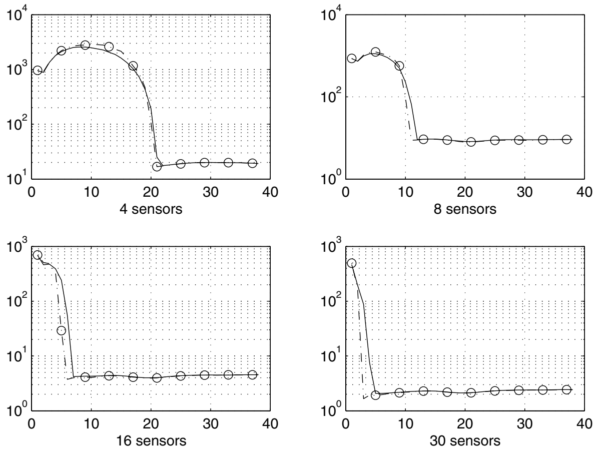

Figure 2. Normalized $trace$ of the error covariance for p = 4,8,16,30 vs. time step, k.

unique matrices at each time step was 7.2 seconds. The normalized steady state error for both the approximated optimal strategy and our strategy versus the number of sensors selected is shown in Fig. 3.

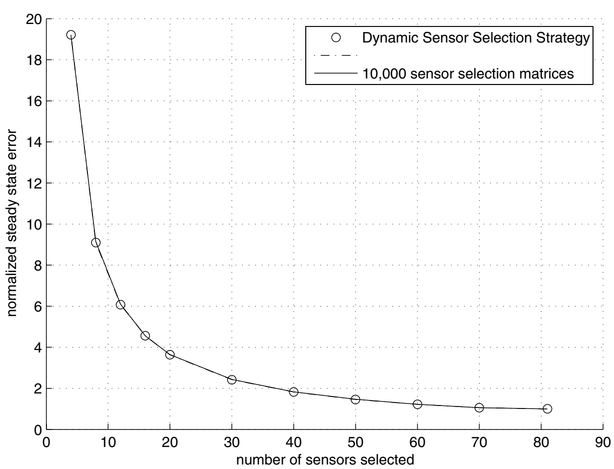

Figure 3. Normalized steady-state error vs. number of selected sensors.

The sensor selection strategy presented in [13] defines, $apriori$, a spanning set of sensor selection matrices which adequately represents the full sensor field and chooses (at each time step) the selection matrix minimizing the predicted next-state entropy. We introduce the criteria for defining a spanning sensor selection matrix set as each sensor must be represented in at least one sensor selection matrix and each sensor selection matrix is unique. Fig. 4 presents the normalized $trace$ of the error covariance for spanning sets containing 25, 100 and 500 unique sensor selection matrices. The solid lines correspond to the different spanning sets, while the circles and dashed line represent the proposed dynamic sensor selection strategy.

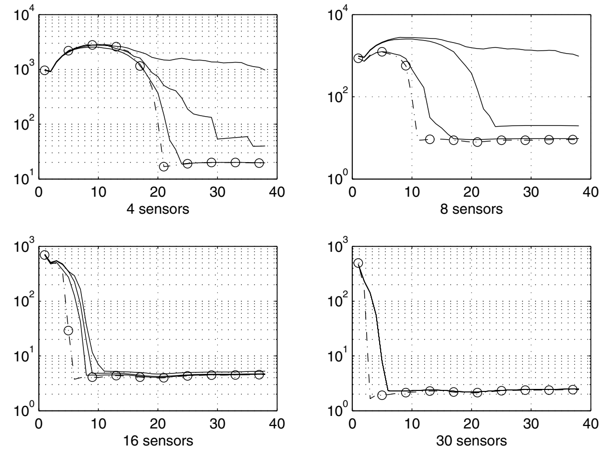

Figure 4. Normalized $trace$ of the error covariance vs. number of selected sensors and size of spanning sensor set.

In terms of computation time, the average values for 25, 100, and 500 unique sensor selection matrices were 0.07, 0.28, and 1.28 seconds respectively. Although the computation times are comparable to those for the proposed sensor selection strategy, the performance (particularly for small p) does not perform as well in the transient.

To compare the sensor selections generated by both the optimal strategy and the proposed strategy, the sensor field was scaled down to 25 sensors. Fig. 5 shows 40 sensor selection steps for $n = 25$ and $p = 5$. The o's represent a selection made by the proposed strategy, while the x's correspond to the optimal selection. In 200 sensor selections (5 selections at each time step), the proposed strategy chose the same sensor as the optimal strategy 185 times (92.5% accuracy).

6. Discussion

This paper introduces a dynamic sensor selection strategy for state estimation in large-scale wireless networks, based on a relaxation-optimization approach. The strategy is compared to other sensor selection strategies for a convection-dispersion example.

The main contribution consists in providing a suboptimal solution for the sensor selection. Despite the two approximations adopted, namely matrix inversions and convex relation, the proposed solution yields comparable performance

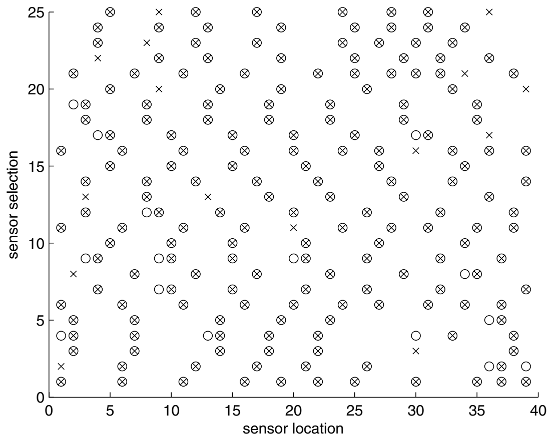

Figure 5. Sensor selection (5 sensors) vs. time step and strategy

and better computational efficiency with respect to both the optimal algorithm and state-of-the-art approximations. The performance of the proposed strategy very closely follows the performance of the approximated optimal strategy both in the transient (Fig. 2) and in the steady state (Fig. 3). In terms of sensor selection, the proposed strategy almost always selects the 1-step optimal solution as shown by Fig. 5. When compared to a strategy using only a spanning set of sensor selection matrices, the proposed strategy performs better in the transient for smaller number of selected sensors as shown by Fig. 4.

There are a number of interesting directions stemming from this work. Currently this strategy is only concerned with state estimation problem. In the future, we plan to implement a detection problem for determining source locations and source rates for the CO_2 monitoring problem. Other extensions comprise dynamic selection of the number of sensors to be used at each step. In fact the relaxation provides a natural tool to select the number of sensors in addition to the specific ones. We also plan on including network constraints such as power and delay, which will further constrain the selection process.

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